

Natural Convection Flow with Combined Buoyancy Effects Due to Thermal and Mass Diffusion in a Thermally Stratified Media

S.C. Saha¹, M.A. Hossain²

¹IBAIS University, Dhaka

²Dept. of Mathematics, University of Dhaka

Received: 09.12.2003

Accepted: 14.01.2004

Abstract. We present here a numerical study of laminar doubly diffusive free convection flows adjacent to a vertical surface in a stable thermally stratified medium. The governing equations of mass, momentum, energy and species are non-dimensionalized. These equations have been solved by using an implicit finite difference method and local non-similarity method. The results show many interesting aspects of complex interaction of the two buoyant mechanisms that have been shown in both the tabular as well as graphical form.

Keywords: modelling, diffusion, thermally stratified media, buoyant force.

1 Introduction

Many free convection processes occur in environments with temperature stratification. Good examples are closed containers and environmental chambers with heated walls. Also the free convection flow associated with heat-rejection systems for long duration deep ocean power modules where the ocean environment is stratified, (Yang et. al., [1]). Stratification of fluid arises due to temperature variations, concentration differences or the presence of different fluids. Cheesewrit's [2] work and also of Yang et. al. [3] showed that similar solutions were not possible. This fact is supported by Eichhorn [4] and by Fujii, et. al. [5] and therefore they developed series solutions to account for the nonzero leading edge temperature difference. Eichhorn [4] had calculated

only three terms in the series solution. On the other hand Fujii, et. al. [5] gave both analytical and experimental results for a temperature stratification in which the ambient temperature distribution varies with distance. In the above investigation they also showed that the fourth term in the series solutions are necessary for comparing the experimental results. The experimental and theoretical study in which both the wall temperature and the ambient temperature varied with a power of the distance along the plate was carried out by Piau [6]. His experimental temperature distributions compare well with his theoretical results; in order to make the comparison, the author had to use a nonzero starting length of the surface. Later Chen and Eichhorn [7] considered a finite isothermal vertical plate in a stable thermally stratified fluid. The experimental results of their paper have represented clear information on heat transfer to a vertical cylinder in water for both the unstratified and the stratified cases. Kulkarni, et. al. [8] investigated the problem of natural convection from an isothermal flat plate suspended in a linearly stratified fluid medium using the Von-Karman-Pohlhausen integral solution method.

The case of non-similar laminar natural convection from a vertical flat plate placed in a thermally stratified medium was studied by Venkatachala and Nath [9]. For getting the desired results they used implicit finite difference scheme developed by Keller and Cebeci [10]. They also used the perturbation series expansion and local non-similarity methods.

Gebhart and Pera [11] presented similarity solutions and investigated the laminar stability of natural convection flows driven by thermal and concentration buoyancy adjacent to flat vertical surfaces. They also presented an excellent summary of this class of doubly diffusive natural convection. Pera and Gebhart [12] extended their previous work flows from horizontal surfaces.

In the above studies the effect of stable ambient stratification on heat and mass transfer was not considered. In the cooling ponds, lakes, solar ponds and atmosphere a stable thermal stratification in the ambient is usually present. A numerical study of the double-diffusive natural convection flow adjacent to a vertical surface in a thermally stratified ambient was presented by Angirasa and Srinivasan [13]. They used the boundary layer approximation for the problem. For solving the conservative equations of mass, momentum, energy

and species they used an explicit finite-difference scheme.

In this paper the conservative equations of mass, momentum, energy and species have been solved by using implicit finite-difference scheme and local non-similarity method. The results show many interesting aspects of complex interaction of the two buoyant mechanisms that have been shown in both the tabular as well as graphical form.

2 Formulation of the problem

Let us consider the two dimensional steady boundary layer flow, heat transfer and mass transfer of a viscous incompressible fluid along an isothermal vertical finite plate immersed in a stable thermally stratified fluid. The coordinate system and the flow configuration are shown in Fig. 1.

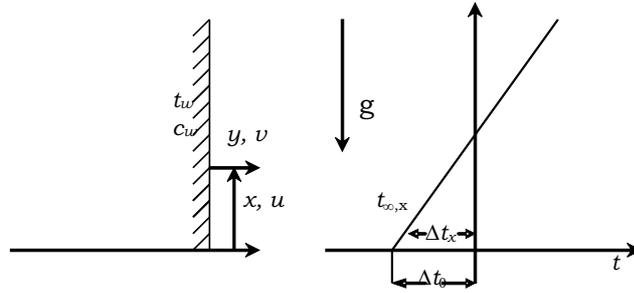


Fig. 1. The flow configuration and the coordinate system.

Using Boussinesq approximations, we obtain the following mass, momentum, energy and species conservation equations for laminar flow adjacent to a flat vertical surface.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(t - t_{\infty,x}) - g\beta^*(c - c_{\infty,0}), \quad (2)$$

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \alpha \frac{\partial^2 t}{\partial y^2}, \quad (3)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} \quad (4)$$

with the boundary conditions

$$\begin{aligned} u = v = 0, \quad t = t_w \quad \text{at} \quad y = 0, \\ u = 0, \quad t = t_{\infty,x}, \quad c = c_{\infty,0} \quad \text{as} \quad y \rightarrow \infty \end{aligned} \quad (5)$$

where u and v are the x - and y -components of the velocity field, respectively, g is the acceleration due to gravity, β^* is the volumetric coefficient of concentration. Here the volumetric coefficient due to temperature β must be positive but β^* may have either sign. If the molecular weight of the species is higher than the solution then β^* is positive and vice versa. Hence we see in equation (2), the two buoyant mechanisms aid each other when the quantities $\beta(t_w - t_\infty)$ and $\beta^*(c_w - c_\infty)$ have opposite signs and oppose each other when they have the same sign. t_w is the temperature of the wall and $t_{\infty,x}$ is the ambient temperature of the fluid. $c_w - c_\infty$ is the difference between species concentration of the boundary layer and the ambient concentration. α is the thermal diffusivity and D is the molecular diffusivity of the species concentration.

The non-dimensional variables can be written as follows:

$$\begin{aligned} X = x \left(\frac{g\beta\Delta t_0}{v^2} \right)^{1/3}, \quad Y = y \left(\frac{g\beta\Delta t_0}{v^2} \right)^{1/3}, \\ U = \frac{u}{(vg\beta\Delta t_0)^{1/3}}, \quad V = \frac{v}{(vg\beta\Delta t_0)^{1/3}}, \\ T = \frac{t - t_{\infty,x}}{t_w - t_{\infty,0}}, \quad C = \frac{c - c_{\infty,0}}{c_w - c_{\infty,0}} \end{aligned} \quad (6)$$

where $\Delta t_0 = t_w - t_{\infty,0}$ ($t_{\infty,0}$ is constant).

The non-dimensional conservative equations are then obtained as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (7)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + T - BC, \quad (8)$$

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} + SU = \frac{1}{\text{Pr}} \frac{\partial^2 T}{\partial Y^2}, \quad (9)$$

$$U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{\text{Sc}} \frac{\partial^2 C}{\partial Y^2} \quad (10)$$

where $B = \beta^*(c_w - c_\infty)/\beta(t_w - t_\infty)$ is defined as the buoyancy ratio and $S = (1/\Delta t_0) dt_{\infty,x}/dx$ as the thermal stratification parameter. Pr is the Prandtl number defined by ν/α and Sc is the Schmidt number defined by ν/D .

We obtain the boundary condition for temperature at the wall in non-dimensional form as follows:

$$T = \frac{t_w - t_{\infty,x}}{t_w - t_{\infty,0}} = 1 - \frac{t_{\infty,x} - t_{\infty,0}}{t_w - t_{\infty,0}}. \quad (11)$$

Since $t_{\infty,x}$ is a linear function

$$T = 1 - \frac{1}{\Delta t_0} \frac{dt_{\infty,x}}{dX} X = 1 - SX. \quad (12)$$

For linear thermal stratification S is constant and for other variation it can be represented as a function of X . The boundary conditions (5) then become

$$\begin{aligned} U = V = 0, \quad T = 1 - SX \quad \text{at} \quad Y = 0, \\ U = T = C \rightarrow 0 \quad \text{as} \quad Y \rightarrow \infty. \end{aligned} \quad (13)$$

Equation (7)–(8) subject to the boundary conditions (13) had been investigated by Angirasa and Srinivasan [13] employing the explicit finite difference method.

3 Transformation of the equations

Let us consider the following transformations

$$\begin{aligned} \psi = X^{3/4} f(X, \eta), \quad \eta = Y X^{-1/4}, \\ T(X, Y) = \theta(X, \eta), \quad C(X, Y) = \phi(X, \eta) \end{aligned} \quad (14)$$

where ψ is the stream function, defined by

$$U = \frac{\partial \psi}{\partial Y} \quad \text{and} \quad V = -\frac{\partial \psi}{\partial X} \quad (15)$$

which satisfies the equation of continuity (5). In (14), f , θ and ϕ are the non-dimensional stream function, temperature and concentration functions, respectively and η is the pseudo-similarity variable.

Applying the above transformations we get the following non-similarity equations:

$$f''' + \frac{3}{4} f f'' - \frac{1}{2} f'^2 + \theta - B\phi = X \left(f' \frac{\partial f'}{\partial X} - f'' \frac{\partial f}{\partial X} \right), \quad (16)$$

$$\frac{1}{\text{Pr}} \theta'' + \frac{3}{4} f \theta' - SX f' = X \left(f' \frac{\partial \theta}{\partial X} - \theta' \frac{\partial f}{\partial X} \right), \quad (17)$$

$$\frac{1}{\text{Sc}} \phi'' + \frac{3}{4} f \phi' = X \left(f' \frac{\partial \phi}{\partial X} - \phi' \frac{\partial f}{\partial X} \right) \quad (18)$$

with boundary conditions

$$\begin{aligned} f(X, 0) = f'(X, 0) = 0, \quad \theta(X, 0) = 1 - SX, \quad \phi(X, 0) = 1, \\ f'(X, \infty) = \theta(X, \infty) = \phi(X, \infty) = 0. \end{aligned} \quad (19)$$

In the present investigation we integrate the set of equations (16)–(19) employing two methods; namely the implicit finite difference method together with the Keller-box elimination techniques and the local non-similarity method. The methods of solution are discussed in the following sections.

Once we know the values of f , θ and ϕ and their derivatives, we may calculate the values of the quantities of physical interest such as the local Nusselt number, Nu_x and the local Sherwood number Sh_x from the following relations against X , the axial distance along the surface of the plate measured from the leading edge.

The local Nusselt number is

$$Nu_x = -X^{3/4}\theta'(Y, X). \quad (20a)$$

The local Sherwood number is

$$Sh_x = -X^{3/4}\phi'(Y, X). \quad (20b)$$

4 Methods of solution

In the present investigation we shall integrate the equations (16) to (19) for all values of X by implicit finite difference method as well as the local non-similarity method.

Implicit finite difference method (FD). For all X , here we propose to integrate the local non-similarity partial differential equations (16)–(18) subjected to the boundary conditions (19) by implicit finite difference method together with Keller-box elimination technique, which was first introduced by Keller [14]. To begin with the partial differential equations (16)–(18) are first converted in to a system of first order equations. Then these equations are expressed in finite difference forms by approximating the functions and their derivatives in terms of the center difference. Denoting the mesh points in the

X, η -plane by X_i and η_j where $i = 0, 1, \dots, M$ and $j = 0, 1, 2, 3, \dots, N$ central difference approximations are made, such that those equations involving X explicitly are centered at $(X_{i-1/2}, \eta_{j-1/2})$ and the remainder at $(X_i, \eta_{j-1/2})$ where $\eta_{j-1/2} = (\eta_j - \eta_{j-1})/h_j$ etc. This results in a non-linear difference equation for the unknowns at X_i in terms of their values at X_{i-1} . To solve resulting equations, Newton's iterations technique together with Keller-box method is then introduced. Recently this method has been discussed in more detailed and was used efficiently by Hossain et. al. [15] in studying the effect of oscillating surface temperature on the natural convection flow from a vertical flat plate. To initiate the process with $X = 0$, we first prescribe the profiles for the functions f, f', f'' and θ, θ' and ϕ, ϕ' obtained from the solution of the equations (16)–(19) by putting $X = 0$. These profiles are then employed in the Keller-box scheme with secondary accuracy to march step by step along the boundary layer. For a given X , the interactive procedure is stopped to get the final velocity, temperature and concentration distributions when the difference in computing the velocity, the temperature and the species concentration in the next procedure becomes less than 10^{-5} , i.e. $|\delta f^i| \leq 10^{-5}$ where the subscript i denotes the number of iterations. Throughout the computations, instead of using equal grid in the η -direction, non-uniform grids have been incorporated considering $\eta = \sinh(j/i)$. This consideration has saved a lot of computational times and on-board memory space. In the computations, the maximum η_e ranged up to 25.0, X ranging from 0.0 to 100.0.

Local non-similarity method (LNS). The local non-similarity method was developed by Sparrow and Yu [16] and has been applied by many investigators, for example Minkowycz and Sparrow [17], Hossain [18], to solve various non-similar boundary layer problems. This method embodies two essential features. First the non-similar solution at any specific stream wise location is found (i.e. each solution is locally autonomous). Second, the local solutions are found from differential equations. These equations can be solved numerically by well-established techniques, such as forward integration (e.g. a Rungr-Kutta scheme) in conjunction with a shooting procedure to determine the unknown boundary conditions at the wall. The method also allows some

degree of self-checking for accuracy of the numerical results.

In the local non-similarity method, all the terms in the transformed conservation equations are retained, with the X derivatives discussed by the new functions $f_1 = \partial f / \partial X$, $\theta_1 = \partial \theta / \partial X$, $\phi_1 = \partial \phi / \partial X$. These represent three additional unknown functions, therefore it is necessary to deduce three further equations to determine the f_1, θ_1 and ϕ_1 . This is accomplished by creating subsidiary equations by differentiation of the transformed conservation equations and boundary conditions (i.e. f_1, θ_1 and ϕ_1 system of equations) with respect to X . The subsidiary equations for f_1, θ_1 and ϕ_1 contain terms $\partial f_1 / \partial X$, $\partial \theta_1 / \partial X$, $\partial \phi_1 / \partial X$ and their η derivatives. When these terms are ignored the system of equations for $f, \theta, \phi, f_1, \theta_1$ and ϕ_1 reduces to a system of ordinary differential equations that provides locally autonomous solutions in the stream wise direction. This form of the local non-similarity method is referred to as the second level of truncation, because approximations are made by dropping terms in the second level equation.

To carry the local non-similarity method to the third level of truncation, all terms are retained in both the f, θ, ϕ and f_1, θ_1, ϕ_1 equations. The X derivatives appearing in the f_1, θ_1 and ϕ_1 are disguised by introducing $f_2 = \partial f_1 / \partial X = \partial^2 f / \partial X^2$, $\theta_2 = \partial \theta_1 / \partial X = \partial^2 \theta / \partial X^2$, $\phi_2 = \partial \phi_1 / \partial X = \partial^2 \phi / \partial X^2$. The f_1, θ_1 and ϕ_1 and their boundary conditions are then differentiated with respect to X to obtain three additional equations for the functions f_2, θ_2 and ϕ_2 . In these new equations, terms involving $\partial f_2 / \partial X$, $\partial \theta_2 / \partial X$, $\partial \phi_2 / \partial X$ and their η derivatives are deleted, so that once again a locally autonomous system of ordinary differential equations for $f, \theta, \phi, f_1, \theta_1, \phi_1, f_2, \theta_2$, and ϕ_2 can be derived.

The procedure as described above in the formulation of the local non-similarity method can result in a large number of ordinary differential equations that may require simultaneous solution. For example, at the third level of truncation there will be nine equations involving $f, \theta, \phi, f_1, \theta_1, \phi_1, f_2, \theta_2$, and ϕ_2 . It is expected that the accuracy of the local non-similarity method results will depend upon the truncation level. Below we give only the equations

valid up to the third level of truncation:

$$f''' + \frac{3}{4}ff'' - \frac{1}{2}f'^2 + \theta - B\phi = X\left(f'\frac{\partial f'}{\partial X} - f''\frac{\partial f}{\partial X}\right), \quad (21)$$

$$\frac{1}{\text{Pr}}\theta'' + \frac{3}{4}f\theta' - SXf' = X\left(f'\frac{\partial \theta}{\partial X} - \theta'\frac{\partial f}{\partial X}\right), \quad (22)$$

$$\frac{1}{\text{Sc}}\phi'' + \frac{3}{4}f\phi' = X\left(f'\frac{\partial \phi}{\partial X} - \phi'\frac{\partial f}{\partial X}\right), \quad (23)$$

$$\begin{aligned} f_1''' + \frac{3}{4}ff_1'' + \frac{7}{4}f''f_1 - 2f'f_1' + \theta_1 - B\phi_1 \\ = X(f'f_2' + f_1'^2 - f''f_2 - f_1''f_1), \end{aligned} \quad (24)$$

$$\begin{aligned} \frac{1}{\text{Pr}}\theta_1'' + \frac{3}{4}f\theta_1' + \frac{7}{4}f_1\theta_1' - f'\theta_1 - Sf' - SXf_1' \\ = X(f_1'\theta_1 + f'\theta_2 - \theta'f_2 - \theta_1'f_1), \end{aligned} \quad (25)$$

$$\begin{aligned} \frac{1}{\text{Sc}}\phi_1'' + \frac{3}{4}f\phi_1' + \frac{7}{4}f_1\phi_1' - f'\phi_1 \\ = X(f'\phi_2 + f_1'\phi_1 - \phi'f_2 - \phi_1'f_1), \end{aligned} \quad (26)$$

$$\begin{aligned} f_2''' + \frac{3}{4}ff_2'' + \frac{7}{2}f_1''f_1 + \frac{11}{4}f''f_2 - 3f'f_2' + \theta_2 - B\phi_2 \\ = X(f_1'f_2' + 2f_1''f_2 - 2f_1''f_2 - f_2''f_1), \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{1}{\text{Pr}}\theta_2'' + \frac{3}{4}f\theta_2' + \frac{7}{4}f_1\theta_2' + \frac{11}{4}f_2\theta_2' - 2f'\theta_2 - 2f_1'\theta_1 - 2Sf_1' - SXf_2' \\ = X(2f_1'\theta_2 + f_2'\theta_1 - 2\theta_1'f_2 - \theta_2'f_1), \end{aligned} \quad (28)$$

$$\begin{aligned} \frac{1}{\text{Sc}}\phi_2'' + \frac{3}{4}f\phi_2' + \frac{7}{2}f_1\phi_2' + \frac{11}{4}f_2\phi_2' - 2f'\phi_2 - 2f_1'\phi_1 \\ = X(2f_1'\phi_2 + f_2'\phi_1 - 2\phi_1'f_2 - \phi_2'f_1). \end{aligned} \quad (29)$$

The boundary conditions are

$$\begin{aligned} f(X, 0) = f'(X, 0) = 0, \quad \theta(X, 0) = 1 - SX, \quad \phi(X, 0) = 1, \\ f_1(X, 0) = f_1'(X, 0) = f_2(X, 0) = f_2'(X, 0) = 0, \\ \theta_1(X, 0) = -S, \quad \theta_2(X, 0) = \phi_1(X, 0) = \phi_2(X, 0) = 0, \\ f'(X, \infty) = f_1'(X, \infty) = f_2'(X, \infty) = 0, \\ \theta_1(X, \infty) = \theta_2(X, \infty) = \phi_1(X, \infty) = \phi_2(X, \infty) = 0. \end{aligned} \quad (30)$$

At the third level of truncation, equations (28)–(29), the terms with $\partial f_2/\partial X$, $\partial \theta_2/\partial X$, $\partial \phi_2/\partial X$ have been neglected. It can be seen that equations (21)–(30) form a coupled linear system of ordinary differential equations

taking as a parameter. Equations (21)–(30) are solved numerically, employing the sixth order implicit Runge-Kutta-Butcher initial value problem solver along with Nachtsheim-Swigert iteration technique. Here, solutions are obtained, up to the third level of truncation, for different values of Sc and Pr and with X from zero to 10. Results for surface heat transfer and mass transfer are given in the following Table. Comparison between the non-similarity solutions and the finite difference solutions shows that consideration of the above equations up to the third level of truncation is sufficient for the present case.

5 Results and discussions

In this present problem two distinct solution methodologies, namely, (i) the finite difference method together with the Keller-box method for all X , (ii) the local non-similarity method for all X , have been applied to integrate the momentum, energy and concentration equation (16)–(19). Computed results thus obtained in terms of the local Nusselt number and local Sherwood number are shown in tabular form. In Table 1 the numerical values of local Nusselt number and local Sherwood number for $Pr = 0.7$ and 7.0 and $Sc = 0.7$ and 100 against X which are found by finite difference method and local non-similarity method has been shown. We observe that with the increase of X , both local Nusselt number and local Sherwood number are increasing. For increasing values of Schmidt number Sc , both Nusselt number and Sherwood number increase. In the present investigation, we have considered the maximum value of X to be 100 because for higher values of X laminar flow may not be valid. We see that for $S = 0.01$, the ambient temperature is equal to the wall temperature at $X = 100$. If $S > 0.01$, the temperature of a portion at the top of the surface will be less than the ambient.

In Fig. 2a the velocity profiles are shown for $X = 100$ and $Pr = Sc = 0.7$ for various values of the stratification parameter S . $S = 0$ indicates that the environment is unstratified. We have chosen $B = -2$ for the two buoyancies aid each other. An increase in ambient thermal stratification invariably decreases the velocity profile. The temperature profile at $X = 100$ are shown in Fig. 2b for $Pr = Sc = 0.7$ and for the same values of S as above. If thermal stratifica-

Table 1. Numerical values of the Local Nusselt number and Local Sherwood number for different values of the Prandtl number Pr and Schmidt Sc

X	Pr = 7.0 and Sc = 100.0				Pr = 0.7 and Sc = 0.7			
	Local Nusselt Number		Local Sherwood Number		Local Nusselt Number		Local Sherwood Number	
	FD	LNS	FD	LNS	FD	LNS	FD	LNS
0.0	∞	∞	∞	∞	∞	∞	∞	∞
0.2	0.23983	0.2275	0.70224	0.7061	0.14131	0.1424	0.14131	0.1461
0.4	0.40334	0.3827	1.18103	1.1875	0.23766	0.2395	0.23766	0.2457
0.6	0.54669	0.5187	1.60077	1.6096	0.32212	0.3246	0.32212	0.3330
0.8	0.67834	0.6436	1.98625	1.9972	0.39969	0.4028	0.39969	0.4132
1.0	0.80192	0.7608	2.34809	2.3611	0.47251	0.4762	0.47251	0.4885
2.0	1.34866	1.2796	3.94901	3.9708	0.79466	0.8008	0.79466	0.8215
3.0	1.82797	1.7343	5.35250	5.3820	1.07708	1.0854	1.07708	1.1134
4.0	2.26816	2.1520	6.64141	6.6781	1.33645	1.3468	1.33645	1.3816
5.0	3.07427	2.5440	7.85132	7.8947	1.57992	1.5922	1.57992	1.6332
6.0	3.45106	2.9168	9.00179	9.0515	1.81143	1.8255	1.81143	1.8726
7.0	3.81457	3.2743	10.10506	10.1608	2.03344	2.0492	2.03344	2.1021
8.0	4.16687	3.9192	11.16948	11.2311	2.24764	2.2651	2.24764	2.3235
9.0	4.50950	3.9534	12.20105	12.2684	2.45522	2.4743	2.45522	2.5381
10.0	7.58404	4.2785	13.20430	13.2772	2.65711	2.6777	2.65711	2.7468
20.0	10.27944		22.20689		4.46870		4.46870	
30.0	12.75479		30.09929		6.05689		6.05689	
40.0	15.07842		37.34740		7.51543		7.51543	
50.0	15.07842		44.15123		8.88457		8.88457	
60.0	17.28788		50.62077		10.18644		10.18644	
70.0	19.40671		56.82492		11.43490		11.43490	
80.0	21.45092		62.81058		12.63940		12.63940	
90.0	23.43205		68.61155		13.80673		13.80673	
100.0	25.35878		74.25323		14.94200		14.94200	

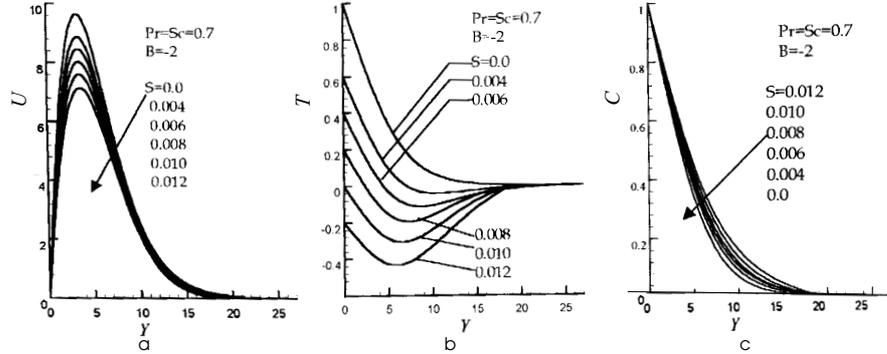


Fig. 2. Velocity (a), temperature (b) and concentration (c) profiles for different stratification parameter S taking $Pr = Sc = 0.7$ and $B = -2$, $X = 100$.

tion is not present the temperature and concentration profiles will be identical when $Pr = Sc$. We observe that for $S = 0.01$ and 0.012 , values of the non-dimensional temperature are negative within the boundary layer. Because, for $S = 0.01$ the temperature difference between the surface and the ambient at $X = 100$ is zero. But fluid coming up from below with the flow sustained by the other buoyant force will have a temperature that is considerably less than that of the surface or the ambient. This is true for $S = 0.012$ also accept that the non-dimensional temperature at the surface is -0.2 and it drops further before asymptotically reaching zero. When $S < 0.01$ there is positive thermal buoyancy at $X = 100$ but in the outer regions the temperatures are still negative. For higher values of S the temperature in the ambient increases rapidly with height.

In Fig. 2c the effect of thermal stratification on concentration boundary layers is presented. For $S = 0.012$ the increase in concentration boundary layer thickness is almost double that of $S = 0.0$ for $Pr = Sc = 0.7$, thus indicating strong influence of thermal stratification on species diffusion. The velocity profiles are qualitatively agreed with those of Angirasa and Srinivasan [13].

6 Conclusions

In this paper we have investigated problems on natural convection flow from a vertical plate placed in a stratified media.

Investigation has been made to the natural convection flow with combined buoyancy effects due to thermal and mass diffusion in a thermally stratified medium. Implicit finite difference method and the local non-similarity method, are employed and investigate the present problem for values of the distance variable X in the interval $[0, 100]$ for fluid having values of $Pr = 0.7$ and for different values of the stratification parameter S . We may draw the following conclusions for the present study:

- For increasing the values of X both local Nusselt number and local Sherwood number are increasing.
- Ambient thermal stratification is found to decrease the local buoyancy levels significantly, that reduces the velocities and increases the concentrations.
- The temperature defect is more pronounced in doubly diffusive free convection flow.

References

1. Yang, K.T., Szewczyk A.A., Novotny J.L. "Problems in free convection of heat-rejection systems", *Proceedings of the Symposium on deep Submergence Propulsion and Marine systems*, AIAA Chicago Section, 1966
2. Cheesewright R. "Natural convection from a plane vertical surface in non-isothermal surroundings", *International journal of Heat and Mass transfer* **10**, p. 1847–1859, 1967
3. Yang K.T., Novotny J.L., Cheng Y.S. "Laminar free convection from a non-isothermal plate immersed in a temperature stratified medium", *International journal of Heat and Mass transfer*, **15**, p. 1097–1109, 1972
4. Eichhorn R. "Natural convection in a thermally stratified fluid", *Progress in Heat and Mass transfer*, **2**, Pergamon Press, p. 41-53, 1969
5. Fujii T., Takeuchi M., Morioka I. "Laminar boundary layer of free convection in a temperature stratified environment", *Proceedings, Fifth International heat transfer Conference, Tokyo, NC2.2*, p. 44–48, 1974
6. Piau J.M. "Influence des variations des propriétés physiques et de la stratification en convection naturelle", *International journal of Heat and Mass transfer*, **17**, p. 465–476, 1974

7. Churf C.C., Eichhorn R. "Natural convection from a vertical surface to a thermally stratified fluid", *ASME Journal of heat transfer*, **98**, p. 446–451, 1976
8. Kulkarni A.K., Jacobs H.R., Hwang J.J. "Similarity solutions for natural convection flow over an isothermal vertical wall immersed in thermally stratified medium", *Int. J. Heat Mass transfer*, **130**(4), p. 691–698, 1986
9. Venkatachala B.J., Nath G. "Non-similar laminar natural convection in a thermal stratified fluid", *Int. J. Heat Mass Transfer*, **24**(11), p. 1848–1850, 1981
10. Keller H.B., Cebeci T. "Accurate numerical methods for boundary layer flows, part II, Two dimensional turbulent flow", *AIAAJ.*, **10**, p. 1193–1199, 1972
11. Gebharf B., Pera L. "The Nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion", *Int. J. Heat Mass Transfer*, **14**, p. 2025–2050, 1971
12. Pera L., Gebhart B. "Natural convection flows adjacent to horizontal surfaces resulting from the combined buoyancy effects of thermal and mass diffusion", *Int. J. Heat Mass Transfer*, **15**, p. 269–278, 1972
13. Angirasa D., Srinivasan J. "Natural convection flows due to the combined buoyancy of heat and mass diffusion in a thermally stratified medium", *Int. J. of Heat Transfer*, **III**, p. 657–663, 1989
14. Keller H.B. "Numerical method in boundary layer theory", *Annual Rev. Fluid Mech*, 1978
15. Hossain M.A., Banu N., Nakayama A. "Non-Darcy forced convection boundary layer flow over a wedge embedded in a saturated porous medium", *Numerical Heat Transfer*, **26**, Part-A, p. 399–414, 1994
16. Sparrow E.M., Yu H.S. "Local non-similarity thermal boundary layer solutions", *Journal of Heat Trans., Trans. ASME, Series C*, **93**, p. 328–334, 1971
17. Milikowycz W.J., Sparrow E.M. "Numerical solution scheme for local non-similarity boundary layer analysis", *Numerical Heat transfer*, **I**, p. 69–85, 1978
18. Hossain M.A. "Viscous and Joule heating effects on MHD-free convection flow with variable plate temperature", *Int. J. Heat Mass Transfer*, **35**(12), p. 3485–3487, 1992