

Chaos and Bifurcation Control Using Nonlinear Recursive Controller

A.M. Harb, M.A. Zohdy

Electrical and Computer Engineering Department
Oakland University, Rochester, MI 48307, USA
aharb@just.edu.jo

Received: date 26.06.2002

Accepted: date 13.11.2002

Abstract: Chaos and bifurcation control is achieved by nonlinear controller that is able to mitigate the characteristics of a class of nonlinear systems that are experiencing such phenomenon. In this paper, a backstepping nonlinear recursive controller is presented. Comparison has been made between it and a Pole Placement controller. The study shows the effectiveness of the proposed control under various operating conditions.

Keywords: nonlinear theory, bifurcation, chaos, backstepping, pole placement, Lyapunov, control systems.

1 Introduction

Controlling chaos as well as bifurcation has been proposed in the past decade. Emphasis has been usually placed on design techniques which result in prescribed nonlinear dynamics for classes controlled processes. In some applications, chaos can be useful while in others it might be detrimental. For example, chaos in power systems [1–7] and in mechanical systems is objectionable. On the other hand, chaos is being proposed to be utilized in communication systems; in [8] and others investigation the use of chaos in synchronization, and information transmission is considered communication systems.

Bifurcation control deals with using a control input to modify the characteristics of a parameterized nonlinear system. The control can thus be static or dynamic feedback control, or open loop control. The objective can be

stabilization and reduction of the amplitude of bifurcation orbital solutions, optimization of a performance index near bifurcation, reshaping of the bifurcation diagram or a combination of these [2]. The control $u(x)$ laws derived and used in [2] e.g. transform an unstable subcritical bifurcation point into a stable supercritical one. Several applications of these results have been further conducted. Among these are prevention of voltage collapse in electrical power system [3], and subsynchronous resonance in power system [1].

Bifurcation control are used in [9], these controllers are designed to control the bifurcation route that leads to chaos. Many researchers are proposed other methods to control chaos, Ott [10] and Ott et. al. [11]. In Hubler [12], Hubler and Luscher [13], and Jackson [14] the methods are based on classical control. In this paper, a new recursive backstepping nonlinear controller is proposed. A comparison is made between it and the pole placement control design.

The paper is organized as follows; in section 2, a mathematical nonlinear dynamical system and the methodology of design the backstepping nonlinear as well as the pole placement controllers are discussed. A numerical simulation is placed in section 3. Finally, section 4, concluded the paper.

2 Mathematical Model and Control Design

As a simple yet practical nonlinear system example, we have chosen the following third order state representation.

$$\dot{x}_1 = -x_3, \tag{1}$$

$$\dot{x}_2 = x_1 - x_2 + u, \tag{2}$$

$$\dot{x}_3 = 3.1x_1 + x_2^2 + F_L x_3 + u_2, \tag{3}$$

where $F_L = 0.5$, is a parameter and u_1 and u_2 are control signals.

2.1 A recursive backstepping controller. Let $x_{1d} = 0$, $x_{2d} = c_1 e_1$, and $x_{3d} = c_2 e_1 + c_3 e_2$, where subscript d refer to desired values. Define the error signals as follows:

$$e_1 = x_1 - x_{1d}, \tag{4}$$

$$e_2 = x_2 - x_{2d}, \tag{5}$$

$$e_3 = x_3 - x_{3d}. \quad (6)$$

Substituting equations (4)–(6) into equations (1)–(3), one obtain:

$$\dot{e}_1 = -e_3 - c_2 e_1 - c_3 e_2, \quad (7)$$

$$\dot{e}_2 = (1 - c_2 + c_2^2) e_1 + (c_2 c_3 - 1) e_2 + c_2 e_3 + u_1, \quad (8)$$

$$\dot{e}_3 = 3.1 e_1 + (e_2 + c_2 e_1)^2 + F_L (e_3 + c_2 e_1) + c_2 (e_3 + c_2 e_1) + u_2, \quad (9)$$

Let:

$$V = \frac{1}{2} \sum_1^3 k_i e_i^2. \quad (10)$$

The time derivative for equation (10) is:

$$\dot{V} = k_1 e_1 \dot{e}_1 + k_2 e_2 \dot{e}_2 + k_3 e_3 \dot{e}_3. \quad (11)$$

Substituting equations (7)–(9) into equation (11), and choosing the following parameters to be: $c_1 = 1$, $c_2 = -1 - F_L$, $c_3 = 0$, yields;

$$u_1 = -k_4 e_2 - c_2 e_3, \quad (12)$$

$$u_2 = -k_5 e_3 + (e_2 + c_2 e_1)^2 \quad (13)$$

This control law guarantee the negative definitiveness of V in equation (1)

2.2 Pole placement controller. Consider the system of equations (1)–(3) as:

$$\dot{X} = F(X, F_L) \quad (14)$$

Let;

$$X = \Delta X + X^*(F_L^*), \quad (15)$$

where X is function of the control parameter F_L .

Substituting equation (15) into equation (14) and using Taylor series expansion and linearized the resulting equation around unstable fixed point, $X = X^*(F_L^*)$, and keeping the linear term only. One obtain:

$$\Delta \dot{X} = A \Delta X + b \Delta F_L, \quad (16)$$

where $\Delta X = X - X^*(F_L^*)$, $\Delta F_L = F_L - F_L^*$, $A = \frac{\partial F}{\partial X}(X^*, F_L^*)$, and

A.M. Harb, M.A. Zohdy

$$b = \frac{\partial F}{\partial F_L}(X^*, F_L^*).$$

Now, let us consider the case of linear state feedback;

$$\Delta F_L = K^T \Delta X, \quad (17)$$

substituting equation (17) into equation (16), yields;

$$\Delta \dot{X} = [A + bK^T] \Delta X, \quad (18)$$

or,

$$\Delta \dot{X} = A_c \Delta X, \quad (19)$$

where $A_c = A + bK^T$ is the closed loop matrix.

So, the objective is to design a linear feedback controller, such that the system is stable. To do that, a closed loop matrix, which has negative eigenvalues, is selected. The signal control that to be added to equation (3) is given by:

$$u = K^T [X] = k_1 x_1 + k_2 x_2 + k_3 x_3. \quad (20)$$

A designed controller is $K = [2.5273 \ 1.0976 \ -6.5854]^T$

3 Numerical Simulations

For equations (1)–(3), the signals u_1 and u_2 are both zero, figs. 1(a) and 1(b) shows the time history and state-plane, respectively. Figs. 1(a) and 1(b) show the chaotic behaviour of the system for the critical parameter $F_L = 0.5$. In the next two subsections, the above mentioned controllers are presented.

3.1 A recursive backstepping controller. In this case, the signals u_1 and u_2 given in equations (11) and (12) are added to equations (2) and (3), respectively. Figs. 2(a, b, c, and d) show the time history as well as the state-plane for both, system state and control signals u_1 and u_2 . By comparing the simulation results of figs. 1(a, b) to that of figs. 2 (a-d) it is clear that the system has recovered from its chaotic behaviour and now exhibits a stable performance. This result was achieved by designed signals shown effective in controlling the bifurcation as well as chaos behaviours.

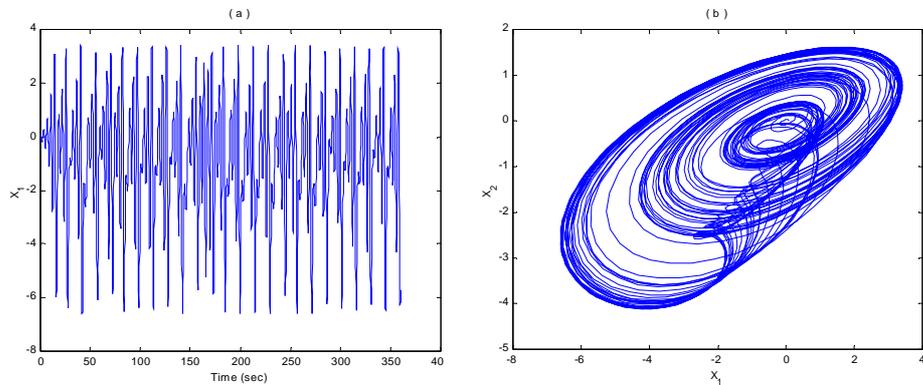


Fig. 1. Time and state trajectory (without control)

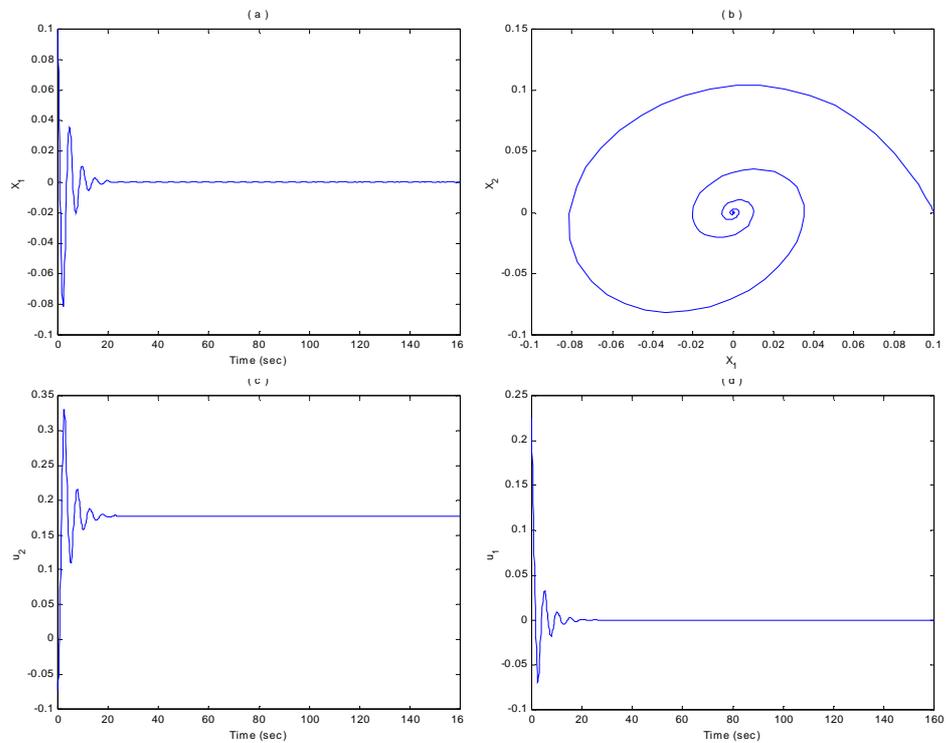


Fig. 2. Time and state trajectory (backstepping controller)

3.2 Pole placement controller. For this case, only a single control signal u is designed in section 2.2. This signal u from equation (20) is added to equation (3). Figs. 3(a-c) shows the time history and state-plane simulations. A comparison between figs. 1(a,b) and figs. 3(a-c) shows that the system has also

recovered from its chaotic behaviour. However, this linearized controller is effective only about its chosen operation point.

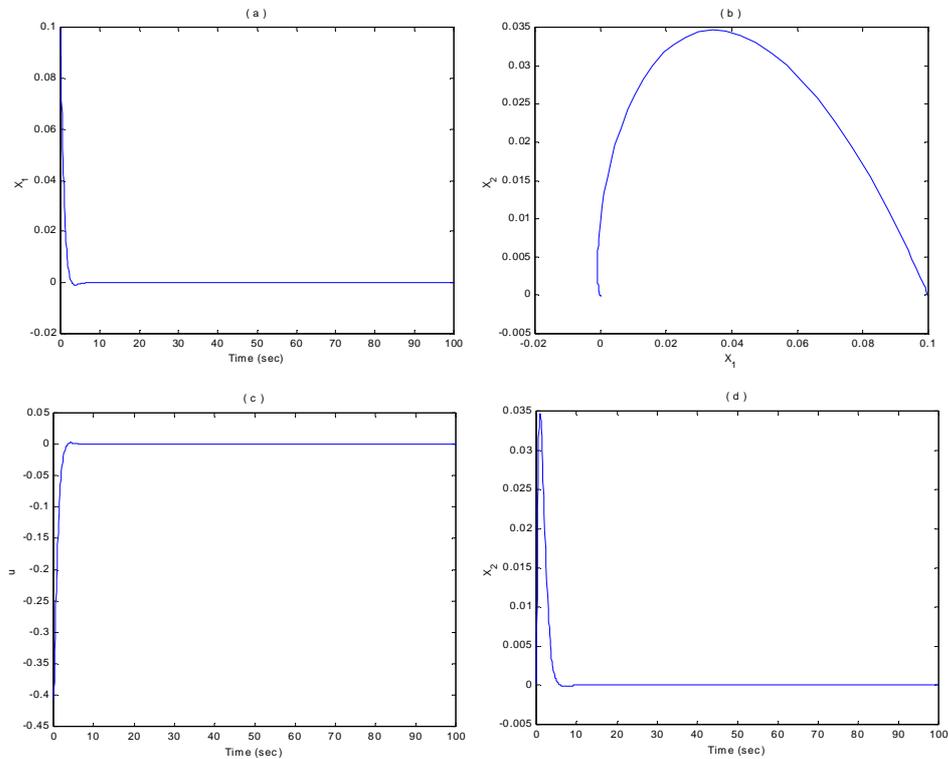


Fig. 3. Time and state trajectory (pole placement controller)

4 Conclusion

In this paper, we have discussed the application of new nonlinear recursive controllers on typical chaotic behaviour. As mentioned, controlling bifurcation as well as chaos has been rapidly advancing in the last decade. Thus, emphasis has been placed on control design techniques which result in prescribed nonlinear performance dynamics for practical controlled processes. This study have shown that a nonlinear recursive controller is effective in controlling an undesirable chaotic behaviour as well as original bifurcations.

References

1. Abed E.H., Fu J.-H. Local feedback stabilization and bifurcation control, I. Hopf bifurcation, *Systems and control letters*, **7**, 11–17, 1986
2. Wang H.O., Abed E.H. Control of nonlinear phenomena at the inception of voltage collapse. In: *Proc. 1993 American control conference, San Fransisco June, 2071–2075*, 1993
3. Harb A.M., Nayfeh A.H., Chin C., Mili L. On the Effects of Machine Saturation on Subsynchronous Oscillations in Power Systems, *Electric Machines and Power Systems Journal*, **28**(11), 1999
4. Abed E.H., Varaiya P.P. Nonlinear oscillations in power systems, *Int. J. of Electric Power and Energy System*, **6**, 37–43, 1989
5. Abed E.H., Alexander J.C., Wang H., Hamdan A.H., Lee H.C. Dynamic bifurcation in a power system model exhibiting voltage collapse, In: *Proceedings of the 1992 IEEE International Symposium on Circuit and Systems, San Diego, C.A.*, 1992
6. Abed E.H., Fu J.-H. Local feedback stabilization and bifurcation control, I. Hopf bifurcation, *System and Control Letters*, **7**, 11–17, 1986
7. Abed E.H., Fu J.-H. Local feedback stabilization and bifurcation control, II Stationary bifurcation, *System and Control Letters*, **8**, 467–473, 1987
8. Harb A.M., Zohdy M.A. Synchronization of two chaotic systems as applied in communication systems, submitted to *Int. Journal of Nonlinear Dynamics*, Aug., 2001
9. Nayfeh A.H., Balachandran B. *Applied Nonlinear Dynamics*, John Wiley, New York, 1994
10. Ott E. *Chaos in Dynamical Systems*, Cambridge University Press, Cambridge, England, 1993
11. Ott E., Grebogi C., Yorke J.A. Controlling chaotic dynamical systems, In: *CHAOS: Soviet-American Perspectives on Nonlinear Science*, Campbell D.K. (Ed.), 153–172, 1990a
12. Hubler A. Adaptive control of chaotic systems, *Helv. Phys. Acta*, **62**, p. 343–346, 1989
13. Hubler A., Luscher E. Resonant stimulation and control of nonlinear oscillators, *Naturwissenschaften*, **76**, p. 67–69, 1989
14. Jackson E.A. Control of dynamic flows with attractors, *Phys. Rev.* **A44**, p. 4839–4853, 1991