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Numerical Treatment of short Laser Pulse Compression in transient stimulated Brillouin Scattering

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Abstract

Results of numerical investigation of the compression process of short pulses during essentially nonstationary SBS are presented, taking into account the diffraction and focusing of the interacting light beams and the finite oscillation period and lifetime of hypersound waves in nonlinear media.

Keywords: short laser pulse, transient stimulated scattering, compression

1 Introduction

During stimulated Brillouin scattering (SBS) of short laser pulses at optimum focusing conditions, efficient pulse compression and generation of Stokes pulses with duration in sub-nanosecond [1, 2] and even picosecond [3, 4] ranges is possible. SBS pulse compressors are widely used nowadays in powerful laser systems [4-10]. Therefore, detailed analysis of efficiency of scattering and spatio-temporal structure of the scattered pulses depending upon the energy, the shape and the duration of the pump pulses, the focusing geometry and physical parameters of the

SBS media is of great interest. Although attempts are still being made to investigate analytically the principles of compression using greatly simplified equations of backward SBS in the approximation of plane-waves of intensity and taking into account the so-called wave nonstationarity only [11], inclusion of the material nonstationarity into the research is possible by numerical methods only.

Since the experimentally achieved durations of the compressed pulses T_S are smaller not only than the lifetime T_R of the hypersound waves induced in the process of the stimulated scattering, but also than their oscillation period T_B as well [1-5], among other things, the second partial derivative with respect to time should be preserved in the material equation for the complex amplitude of hypersound oscillations [12-17]. However, in most works dealing with the pulse compression in nonstationary SBS, the treatment is limited to preserving the first derivative and the term describing the decay of hypersound oscillations. Only in relatively recent work [18], attention was drawn again to the necessity to preserve the second derivative when analyzing the possibility to compress the Stokes pulses to durations shorter than the period of the hypersound oscillations, using the plane-wave approximation for analysis of the compression regime [17].

At the same time, for correct description of the experimental results, it is necessary in principle to explore the so-called generation regime of the Stokes radiation from level of the distributed noise of spontaneous scattering of the focused radiation [12-16]. It is quite natural to take into account significant variation (of the order of 20 times) of the transverse sizes of the pump beam and the Stokes beam achieved in the SBS compression processes by expanding the fields into the series of Laguerre-Gaussian modes [12-16, 19], which has been done already for quite some time. Besides, a parallel calculation algorithm has been implemented [20], using the expansion into the Laguerre-Gaussian modes. Therefore, the statements of the authors of recent works [21, 22] that a unified three-dimensional model encompassing finally all the aspects of SBS, particularly, the nonstationarity and the transverse effects, has been created in these works only, looks even more surprising.

It should be noted that only the first temporal derivative is taken into account in the material equation in these works, and the Laguerre-Gaussian mode expansion is also used for description of distribution of focused axially-symmetric fields. Besides, these works treat only the well-known pre-compression of the leading edge of the pulses during the nonstationary scattering regime rather than the compression regime. The main substance, however, is that these works conclude, following the works dealing with the stationary SBS in focused axially-symmetric beams [23, 24] where the Laguerre-Gaussian mode expansion was also applied, that the transverse dimensions of the Stokes beam exceed the size of the pump beam beginning from the focal waist and up to the exit from the cell in the regime of generation from the noise level of spontaneous scattering. These results contradict those obtained in our previous works [13-16,25].

Therefore, in this work, a novel algorithm for numerical solution of equations of strongly nonstationary SBS in focused beams was developed (without application of the Laguerre-Gaussian mode expansion) and numerical investigation of compression dynamics of short laser pulses was performed, where the main emphasis was on the changes of conversion efficiency and quality of the Stokes beams as the energy of the pump pulses grow, and the dependencies of the spatio-temporal characteristics of the compressed Stokes pulses upon the lifetimes and periods of hypersound oscillations.

2 Equations of strongly nonstationary SBS with focused beams

The main part of an SBS compressor (Fig. 1) is a long cuvette with glass facets containing liquid or gaseous Brillouin medium. The circularly polarized collimated laser beam with radius w_{L0} is focused by a lens with focal length F in such a way that the beam waist is located inside the cuvette. If the length $Z_F = nF$ is close or larger to half the spatial length of the laser pulse inside the

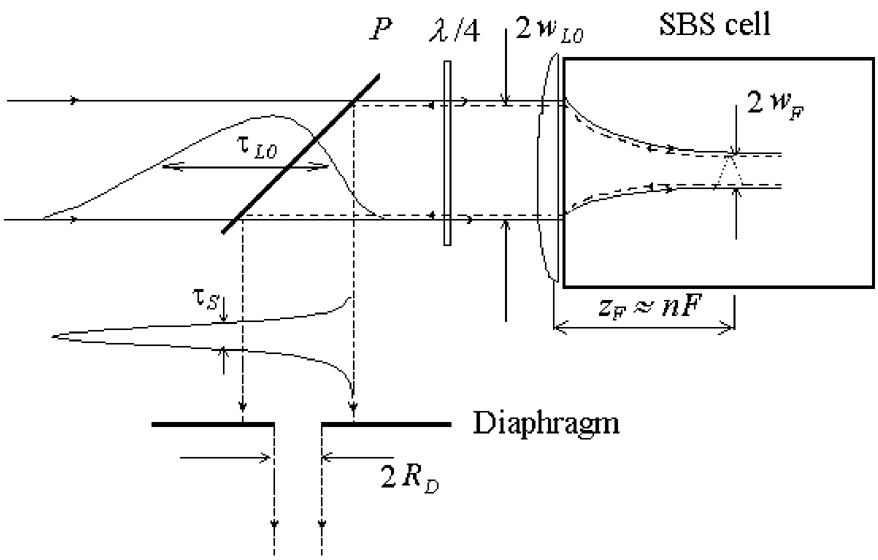

Fig.1 The optical scheme of an SBS compressor

cell $v_L T_{L0}$ and the convergence angle of the focused beam inside the cuvette is chosen appropriately, a short Stokes pulse (generated at the waist from the noise of spontaneous scattering) is efficiently amplified due to SBS while propagating against the laser pulse. In this way, Stokes pulses are generated in the compressor whose duration is smaller by a compression ratio $N = T_{L0}/T_S$ than that of the incident laser pulse, and their energy exceeds the level of half or even more the initial energy of the laser pulse. After traversing the polarization decoupling system consisting of quarter- wave plate $\lambda/4$ and dielectric polarizer P, the Stokes pulses exit from the SBS- compressor, and propagate through the diaphragm with radius R_D.

The laser and the Stokes optical waves and the sound wave propagating along the cell (Z axis) interact in the SBS process:

$$E_L(R,Z,T)=\frac{1}{2}\left(A_L(R,Z,T)e^{i(k_L Z-\omega_L T)}+c.c.\right), \quad (1a)$$

$$E_S(R,Z,T)=\frac{1}{2}\left(A_S(R,Z,T)e^{-i(k_S Z+\omega_S T)}+c.c.\right), \quad (1b)$$

$$\tilde{\rho}(R,Z,T)=\frac{1}{2}\left(U(R,Z,T)e^{i(qZ-\Omega T)}+c.c.\right). \quad (1c)$$

Here, T is the temporal coordinate, $R=\left(X^2+Y^2\right)^{1/2}$ and Z are the spatial coordinates, A_L, A_S and U are the slowly varying complex amplitudes of these waves, k_L, k_S, $q=k_L+k_S$ are their wavenumbers, ω_L, ω_S, $\Omega=\omega_L-\omega_S$ are their cyclic frequencies. To describe the nonlinear interaction of the waves in case when the Stokes pulse duration is shorter not only than the sound wave lifetime T_R but also shorter than its period T_B, strongly nonstationary SBS equations of cylindrical beams for slowly varying amplitudes of laser, Stokes and sound waves, containing the second derivative in the sound wave equation and the terms describing the influence of Kerr nonlinearity of refraction index of the medium were used [12-18]:

$$\frac{1}{v_L}\frac{\partial A_L}{\partial T}+\frac{\partial A_L}{\partial Z}+\frac{\Delta_\perp A_L}{2ik_L}=\frac{i\gamma_e k_L}{4\rho_0 n_L^2}A_S U+i\frac{\omega_L n_2}{2c}\left(|A_L|^2+2|A_S|^2\right)A_L\,, \qquad (2a)$$

$$\frac{1}{v_S}\frac{\partial A_S}{\partial T}-\frac{\partial A_S}{\partial Z}+\frac{\Delta_\perp A_S}{2ik_S}=\frac{i\gamma_e k_S}{4\rho_0 n_S^2}A_L U^*+i\frac{\omega_S n_2}{2c}\left(2|A_L|^2+|A_S|^2\right)A_S\,, \qquad (2b)$$

$$\frac{iT_B}{4\pi}\left(\frac{\partial^2 U}{\partial T^2}+\frac{1}{T_R}\frac{\partial U}{\partial T}\right)+\frac{\partial U}{\partial T}+\frac{U}{2T_R}=\frac{\gamma_e q^2}{16\pi\Omega i}A_L A_S^*+U_f\,. \qquad (2c)$$

where $\Delta_\perp=\frac{1}{R}\cdot\frac{\partial}{\partial R}\left(R\frac{\partial}{\partial R}\right)$ is transverse Laplacian, γ_e is the electrostriction coefficient, ρ_0 is the unperturbed density of the nonlinear medium, $v_{L,S}$ are the group velocities of the laser and the Stokes waves in the medium, $n_{L,S}$ are the refraction indices for the laser and the Stokes beam, n_2 is the nonlinear refraction index, U_f is the amplitude of the thermal fluctuations of the medium. It was assumed that the resonance condition $qv_L=\Omega$ holds in the SBS generation regime when deriving (2c). The domain of the solution of Eqs. (2) is $0\le T\le T_m$, $0\le Z\le L$, $0\le R\le R_m$, where T_m is the time interval during which the compression process is considered, L is the length of the medium along the wave propagation direction, R_m is the distance from the laser beam axis to the boundary of the medium, along the direction transverse to the beam axis. The initial conditions of Eqs. (2) are: $A_{L,S}(R,Z,T=0)=0$, $U(R,Z,T=0)=2T_R U_f$. The boundary conditions are $\partial A_{L,S}(R=0,Z,T)/\partial R=0$, $A_{L,S}(R=R_m,Z,T)=0$, $A_S(R,Z=L,T)=0$, and $A_L(R,Z=0,T)=A_{L0}(R,T)$ is the initial laser pulse of known amplitude.

Equations (2) were used in normalized form for numerical solution. Since the diameter of the beam focused into the cuvette is many times (by a factor of 20 and more) smaller at the beam waist than its diameter at the entrance to the cell, the adaptive transformation of the transverse coordinate R proposed in Ref. [26] was used:

$$\overline{R} = R\Big/\left(1+\left((Z-Z_F)/Z_R\right)^2\right)^{1/2} = R/\rho(Z)\,, \tag{3}$$

where Z_F is the distance from the entrance to the medium to the laser beam waist, $Z_R = k_L w_F^2/2$ is the Rayleigh length, and w_F is the beam radius at the waist. After applying the transformation (3), the normalized strongly nonstationary SBS equations read as follows:

$$\frac{\partial e_L}{\partial t}+\frac{\partial e_L}{\partial z}-\frac{i\mu}{\rho^2(z)}\left(\frac{\partial^2 e_L}{\partial r^2}+\frac{1}{r}\frac{\partial e_L}{\partial r}\right)+\frac{\left(ir^2/4\mu\right)+(z-z_F)}{z_R^2+(z-z_F)^2}e_L =$$
$$= i\Gamma_o e_S u + i\eta\left(|e_L|^2+2|e_S|^2\right)e_L\,, \tag{4a}$$

$$\frac{\partial e_S}{\partial t}-\frac{\partial e_S}{\partial z}-\frac{i\mu}{\rho^2(z)}\left(\frac{\partial^2 e_S}{\partial r^2}+\frac{1}{r}\frac{\partial e_S}{\partial r}\right)+\frac{\left(ir^2/4\mu\right)-(z-z_F)}{z_R^2+(z-z_F)^2}e_S =$$
$$= i\Gamma_o e_L u^* + i\eta\left(2|e_L|^2+|e_S|^2\right)e_S\,, \tag{4b}$$

$$\frac{i\tau_B}{4\pi}\left(\frac{\partial^2 u}{\partial t^2}+\frac{1}{\tau_R}\frac{\partial u}{\partial t}\right)+\frac{\partial u}{\partial t}+\frac{u}{2\tau_R} = ie_L e_S^* + u_f\,. \tag{4c}$$

where $t = T/T_0$, $T_0 = L/v_L$, $z = Z/L$, $r = \overline{R}/R_0$, are normalized independent variables where $R_0 = R_m\Big/\sqrt{1+(Z_F/Z_R)^2}$, $e_L = \dfrac{A_L}{A_0}\exp\left[\dfrac{i}{4\mu}\dfrac{(z-z_F)}{z_R^2}r^2\right]$, $e_S = \dfrac{A_S}{A_0}\exp\left[\dfrac{i}{4\mu}\dfrac{(z-z_F)}{z_R^2}r^2\right]$ are the slowly varying complex amplitudes of the laser and the Stokes waves normalized to the characteristic amplitude A_0, $z_F = Z_F / L$, $z_R = Z_R / L$, $u = U/U_0$ is the normalized sound wave amplitude, $\mu = L/(2k_L R_0^2)$, $\tau_B = T_B v_L / L$ is the normalized sound wave period, $\tau_R = T_R v_L / L$ is the normalized sound wave lifetime, $\Gamma_0 = g I_0 v_L / 4T_R$ is the coefficient of the nonlinear coupling ($g = \frac{\omega_L^2 \gamma_e^2 T_B}{n_L \rho_0 c^3 v}$ is the SBS amplification coefficient, v is the velocity of hypersound, $I_0 = c n_L |A_0|^2 / 8\pi$), $\eta = 8\pi^2 n_2 I_0 L / c n_L \lambda_L$ is the nonlinearity coefficient of the refraction index, u_f is the normalized amplitude of thermal fluctuations of the medium. The limits of variation of the normalized variables are: $0 \le t \le t_m$, $0 \le z \le 1$, $0 \le r \le 1$, where $t_m = T_m v_L / L$.

3 Numerical method

For numerical solution of Eq. (4), the split step method was used whose scheme is presented on Fig. 2. The essence of this method is

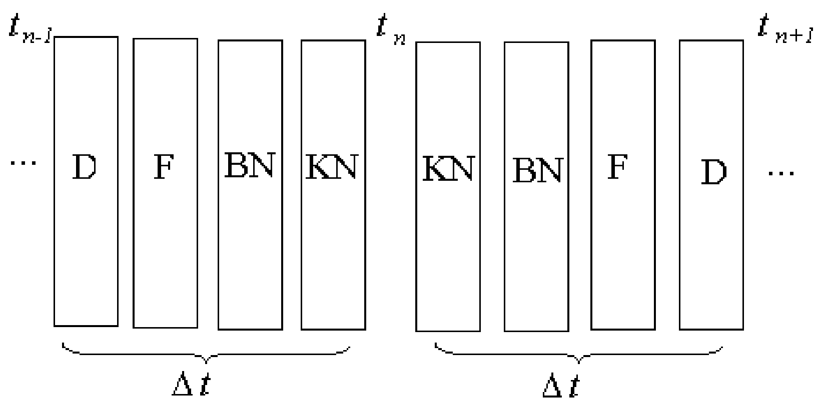

Fig.2 The scheme of the numerical method, $t_n = n\Delta t$, $n = 0 \div N_T$, $\Delta t\, N_T = t_m$

division of simultaneous wave diffraction and processes of nonlinear interaction into consecutive sequence.It is assumed that, in every discrete time step Δt, the diffraction (D), the focusing (F), the Brillouin nonlinearity (BN) and the Kerr nonlinearity (KN) take place sequentially. In order to increase the accuracy of the method to the second order, the symmetrical scheme [27] is used, where the mentioned processes are applied in reverse order for consecutive time steps.

Diffraction step. In this step, the equations of wave propagation and diffraction are being solved for the amplitudes of the laser and the Stokes waves:

$$\frac{\partial e_L}{\partial t}+\frac{\partial e_L}{\partial z}-\frac{i\mu}{\rho^2(z)}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial e_L}{\partial r}\right)=0, \qquad (5a)$$

$$\frac{\partial e_S}{\partial t}-\frac{\partial e_S}{\partial z}-\frac{i\mu}{\rho^2(z)}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial e_S}{\partial r}\right)=0. \qquad (5b)$$

The finite-difference method is applied. The grid with constant step along the z axis and increasing density toward the beam axis along the r coordinate is used in the domain $0 \le z \le 1$, $0 \le r \le 1$: $z_j = j\,\Delta z$, $j = 0 \div N_Z$, $\Delta z\, N_Z = 1$, $r_k = \left(e^{\alpha y_k}-1\right)/\left(e^{\alpha}-1\right)$, $k = 0 \div N_R$; $y_k = k\,\Delta r$; $\Delta r\, N_R = 1$; $\alpha \ge 1$.

The values of the amplitudes at the grid nodes $e^n_{L\,j,k} \equiv e_L(r_k, z_j, t_n)$, $e^n_{S\,j,k} \equiv e_S(r_k, z_j, t_n)$ were calculated using the following implicit finite-difference scheme [28]:

$$\frac{e^{n+1}_{L\,j,k} - e^{n}_{L\,j-1,k}}{\Delta t} - \frac{i\mu}{\rho^2\left(z_{j-0.5}\right)}\Lambda\left((1-\beta)e^{n+1}_{L\,j,k} + \beta e^{n}_{L\,j-1,k}\right) = 0\,, \tag{6a}$$

$$\frac{e^{n+1}_{S\,j-1,k} - e^{n}_{S\,j,k}}{\Delta t} - \frac{i\mu}{\rho^2\left(z_{j-0.5}\right)}\Lambda\left((1-\beta)e^{n+1}_{S\,j-1,k} + \beta e^{n}_{S\,j,k}\right) = 0\,. \tag{6b}$$

It was assumed here that $\Delta t = \Delta z$, $z_{j-0.5} = z_j - 0.5\cdot\Delta z$ and the coefficient β can assume the values from the interval $0 \le \beta \le 1$. When $\beta = 0$, we have a completely implicit scheme, and in case $\beta = 0.5 -$ the Crank-Nicholson scheme.

The following finite-difference approximation was used for the transverse Laplacian:

$$\Lambda e_k = \frac{2}{\Delta r_{k+0.5} + \Delta r_{k-0.5}}\left(\frac{e_{k+1} - e_k}{\Delta r_{j+0.5}} - \frac{e_k - e_{k-1}}{\Delta r_{j-0.5}} + \frac{1}{r_j}\frac{e_{k+1} - e_{k-1}}{2}\right), \tag{7}$$

where $\Delta r_{k+0.5} = r_{k+1} - r_k$, $\Delta r_{k-0.5} = r_k - r_{k-1}$. From the boundary conditions, $e_{-1} = e_1$, $e_{N_R} = 0$. The backsubstitution algorithm [28] was used for solution of the set of the linear algebraic equations.

Focusing step. The following equations are solved in this step:

$$\frac{\partial e_L}{\partial t} + \frac{\left(ir^2/4\mu\right) + \left(z - z_F\right)}{z_R^2 + \left(z - z_F\right)^2}e_L = 0\,, \tag{8a}$$

$$\frac{\partial e_S}{\partial t} + \frac{\left(ir^2/4\mu\right) - \left(z - z_F\right)}{z_R^2 + \left(z - z_F\right)^2}e_S = 0\,. \tag{8b}$$

The exact solutions of these equations are:

$$e_{L\,j,k}^{n+1} = e_{L\,j,k}^{n} \exp\left[-\frac{\left(ir_k^2 / 4\mu \right) + \left(z_{j-0.5} - z_F \right)}{z_R^2 + \left(z_{j-0.5} - z_F \right)^2} \Delta t \right], \tag{9a}$$

$$e_{S\,j,k}^{n+1} = e_{S\,j,k}^{n} \exp\left[-\frac{\left(ir_k^2 / 4\mu \right) - \left(z_{j+0.5} - z_F \right)}{z_R^2 + \left(z_{j+0.5} - z_F \right)^2} \Delta t \right]. \tag{9b}$$

Brillouin nonlinearity. The following set of equations is solved:

$$\frac{\partial e_L}{\partial t} = i\Gamma_o e_S u\,, \tag{10a}$$

$$\frac{\partial e_S}{\partial t} = i\Gamma_o e_L u^*\,, \tag{10b}$$

$$\frac{i\tau_g}{4\pi} \left(\frac{\partial^2 u}{\partial t^2} + \frac{1}{\tau_R} \frac{\partial u}{\partial t} \right) + \frac{\partial u}{\partial t} + \frac{u}{2\tau_R} = ie_L e_S^* + u_f\,. \tag{10c}$$

The predictor-corrector method proposed and analyzed in [29, 30] is used:

$$Lu_{j,k}^{p} = ie_{L\,j,k}^{n} \left(e_{S\,j,k}^{n} \right)^* + u_{f\,j,k}^{n}\,, \tag{11a}$$

$$\frac{e_{L\,j,k}^{n+1} - e_{L\,j,k}^{n}}{\Delta t} = i\Gamma_0 \left(\frac{e_{S\,j,k}^{n+1} + e_{S\,j,k}^{n}}{2} \right) \left(\frac{u_{j,k}^{p} + u_{j,k}^{n}}{2} \right), \tag{11b}$$

$$\frac{e_{S\,j,k}^{n+1}-e_{S\,j,k}^{n}}{\Delta t}=i\Gamma_0\left(\frac{e_{L\,j,k}^{n+1}+e_{L\,j,k}^{n}}{2}\right)\left(\frac{u_{j,k}^{p}+u_{j,k}^{n}}{2}\right)^{*}, \tag{11c}$$

$$Lu_{j,k}^{n+1}=i\left(\frac{e_{L\,j,k}^{n+1}+e_{L\,j,k}^{n}}{2}\right)\left(\frac{e_{S\,j,k}^{n+1}+e_{S\,j,k}^{n}}{2}\right)^{*}+u_{f\,j,k}^{n+1}, \tag{11d}$$

$$Lu^{n+1}=\begin{cases}\dfrac{i\tau_g}{4\pi}\dfrac{y^{n+1}-y^{n}}{\Delta t}+\left(1+\dfrac{i\tau_g}{4\pi\tau_R}\right)\dfrac{y^{n+1}+y^{n}}{2}+\dfrac{u^{n+1}+u^{n}}{4\tau_R} \\ \dfrac{u^{n+1}-u^{n}}{\Delta t}=\dfrac{y^{n+1}+y^{n}}{2}\end{cases}. \tag{11e}$$

Kerr nonlinearity. The following set of equations is solved for description of interaction of the optical waves due to refraction index nonlinearity:

$$\frac{\partial e_L}{\partial t}=i\eta\left(|e_L|^2+2|e_S|^2\right)e_L, \tag{12a}$$

$$\frac{\partial e_S}{\partial t}=i\eta\left(2|e_L|^2+|e_S|^2\right)e_S. \tag{12b}$$

The exact solution of these equations is:

$$e_{L\,j,k}^{n+1}=e_{L\,j,k}^{n}\exp\left[i\eta\left(\left|e_{L\,j,k}^{n}\right|^2+2\left|e_{S\,j,k}^{n}\right|^2\right)\Delta t\right], \tag{13a}$$

$$e_{S\,j,k}^{n+1}=e_{S\,j,k}^{n}\exp\left[i\eta\left(2\left|e_{L\,j,k}^{n}\right|^2+\left|e_{S\,j,k}^{n}\right|^2\right)\Delta t\right]. \tag{13b}$$

The theoretical accuracy of the above discussed symmetrical split-step method is ($\Delta t^2, \Delta r^2$), when the step is $\Delta r = \text{const}$. During the simulation, the calculation accuracy was being checked by verifying the conservation of the total energy of the optical waves, so that the energy deviation would not exceed several percents. Accuracy of solution of the linear diffraction problem was estimated by comparing the numerical solution with the known exact solution for the Gaussian beam. Accuracy of the total solution of the nonlinear problem was estimated by determining by how much the solution changes when the step Δz or Δr is reduced twice.

4 Modelling results

The normalized equations (4) and the above discussed numerical split step method was applied for modeling of compression of laser pulses with wavelength of λ_L=1.064 μm and the envelope with a steeper leading edge:

$$e_{L0}(r, z=0, t) = \sqrt{2e}\,\frac{t}{\alpha\tau_{L0}}\exp\left(-\frac{t^2}{(\alpha\tau_{L0})^2}\right)\exp\left(-\frac{r^2}{w_{L0}^2}\right) \qquad (14)$$

(here, the constant α=1.2245, τ_{L0} is the pulse duration at half the maximum intensity level, w_{L0} is the laser beam radius at 1/e of the maximum amplitude), at the optimum focusing conditions, for the cases when the durations of the input laser pulse are T_{L0}= 2 ns (W_{L0}= 0.14 cm, F= 19 cm) and T_{L0}=1 ns (W_{L0}= 0.09 cm, F = 9 cm), in the Brillouin medium whose parameters are as follows: the sound wave relaxation time T_R= 0.4 ns and the period T_B= 0.75 ns, refraction index n = 1.27, the gain coefficient $g = 4.5\cdot 10^{-9}$ cm/W, which corresponds to the medium FC-75

[3,4]. It should be noted that, for different durations of the pump pulses, the lengths of the cuvettes used L_j were also different (T_{L0}= 1 ns - $L_1 = 15$ cm, $T_{01} = 0.628$ ns and T_{L0}= 2 ns - $L_2 = 31$ cm, $T_{02} = 1.327$ ns), therefore, the normalization times $T_{0j} = L_j / v_L$ were different as well. Therefore, the Stokes pulses having the same normalized durations might have different durations in absolute units.

First, the dependencies of the energetic reflection coefficient

$$R = \int_0^{r_m} H_S(r) r dr \Big/ \int_0^{r_m} H_L(r) r dr , \tag{15}$$

where $H_{L,S}(r) = \int_{-\infty}^{\infty} I_{L,S}(r, z = 0, t) dt$ the energy densities of the laser and the Stokes pulses at the entrance plane $z = 0$ of the nonlinear medium, durations at half the maximum τ_P of the normalized power envelopes of the Stokes pulse

$$P_s(z = 0, t) = 2\pi \int_0^{r_m} \left| e_S(r, z = 0, t) \right|^2 r dr \tag{16}$$

and durations at half the maximum τ_I of the normalized intensity envelope at the beam axis $I_S(r = 0, z = 0, t) = \left| e_S(r, z = 0, t) \right|^2$, and the Stokes beam quality parameters $M^2(t)$ and $\langle M^2(t) \rangle$ upon the parameters of the incident laser pulses were investigated. For characterization of the beam quality, the widely used laser beam quality parameter M^2 that characterizes the beam divergence in the far zone compared to that of the ideal Gaussian beam of the same radius [7,16,17], was used. For the ideal Gaussian beam, $M^2=1$, and for all other beams, $M^2>1$. For the known transverse amplitude distribution $e_S(r,z,t)$, the beam quality parameter values are calculated using the following expression [16,17]:

$$M^2(t)=\left\{\int_0^\infty\left|\frac{\partial e_S}{\partial r}\right|rdr\int_0^\infty|e_S|^2r^3dr-\frac{1}{4}\left|\int_0^\infty r^2\left[\frac{\partial e_S}{\partial r}e_S^*-e_S\frac{\partial e_S^*}{\partial r}\right]dr\right|^2\right\}^{1/2}\Bigg/\int_0^\infty|e_S|^2rdr \quad (17)$$

The transverse amplitude distribution of a pulse is changing in time, therefore, the quality parameter is a function of time $M^2(t)$ that characterizes the beam quality at every moment in time. For characterization of the entire pulse, the power-weighted $P(t)$ time-averaged quality parameter [16]

$$\langle M^2\rangle=\int_{-\infty}^{\infty}M^2(t)P_S(t)dt\Bigg/\int_{\infty}^{\infty}P_S(t)dt \quad (18)$$

was used.

The developed software provided the possibility to record and analyze the values of $e_{L,S}(r_j,z_k,t_m)$ and $u(r_j,z_k,t_m)$ in a number of planes z_k. Fig. 3 presents the three-dimensional distribution and isolines of the square of the modulus of the complex amplitude of the Stokes pulse at the exit ($z=0$) from the SBS cell. It is seen that the compressed pulse has a rather complex characteristic spatio-temporal structure [16,17,19] containing off-axis sub-pulses in the trailing part of the pulse. Their spatial and temporal separation makes it possible to extract by means of a simple diaphragm (Fig. 1) the main sub-pulse whose duration is essentially shorter than the integral duration of the entire pulse.

The pulse quality degradation is determined by a complex spatio-temporal dynamics of the Stokes pulse buildup, that is clearly reflected by the intensity envelope and its isolines of the Stokes pulse. It is seen that, as the Stokes pulses of so short duration build up, the prolonged trailing edge of the pulse is significantly distorted, and a ring-shaped structure emerges in it. Therefore, the quality of the pulse decreases considerably (Fig. 4a), and a peak builds up at the center of the near field energy density distribution (Fig. 4b). The energy density distribution at the far field is close to Gaussian (Fig. 4b).

The dependencies of the Stokes pulse durations τ_P and τ_I, the quality parameter and the reflection coefficient upon the incident laser pulse energy were investigated. The results are presented on Fig. 5. As seen from Fig. 4, the temporal shape of the Stokes pulses has in general a structure comprising a number of sub-pulses. The second and subsequent sub-pulses appear in the temporal shape of the pulses of instantaneous power even when the SBS threshold is exceeded only slightly (Fig. 5a).

The first most intense peak is always separated from the following ones by a sag where the power is less than half of the peak power. As it was seen from Fig. 3, this axial peak is spatially separated from the subsequent off-axis power peaks, therefore, they can be in principle distinguished. Therefore, it is reasonable to introduce the duration of the first sub-pulse only $\tau_P^{(1)}$ and the total duration of the pulse of the instantaneous power at half the maximum level $\tau_P^{(2)}$. It can be seen that the durations of the first power sub-pulse $\tau_P^{(1)}$ and of the intensity at the beam axis τ_I decrease monotonously as the laser pulse energy increases

(Fig. 5a). The maximum compression (the ratio $\tau_{L0}/\tau_P^{(1)}$ and τ_{L0}/τ_I) for the 18 mJ energy is more than 30. Decrease of the overall duration $\tau_P^{(2)}$ is somewhat smaller, but still sufficiently noticeable (~20 times). As the laser pulse energy increases, the Stokes pulse energy increases monotonously as well (Fig. 5b). The maximum reflection coefficient exceeds 75%. For high energies, a saturation of the compression and the reflection coefficient can be observed. As the laser pulse energy increases, the quality parameter $\langle M^2 \rangle$ of the Stokes pulse increases (Fig. 5b), i.e., its quality degrades significantly.

Later, it was investigated how the compression process is influenced by the sound wave period. It has been determined that, as the sound wave period decreases, more intense energy transfer from the laser pulse to the Stokes pulse is achieved, shorter duration and better pulse quality are obtained. Comparison of the Stokes pulse power and intensity envelopes on Fig. 6 demonstrates that increase of the period results in strongly modulated and prolonged trailing edge of the pulse, although the modulation in the central part of the beam remains weak. Therefore, the modulation of the trailing edge can be reduced, better pulse quality and shorter duration τ_P, close to τ_I, can be achieved by propaga ting the Stokes pulse through a diaphragm whose radius R_D is much smaller than w_{L0} (Fig. 7). The modeling results demonstrate that, after the Stokes pulses propagate through a diaphragm whose radius is $w_{L0}/4$, their duration is only 56 ps, when the laser pulses of 1 ns duration are being compressed (Fig. 7b). However, a considerable part of the Stokes pulse energy is lost (86% in this case) due to small diameter of the diaphragm compared to the incident laser beam diameter; therefore, further amplification of these pulses is necessary.

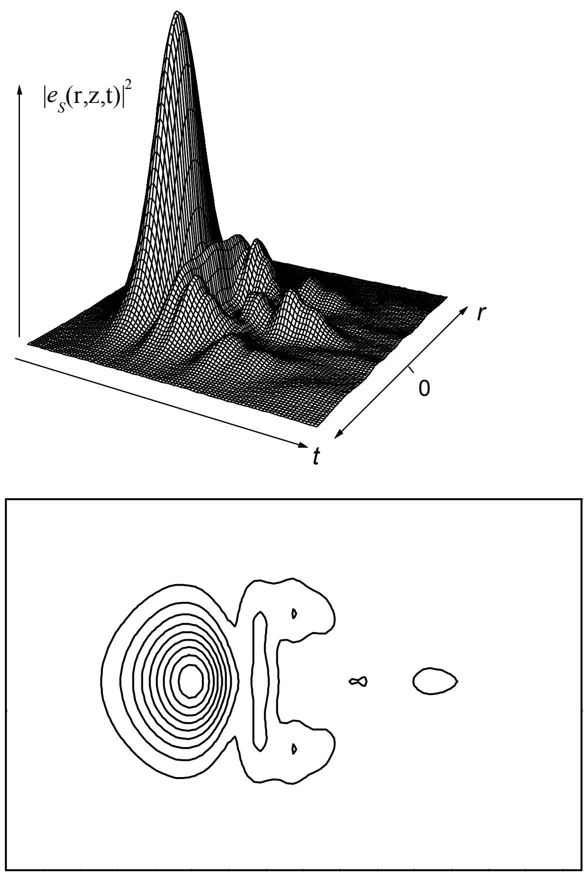

Fig.3. Spatio-temporal structure of compressed Stokes pulses at the exit of SBS-cell for the input laser pulse with energy $E_{L0} = 12$ mJ, duration $T_{L0} = 2$ ns, hypersound wave period $T_B = 0.75$ ns and its lifetime $T_R = 0.4$ ns.

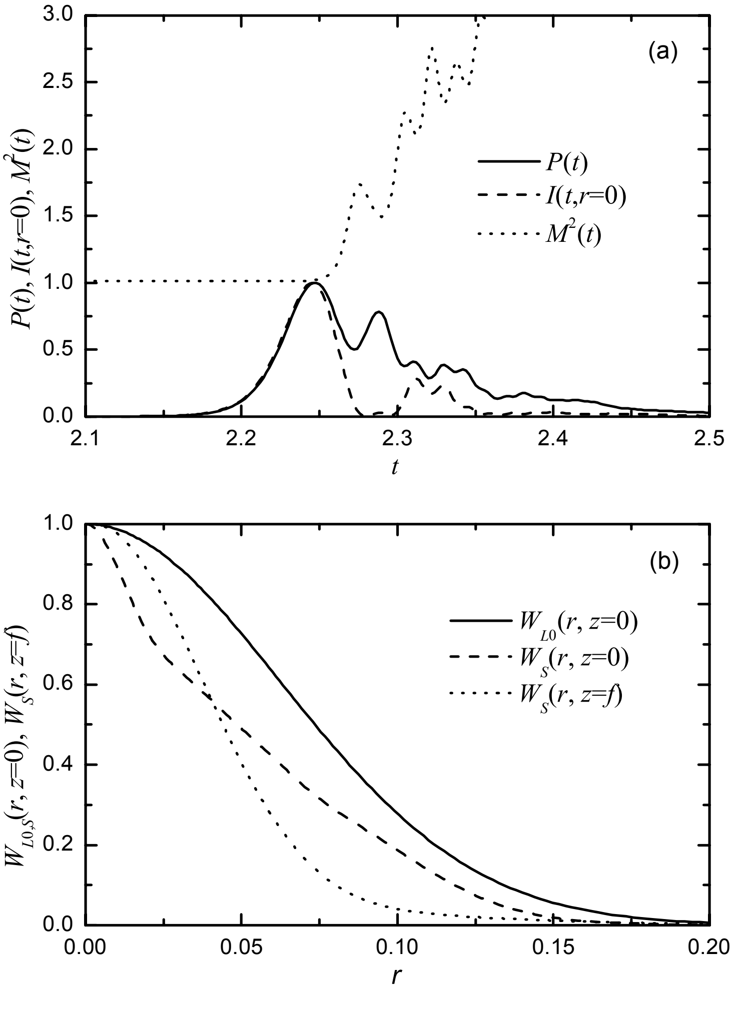

Fig.4. Temporal dependences a) of intensity on the beam axis $I(r=0,t)$, the power $P(t)$ and factor $M^2(t)$ of the Stokes pulse at the cell output and b) transverse distributions of the energy density $W_{L0}(r,z=0)$ and Stokes $W_S(r,z=0)$ pulses at the input and output of the cell and at the focal plane of the focusing lens $W_S(r,z=f)$, at the input laser pulse energy $E_{L0}=18$ mJ, the duration $T_{L0}=2$ ns and the hypersound wave period $T_B=0.75$ ns and lifetime $T_R=0.4$ ns.

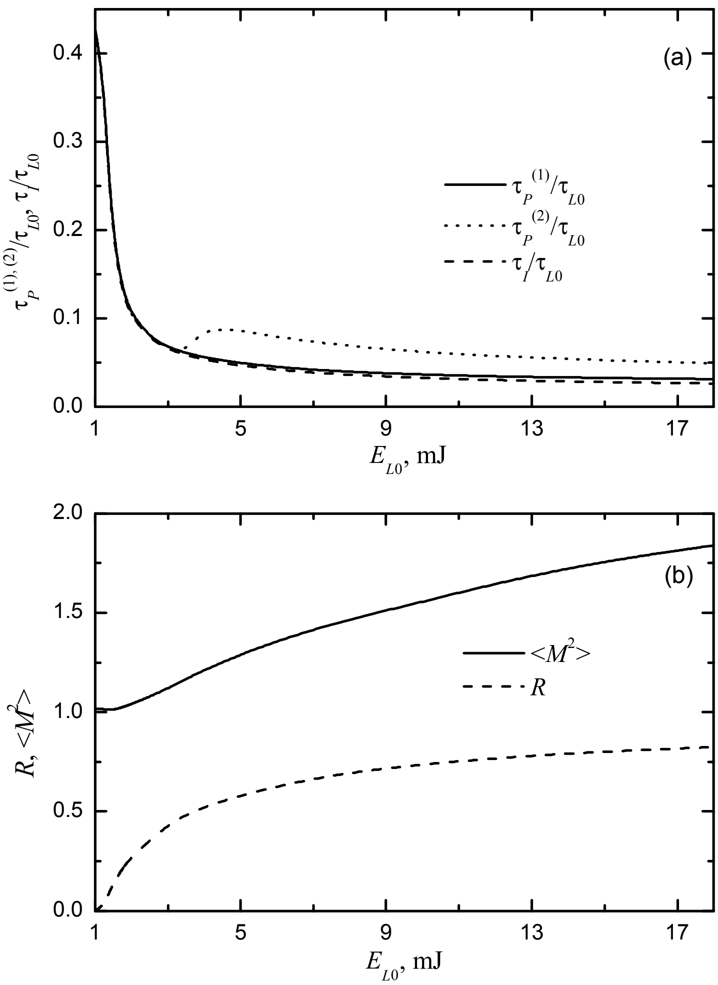

Fig.5. The dependences a) of relative durations (at half the maximum) of the compressed Stokes pulses intensity at the beam axis τ_I / τ_{L0} and by instantaneous power of the first peak only $\tau_P^{(1)} / \tau_{L0}$ and of the entire pulse $\tau_P^{(2)} / \tau_{L0}$ and b) of the averaged propagation factor $\left\langle M^2 \right\rangle$ and the energy conversion efficiency *R* on the input laser pulse energy E_{L0} at the duration $T_{L0} = 2$ ns with the hypersound wave period $T_B = 0.75$ ns and lifetime $T_R = 0.4$ ns.

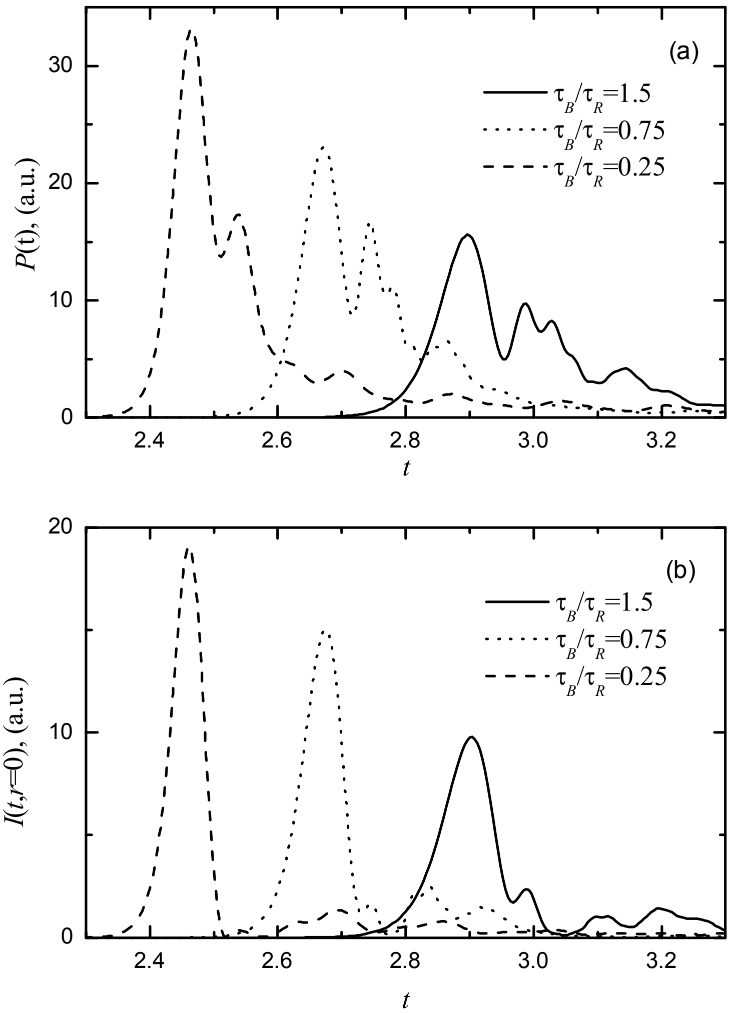

Fig.6. Temporal dependences a) of the instantaneous power $P(t)$ and b) of the intensity on the beam axis $I(t, r=0)$ of the Stokes pulse at the cell output upon the hypersound wave period T_B at the input laser pulse energy $E_{L0} = 12$ mJ, the duration $T_{L0} = 1$ ns and constant lifetime $T_R = 0.4$ ns.

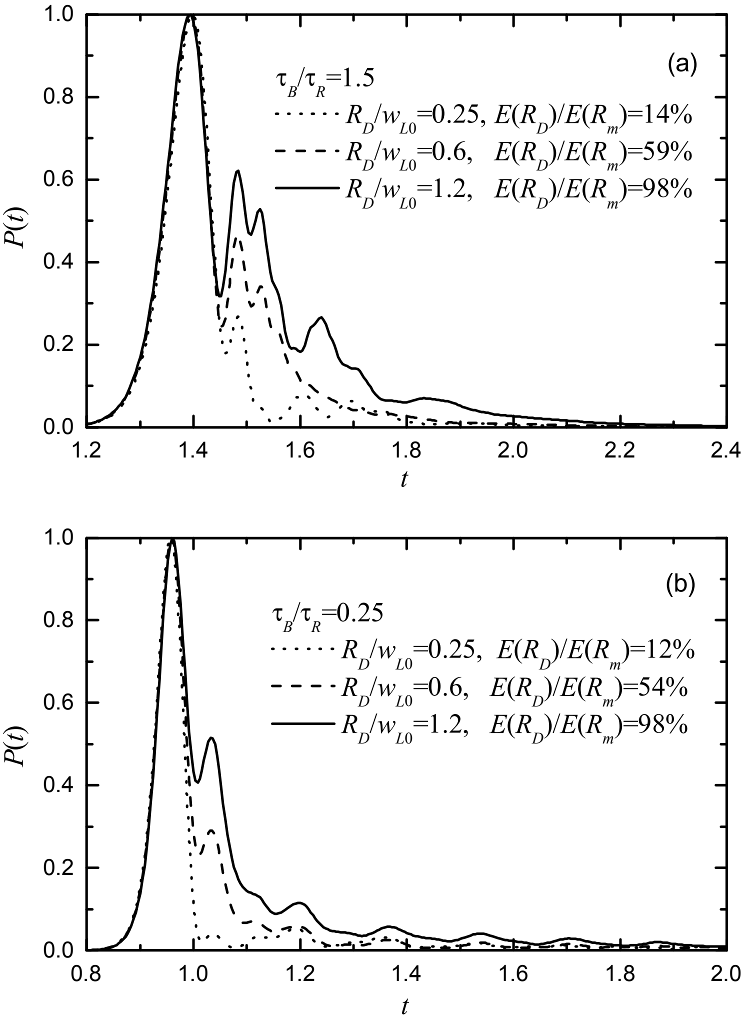

Fig.7. The envelopes of the instantaneous power $P(t)$ of Stokes pulses propagated through diaphragms of different radii and the hypersound period T_B at the input laser pulse energy $E_{L0} = 12$ mJ, the duration $T_{L0} = 1$ ns, beam waist $w_{L0} = 0.09$ cm and lifetime $T_R = 0.4$ ns.

Investigation of how the compression is influenced by Kerr nonlinearity of the refraction index of the medium has shown that the reflection coefficient changes insignificantly, but the Stokes pulse duration increases at high energies of laser pulse, although the duration at the beam axis decreases, because the most intense central part of the pulse is being focused most sharply. The quality of the central part of the pulse degrades to a large extent, therefore, the quality of the entire pulse degrades more strongly as well.

5 Conclusion

A novel algorithm is proposed in this work for numerical analysis of essentially nonstationary SBS in focused beams. The algorithm is based on a certain transformation of variables that makes it possible to apply the standard finite-difference methods also for sharply focused light beams. Utilization of the split step and the predictor-corrector methods enable to take into account the focusing, the diffraction and the nonlinearities of various physical nature of strongly nonstationary SBS in a uniform manner.

The developed calculation algorithm permitted to investigate the energetic efficiency of the SBS compression process of short laser pulses in case when their initial duration is comparable by an order of magnitude with the lifetimes and the oscillation periods of the hypersound phonons. The presented results demonstrate that separation of the central near-axial part of the Stokes pulses makes it possible to achieve the durations of the Stokes pulses practically in the picosecond range and the compression rate exceeding 30 times.

It should be noted that, in case when the threshold is significantly exceeded, also the overall pulse duration $\tau_P^{(2)}$ is not limited in the focused beams by the half-period of the hypersound as it is in case of the plane-wave approximation [18]. However, in this case, the quality of the compressed pulses degrades significantly (up to $\langle M^2(t) \rangle \sim 2$).

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