Nonlinear Analysis: Modelling and Control, 2002, Vol. 7, No. 2, 123–137

Energetics as Self-Organized System: Methodological Aspects

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Received: date 30.11.2002
Accepted: date 10.12.2002

Abstract. In the article we propose new decision support tools for region energetics development. The method is based on analysing complex system's state and comparing it with the self-organized criticality (SOC) class that characterize the long-term system's stability. We start with urban system modelling using cellular automata (CA) with the best fit to real data and then we raise hypothesis about adequacy of urban and energetics system elements distribution. The urban data analysis shows that it is in SOC state for wide rank of countries. We calculate required parameters for energetics system, which are used in decision making for the system long-term development.

Keywords: urban system, electricity generators system, cellular automata (CA) modeling, self-organized criticality (SOC), strategy formation, decision making.

1 Introduction

Complex systems are spatially and/or temporally extended non-linear systems characterized by collective properties associated with the system as a whole – and that are different from the characteristic behaviours of the constituent parts. While chaos theory is the study of how simple systems can generate complicated behaviour, complexity theory (or the study of complex systems) is really about how a system which is complicated (usually by having many interactions) can lead to surprising patterns when the system is looked at as a whole.

Cellular automata (CA) have been used to provide algorithmically simple models for complex systems [1,2,3], which historically has differential equations modelling approach [1]. They also provide a useful discrete model for a branch of dynamical systems theory studying the emergence of well-characterized collective phenomena such as ordering, turbulence, chaos, symmetry breaking, fractality, etc [14,20].

A short introduction to self-organized systems (SoS) and self-organized criticality (SOC) theory and their stability analysis is given. Analysis of real system statistical data, it's modelling and comparing results with SOC state gives us a comprehensive view of system behaviour. There is shown that only the system in SOC state has long-term structural stability [5,6], therefore we can use the information about systems digression from the SOC state as decision-making tool for long-term strategy development for the system under consideration.

In this paper we propose methodology foundation for using self-organized criticality (SOC) approach in decision-making application for long-term regional power generators (installed capacity) system development. Raising assumption that energetics system structure is essentially influenced by urban system, we utilize urban system CA model as the basic model for electricity generators as the superior system. Because of energetics becomes the most important and most problematic area of human activity in nearest future, the importance of proper decision-making here is evident.

2 Self-Organized and Self-Modifying Systems

A self-organised system is one, which directs its own processes and its own form including, remarkably, its own development. Self-organising systems have two aims:

(1) to reproduce itself and evolution (e.g. Alife; Bak [14], Kauffman [15])
(2) to simulate cognition and problem solving (e.g. Genetic Algorithms; Holland. [22])

Unlike the simplest cases, in which dependencies are readily exhibited, in non-linear cases causal dependencies are not obvious and system behavior can be difficult or impossible to predict [20]. Self-organization allows us to describe the features such systems will manifest, and their natural dynamics. This is done without predicting, or knowing, or caring about, the specific organization of the system. We abstract from the organization, and then describe the macroscopic phenomena statistically, as the expected features of complex systems.

Using a self-organization approach, system development can be understood in terms of general systemic principles that operate in the presence of variety. Viewing dynamics in terms of complex adaptation in self-organizing systems yields non-equilibrium and nonlinear perspectives reinvigorating them as the basis of evolutionary thinking in the new Millennium [19].

Self-modifying systems [21] have been forwarded as an alternative paradigm to dynamical systems and extension to self-organized systems. Self-modifying systems are component system that draw on an open-ended set of different types of components and that produce and destroy their own components during their typical activities. Ecosystems as self-modifying systems produce new variables, for example due to the come and go of elements, due to new environmental contexts, in which hidden properties appear, or due to competition or evolutionary processes.

As self-modifying systems pick up information on-line, it is impossible to map all the relevant properties of the components in advance. Thus, parameters and variables are definable only *a posteriori*. Therefore the paradigms of self-organization and self-modification emphasize process, time and 'becoming'. For short time frames, the dynamical system approach may be valid, but on the large scale the dynamical models break down. On the time scales targeted by sustainability, complex systems may have to be conceived as self-modifying systems, as well as the cognitive (i.e. human individuals) and social systems (e.g. the science system) that observe and interact with them.

We assume that urban and energetic system is one of self-organized and self-modifying systems class in the content of complexity and unpredictability of particular variables dynamics. For the construction of rules and methods analyzing such systems we are obliged to include this point of view for getting effective results.

3 Self-Organized Criticality

The concept of self-organized criticality was introduced to explain the behaviour of the sand pile model. In this model, particles are randomly dropped onto a square grid of boxes. When a box accumulates four particles they are redistributed to the four adjacent boxes or lost off the edge of the grid.

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Procedure "Sandpile" model pseudocode
Repeat
        Random x and y
        Z(x,y):= Z(x,y)+1;
        If Z(x,y) = 4 then
                Z(x±1, y) := Z(x±1, y) + 1;
                Z(x, y±1) := Z(x, y±1) + 1;
                Z(x, y) := Z(x, y) - 4;
Update cells.
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Redistributions can lead to further instabilities with the possibility of more particles being lost from the grid, contributing to the size of each 'avalanche'. These model 'avalanches' satisfied a power-law frequency–area distribution with a slope near unity.

Self-organized critical state describes evolving systems for which log-log distribution $N=m^{-a}$ is proper, where N is amount of cluster elements, m – cluster magnitude, $\boldsymbol{a}$ – parameter. We can encounter meet such systems in different scales and substance organization levels. SOC state is also common to energetics system that is evolving social-economic-technological system.

Other cellular-automata models, including the slider-block and forest-fire models, are also said to exhibit self-organized critical behaviour. It has been argued that earthquakes, landslides, and species extinctions are examples of self-organized criticality in nature. In addition, wars and stock market crashes have been associated with this behaviour. In the basic forest-fire model, trees are randomly planted on a grid of points. Periodically in time, sparks are randomly dropped on the grid. If a spark drops on a tree, that tree and adjacent trees burn in a model fire. The fires are the 'avalanches' and they are found to satisfy power-law frequency–area distributions with slopes near unity. This forest-fire model is closely related to the site-percolation model [13], that exhibits critical behaviour. In the forest-fire model there is an inverse cascade of trees from small clusters to large clusters, trees are lost primarily from model fires that destroy the largest clusters. This quasi steady-state cascade gives a power-law frequency–area distribution for both clusters of trees and smaller fires. The site-percolation model is equivalent to the forest-fire model without fires. In this case there is a transient cascade of trees from small to large clusters and a power-law distribution is found only at a critical density of trees.

4 Statistical Example

For the simple illustration of our methodology aplication to system analysis from point of wiev of self-organized criticality we examine statistical data concerning urban situation in Lithuania and Sweden as well as energetics situation in Lithuania. By collecting statistical data we used information source of Lithuanian statistical yearbook of year 2001, where some data shortage was observed especially for villages with less then 1000 inhabitants..

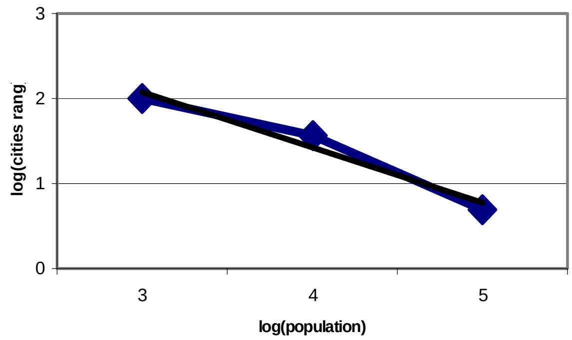


Fig. 1. Distribution of cities rang according to population 2001
log (cities rang)= - 0.65 log(population)

Statistical data about 48 power plants in Lithuania were aquired from energetic sector report from 2000 year annual report of Lithuanian Energy Institute [23]. Again, there are no data about small and very small generators in Lithuania.

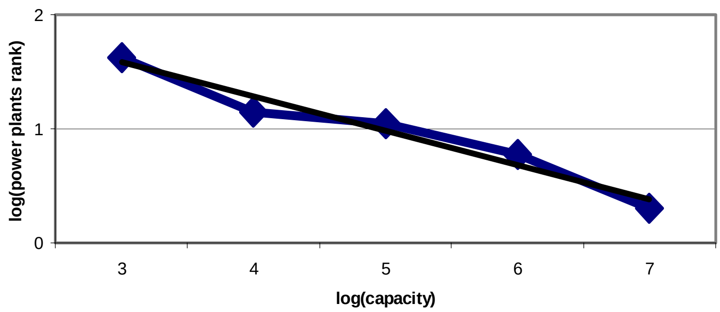


Fig. 2. Distribution of generators rank according to their capacity 2000
log (power plants rank)= - 0.30 log(capacity)

We can see that distribution for both cases in log-log coordinate system approaches to the linear distribution, but the slope is not unit. Analysing urban situation in such countries as Estonia and Sweden we have perfect fit to SOC theory, but we don't have data about generators in these countries. We can conclude that in case of Lithuania the calculated criticality value ***a*=0.**65 indicates that system is in subcritical state and urban dynamics is determined by population density increasing.

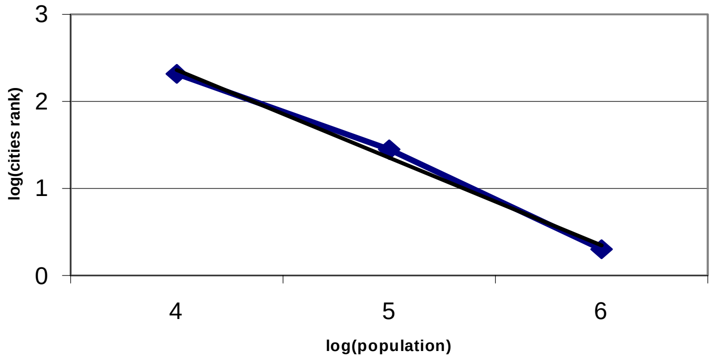


Fig. 3. Distribution of cities rang according to population in Sweden 2000
log (cities rang)= - 1.01 log (population)

5 Cellular Automata Model for Urban System

5.1 Mathematical Model. The finite urban domain S consists of elements s_{ij} of equal size. Each element except those forming the border of the domain S has boundaries with the other eight elements. Each element s_{ij} can be in one of these states: 1) empty land – E_{ij}; 2) river or lake – W_{ij}; 3) road – R_{ij}; 4) forest – F_{ij}; 5) house – H_{ij}. The initial configuration of the domain S changes at discrete time moments according to such a transition rule: Element H_{ij} may emerge only from the element E_{ij}. All the rest elements pass to themselves at discrete time moments. Each element s_{ij} has the weight $w(s_{ij})$[1], which depends on the state of the element. Since the transition rule permits the change only of element E_{ij} further we shall consider the properties of these elements. The k-vicinity of element E_{ij} in the land state is formed of all the elements distant from the E_{ij} no more than by k elements. E.g., 1 - vicinity consists of 9 adjacent elements (including the element E_{ij} itself), etc.

The total amount of elements constituting the k-vicinity

$$w^k(E)=\sum_{l=i-k}^{i+k}\sum_{m=j-k}^{j+k} w(s_{lm}).$$

It should be noted that the larger the k-weight $w(E_{ij})$ of the element, the greater the probability for the land element E_{ij} to pass into the state H_{ij}.

5.2 Algorithm. The states to all the elements of the domain S are attributed so that the domain corresponds to some real or fictitious urban situation. The initial situation will change at discrete time moments. We choose the environment k>0. The k-weight $w^k(E_{ij})$ is computed to each land element E_{ij}. Since the weights of elements making up the k-environment in this model do not depend on the distance to E_{ij}, too large an environment equalize the weights of elements E_{ij}. Therefore, in fact is not to be too large. The sum of k-weights of all elements E_{ij} of the region S is $w^k(S)$, and the k-weight of element E_{ij} is a part $\frac{w^k(L_{ij})}{w^k(S)}$ of the total sum. The interval $w^k(S)$ is consecutively filled by partial intervals $w^k(E_{ij})$. In this interval a uniformly distributed random variable is generated. A random value indicated the interval, to the element of which the transition rule is applied: the state E_{ij} is changed into the state H_{ij}. After this phase stage, the weights of the remaining elements E_{ij} (there are less of them by a unit) are recomputed and a random variable generated again [11].

In recent decades popular region urban development theory of central places [18] requests additional assumptions about landscape metric. Meanwhile proposed CA model [17] automatically generates urban structure with scale symmetry, therefore it best correspond to the natural development of urban system. Urban structure composed during CA model realization has clearly defined conglomerates which correspond to power-law distribution $N=m^{-\boldsymbol{a}}$, where N is amount of cluster elements, m – cluster magnitude, $\boldsymbol{a}$ – parameter that usually characterize the average land market value.

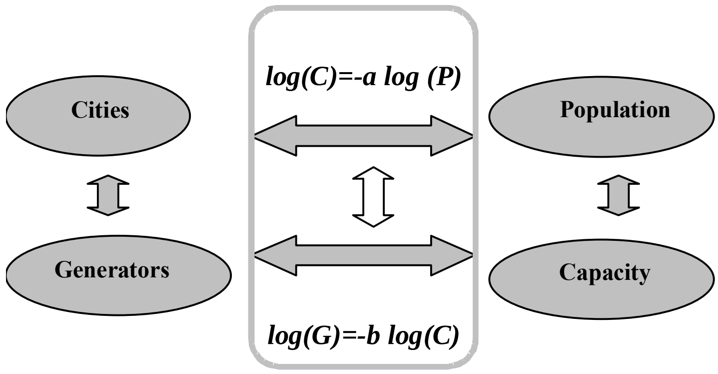
Generally speaking, CA model is algorithmically simple even if requires enormous computational resources. Our suggested CA model is structurally minimal as well, where perturbations and symmetry breaking is included using additional components (roads, rivers and lakes, forest cells). To break the symmetry of initially homogenous conditions there is enough to include one additional element in the initial landscape area.

All parameters that influence conglomerate dynamics in CA model are tuned according to real country or region situation. Statistical data analysis show that some industrial countries' regions have formed cities conglomerates that conditionally don't fit to log-log distribution while urbanized countries log-log distribution has clear profile. We can propose that when regions transit from industrial stage to post-industrial and information stage we have some kind of phase transformation similar to percolation phenomenon when strong centres dominance is excluded. In CA model these transformations reflect in changing parameters weight values that influence dynamical process “centralization-decentralization” axis. Drawing extreme cases, "empty place" cells with 0 weight and "houses" with maximal weight generates centralized urban landscape, meanwhile "empty place" cells with maximal weight and "houses" cells with minimal weight generates decentralized urban landscape. As an example of close to SOC state we get "empty place" cells with weights 3 and "houses" with 800 weight. Distribution of clusters in that situation is similar to Sweden urban situation.

6 Energetics as a Superior System

Innovations in energetics are not so quick as in computer science. Their changeovers happen very slowly, so distribution of generators is quite stable. It means that the directions of investments are very important for systems efficiency, and the adaptivity plays the follow-up role. For the energetic system that is in the SOC state there is much more potential for adaptivity comparing to one which is not in SOC state.

Hypothesis. *For a big enough restricted region with developed energetics urban conglomerates and power generators are both in a SOC state. There is interrelation between both systems.*



According to Hypothesis we breed power plants on urban pattern that was generated using CA model close to the real situation. They should appear with certain probability and certain capacity to form the log-log distribution. The result of generators geographical-capacity distribution is very close to real situation for many countries, but not for all. That means that these countries have to formulate their energetics policies to put the energetics system in SOC state.

Usually energetics system modelling is performed using system dynamics methods and particularly differential equations system. System dynamics based models are designed to support the strategic planning process of a large electric utility as well as for analysis of strategic issues. It computes several performance indices while considering the regulatory compact for the electricity supply industry, the response of the consumers and the values of prospective investors.

Major shortcomings of the dynamical system paradigm are highlighted: dynamical systems are conceptually closed systems requiring a fixed set of *a priori* defined parameters, part of which are parameters of convenience satisfying mathematical needs and part of which are residual parameters that account for noise and system background. Many natural systems are conceived as conceptually open, self-modifying systems, which constantly (‘on-line’) produce novelty and new parameters and which cannot be severed from their environment. Although calibration may adapt models to data sets of the past, it does not assure predictive capacity or validity. While models serve heuristic and theoretical functions and may outline the space of possible behaviour, they may be deficient instruments for the reduction of uncertainty as to future system behaviour.

7 Decision Making for SOC System

Complexity has become interesting to management scholars who value its challenge to reductionism, prediction and equilibrium, as well at its ability to derive interesting emergent properties from simple relations. We step through these and other properties attributed to chaos and complexity to examine which of the properties are actually desirable and whether the approaches actually have that property. For example, we find reductionism generally desirable, but find that complexity may be overly reductionistic for the study of humans. As a matter of comparison, we show that most of the desirable properties attributed to complexity and chaos can be found, sometimes uniquely, in the theory of self-organized systems.

Historically the basic objective of power system planning may be defined as the determination of the commissioning dates of generation and transmission facilities, in order to supply the electric energy market at minimum cost for a given reliability level.

At any time range, the solution of the planning problem involves a compromise between a desired reliability level and the costs incurred to obtain it. This compromise has been traditionally expressed as the minimization of the total cost to supply a given demand forecast. The total cost is the sum of investment, operation and failure costs.

The solution of this complex programming problem usually is done by a decomposition scheme, able to break down this large-scale, mixed-integer stochastic programming problem into two subproblems: a mixed integer decision subproblem and a linear operation subproblem, as shown in fig. 4.

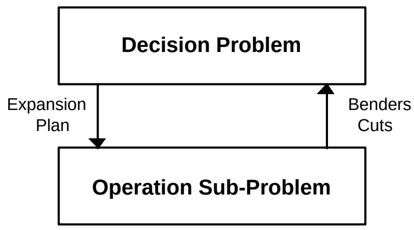


Fig. 4. Structure of the Strategic Model in the Cost Based Planning Framework

Wide-ranging mathematical models are efficiently used in analysis of system behaviour and decision-making. One of the biggest restrictions we encounter in complex economic systems modelling is uncertainty in future systems' transformation. For example, forecast of energy generation dynamics depends not only on consumption forecast, but also on technological innovations which infallible prognosis is impossible. Therefore long-term prediction of many dynamical characteristics is out of sense and usually is replaced by analysis of different scenarios. In this case dynamical models designed using differential equations or statistical methods is one of most popular approach in decision making.

Different forms of uncertainty are at the heart of energetics decision-making, among them epistemic uncertainty, which arises when the normal, disciplinary forms of uncertainty reduction fail and which leads to debate on adequate ways of coping with uncertainty. Correspondingly, modelling for theoretical scientific purposes and modelling for decision-making may follow separate paths. Modelling for decision-making may have to take into account requests for transparency and participation ('deliberation frames analysis') and the validity of model products will be judged according to their capacity of providing context-sensitive knowledge for specific decision problems.

Numerous other methods already exist to help decision-makers peer into the future. Most prominent are statistically based time series and other econometric models. Simple trending can also be an effective tool. Scenario analysis and Delphi are two other widely used methods. To represent uncertainty associated with forecasts, trends, and subjective judgments, one can use probability methods (and generalizations there of), fuzzy sets, certainty factors, and various other techniques. Thus, theoretically, it is possible to provide to environmental decision makers probabilized forecasts - i.e., forecasts of variables that contain probabilities over what values the variables could take in each period into the future.

In theory of *adaptive* decision-making the main task is to find values of dynamical parameters to satisfy local optimum criterion. In contrary, for the long-term strategies there is important to search for the systems global optimal solution. In this case we have to take into consideration the main features of SOC systems such as resistance not only to inner non-linear interactions but also to outer influence, positive innovations and destructive perturbations. We assume that the systems criticality is one of the most important characteristics, which simplify the long-term optimal solutions finding.

The time interval $\boldsymbol{T}$, in which the decision is effective for the parameter $\boldsymbol{g}$, plays significant role in long-term decision-making. The systems efficiency is measured with efficiency function $\boldsymbol{u=u(g,t)}$, where vector $\boldsymbol{g}$ and $\boldsymbol{u}$ in general case is non-linear function. As a rule to find the optimal solution for $\boldsymbol{u}$ is a complicated procedure. An example of function $\boldsymbol{u}$ could be the investments, which we are seeking to minimize in time interval $\boldsymbol{T}$.

After the estimation criticality value $\boldsymbol{a}$ and criticalities quality $\boldsymbol{Q}$ for self-organized system we can get a lot of information that can be used in strategic decision-making. The numerical value of criticality $\boldsymbol{a}$ can be any one from the interval $(0, \infty)$, but qualitatively the self-organized system is described as a subcritical ($0<\boldsymbol{a}<1$), critical ($\boldsymbol{a}\sim 1$) and supercritical ($1< \boldsymbol{a} <\infty$). The criticalities quality $\boldsymbol{Q}$ is estimated after the analysis of every point distance from the line aquaired by using Least Square Method in logarithmic scale.

We know that system's SOC state is maximally adaptive, i.e. the systems time erequired for adaptation to changeable external conditions is minimal. Meanwhile even the small external influence to the system, which is taken out or didn't reach SOC state, can determine enormous losses or total collapse of the system. The presence of various modes in system is basic condition for its adaptive stability.

Decision making in energetics we perceive as a policy of investment in long-term development of energetics economy, considering the structure of generators instead of total dynamic characteristics of produced energy. Generators rank distribution law unambiguously identifies the energetic systems structure. In the ideal case the system of generators should be in the self-organized critical state that corresponds to the naturally formed urban system state. Because of lack of statistical data we were not able to verify presence of this consistent pattern in various countries except in Lithuania. Therefore during the comming research we'll analyze additional statistical data to identify regularities. Accordingly we consider this method as operating *ad hoc* hypothesis.

The proposed method based on CA modelling of urban system is meaningful for decision making in two aspects. First, it gives important geographic characteristics and its dynamics for urban system (structure of clasters distribution), second, the criticality parameter $\boldsymbol{a}$ is important itself for long-term decision-making. When we observe real system deviance from SOC state, the taken decision must re-establish this state. That allows us to answer both questions 'what capacity and how many generators' and 'where to put them'. For example, if capacity of generators is bigger ten regions demand for energy, the location of generator is not important because its destination is energy export.

8 Discussion

After the analyzing urban and generators capacity data we estimate parameters for urban dynamics CA model, which can be taken into account as a useful tool in decision-making for energetics development.

It is obvious that the SOC-based decissioning method is assumptions free for a wide class of systems. In contrary, system dynamics revealed by other mathematical models allow us to perform forecasts of real systems quantitative parameters behaviour which accuracy hardly depends on assumptions done in modelling process.

The critical state in many complex system is considered to be of a natural origin, therefore it has optimal functioning performance in any time and space interval, i.e. dynamical stability gives to the system maximal adaptivity rate to extragenous and intravenous disturbances. Therefore the system present state figures out the character of possible decisions, which have to be made with regard to system further structural transformations.

In present work we suggest that for the complex dynamical systems strategy formation there is enough to restrict to the keeping constant parameter ***a*** (exponent) in log-log distribution and to seek to monotonize this distribution. This requirement narrows the set of possible strategies. For instance, it doesn't admit to develop or eliminate one systems element without changing quantitative expression of neighbouring elements. Otherwise a system natural evolution lasts too long and produces lateral effects therefore generally speaking is not economically effective.

The change of parameter ***a*** means that in a log-log coordinate systems the slope angle of linear function is adequately changing. The long-term unchangeable exponent ***a*** means that energetics system is permanently ready to accept any technological changes. We can suppose for instance that systems resistance to parameters ***a*** increasing expresses systems stability in case of innovations in energy transmission when demand for powerful generators manifests itself, meanwhile resistance to its decreasing means possibility of wide rang innovative modifications for small generators. For example, using superconductors for electricity transmission let us efficiently exploit high capacity generators, because centralized generation reduce energy prices. Progress in solar energetics decrease in general energy demand produced by big power plants.

System in SOC state stays dynamically stable and effectively adjusts to inconstant new environment. Otherwise, if the monotony of log-log distribution is destroyed, instability appears in a part of system as well as in the complete system. Disturbance of monotony is possible when there are new considerable technological changes, there is introduced new technological equipment, but the old one is still functioning or vice versa – the old one disappears and there is any new introduced. Consequently, decisions about system development should be made according to generators size distribution.

Therefore by making changes in regions energetics system there is necessary to respect the structure of installed capacity. Non-monotony of structure shows us that it is in unstable transitive state and strategic planning is directed first of all to liquidation of this situation. In natural systems, which are in lower hierarchical level, SOC state has been acquired naturally during evolutionary process. In artificial systems such as energetics information we can use in decision-making is crucial for long-term systems development.

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