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NEURAL NETWORK CHAOS ANALYSIS

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Abstract

The analysis of a chaos theory, in general, and a neural network chaos paradigm with a concrete interpretation by a simple neural network, in particular, allowed us to set up a new aspect in an artificial neuronal approach of methodology: an analogy between natural chaos experimentally observed in neural systems of the brain and artificial neural network chaos phenomena has been considered. The significance of asymmetry and nonlinearity, which were increased on introducing a restricted N-shaped synaptical relation in a two-mod dynamic model, is emphasized.

There are illustrated the different computational examples of the neural network properties which are expressed by the equillibrium point, stable cycle or chaotic behaviour in strong nonlinear neural systems.

Key words: Neural network, dynamics, chaos theory, chaotic computer, equillibrium state, symmetry, asymmetry.

INTRODUCTION

A neural network chaotic phenomenon is strong connected with better understanding of signal and pattern recognition under noise conditions, the information transform and transmission. The amount of information transmission in chaotic dynamic systems was calculated by Matsumoto and Tsuda [15], [16]. There were proved that a chaotic neural network has an ability of an effective transmission of any information gone from outside. Parisi [19] investigated a problem of discrimination of correct retrieval states and spurious ones in chaotic asymmetric neural networks: the correct states are the time independent ones; the spurious states are the time dependent chaotic ones. Though this hypothesis is not proved yet. Lewenstein and Nowak [13] established that a correct retrieval states appear when they are excited by a small signal, then it is cooled more. When the exitation is frequent, that is the network is heated, the chaotic states appear. Fukai and

Shiino [4] discovered a chaos route via Hoph bifurcations in the neural network model with time delay. The asymmetric neural networks in terms of a mean field theory found chaos through an appearance of homoclinic orbits [25]. The nonlinear dynamics and chaos theory is presented in [1]. Most of the techniques that have been developed so far can be only applied in relatively simple systems. The most of authors recognise that artificial neural networks are a powerful pattern recognition implementation.

Chaos plays an important role in the human brain cognitive functions related to the memory processes. Different versions of self-organizational hypothetical computer are known, such as a synergetic computer by Haken [8], resonance neurocomputers by Grossberg [7], Kryukov [12], the holonic computer by Shimizu [23)], chaotic computers: the chaotic cognitive map by Massyoshi and Nagayoshi [14], the chaotic memory by Nicolis [18], chaotic information processing and chaotic neural networks by Tsuda [25], Riedel et al.[21]. All of them try to use dynamics theory less or more in nonlinear and far from equillibrium states of physical systems as an certain mechanism of explaining brain information processing of higher animals.

According to Tsuda [25], chaos as a irregular motion of deterministic dynamics may have five functional cases in the human brain memory processes. We pay attention to the second, related to the intermediate-term memory, and the fourth and the fifth ones, devoted to the search process and for storage of a new memory, respectively.

The idea of chaos expressed as a parsimonious biological processor is, to our mind, very fruitful. The biological organism surrounded by variable exterior conditions has to achieve two opposite situations: to grow the information processing ability when the processor randomly updates phase space in contrast to the accuracy of modelling, and to diminish the processing ability if the processor updates only cycles or separate points in phase space returning the accuracy of modelling. It is supposed that the parsimonious biological processor based on chaos can match these two situations.

In dynamic systems, chaos is characterised by many factors. One of them is a strange attractor. The second one is based on a pseudo-orbit tracing property which under appropriate conditions can become an approximation of some true orbit with sufficient accuracy. The strange attractor which is of non-uniform character stems from dynamic intermittent systems. The crucial property of intermittent chaotic behaviour is interchange between two frequency mods: high and low ones. Below we present the model which has a slow equation (low frequency) and a fast equation (high frequency).

SYMMETRIC AND ASYMMETRIC CONDITIONS

We discuss the conditions of nonlinear neural network stability under circumstance of symmetric and asymmetric weight matrices. The main neural network (NN) equations representing a change of NN states in dynamics are Cohen and Grossberg [3] equations for short-time memory (STM) (with some generalisation of the Hebb law to underline the stability problem):

- Cohen-Grossberg differential equations

$$\frac{\mathrm{dx_i}}{\mathrm{dt}} = a(x_i)[b(x_i) - \sum_{j=1}^{N} \mathrm{w_{ij}} g_j(x_j)], \quad i = 1, 2, ..., N \tag{1}$$

- Delta equations with the generalised Hebb law [34]

$$\frac{\mathrm{dw_{ij}}}{\mathrm{dt}} = [-D_{\mathrm{w}} \mathrm{w_{ij}} + \eta g_i(x_i) g_j(x_j)] F(g_i(x_i) g_j(x_j)), \quad i, j = 1, 2, ..., N \tag{2}$$

- Global Lyapunov function

$$L(x) = 1/2 \sum_{j,i=1}^{N} \mathrm{w_{ji}} g_j(x_j) g_i(x_i) - \sum_{i=1}^{N} \int_0^{x_i} b_i(\gamma_i) g_i^{'}(\gamma_i) d\gamma_i, \tag{3}$$

where $a(x_i)$ is positive apart from the initial condition (in (3) it is constant and equals to one), $b(x_i)$ is with the opposite sign as x_i, $\mathrm{w_{ji}}$ are the weights of onelayer NN, D_{w} is the decay constant, η is the learning parameter, $g_j(x_j)$ is the inhibitory feedback function which is a monotone nondescreasing one, N is the number of neurons in Hopfield NN [10].

The function $F(\star)$ in (2), i.e. the generalised Hebb law, is derived to improve the learning, and it is a threshold function such that $F(\star) = 0$ if $x \leq \Gamma^-$ and $F(\star)^{'} > 0$ if $x > \Gamma^-$, where Γ^- is the threshold.

Cohen and Grossberg [3] proved the existance of a global pattern formation property and its absolute stability. But the absolute stability is possible only when matrix W with elements $\mathrm{w_{ji}}$ is positive. And then, the competetive neural network converges with probability one [9]. In the case the sign of $\mathrm{w_{ji}}$ elements is arbitrary and matrix W is symmetric, the Lyapunov function (2) exists, and the absolute stability was proved by Lyapunov direct method [3]. We show below the results of simulation confirming these symmetric theses.

In neurophysiological reality, the synapses are known to be more frequently asymmetric. On the average a neuron is connected with all other neurons by the relation approxometely 10^{-6}. Different theoretical aspects of asymmetric neural neworks have been investigated by many artificial intelligence scientists. Parisi [19] emphasizes that the asymmetric NN behaviuor is much more complicated even at zero temperature (without inside noise). He also noticed that, in the cases where the oscilations are caused, the length of cycles can be very great and the route to the chaotic behaviuor is possible.

Asymmetric couplings $\mathrm{w_{jk}} \neq \mathrm{w_{jk}}$ in neural networks have been studied using the generalised Hebb law or excluding (diluting) some connections in a direct or probabilistic way. Asymmetric and/or diluted versions based on a probability were examined in [2], [5], [6], [20], [24]. Almost all the authors have arrived at the conclusion that the spin-glass is destroyed due to essential extent of asymmetry. Although in [11] at specific presentation of asymmetric couplings, spin- glass phase in ferromagnetic field theory was found. And once again in [20] it is supposed that symmetric neural networks can not provide dynamic association, i.e., they lack the ability to retrieve series of patterns at a single recalling input pattern. In order to achieve this, asymmetric couplings in neural networks are needed.

SIMULATION OF CHAOTIC PHENOMENA IN NEURAL NETWORKS

We have performed the simulation of two type mathematical models with a chaotic neural network paradigm. The first is based on two frequency mods [26] but it was essentially modified introducing the generalised Hebb law and more realistic N-shaped synaptic couplings. The second is based upon a dynamic map in a discrete time variable like a simple logistic equation [1].

The first model is organised in the following way. The Cohen-Grossberg equations (2), after simplifying and introducing of a nonlinear N-shaped postsynaptic potential function, become fast (high frequency) equations discretised in time

$$x_i(t+1) = (1 - D_x)x_i(t) + Eg_i(x_i(t)), \tag{4}$$

where

$$g_i(x_i(t)) = \sum_{j=1}^{N} \mathrm{w_{ij}} \rho_j(x_j(t)) + I_i(t), \tag{4a}$$

D_x is the decay parameter of potentials, E is the excitatory rate, $I_i(t)$ is the external input of the ith neuron, $\rho_j(x_j(t))$ is the N-shaped synaptic function further simplified as $\rho(x)$ because of its independence of time and neuron number.

Since an increase in depolarised current and a descrease in the polarised one are saturated in natural dendrite, we introduced artificial restrictions. So we have such a piece-wise polynomial approximation

$$Y = \rho(x) = [x^3 - c_1 x^2 + c_1 x(1 - c_2^2)][c_1(1 - c_2^2)]^{-1} \tag{5}$$

under two types of conditions:

a)

$$Y = \begin{cases} \rho(x) & \text{if} \quad -a < Y < a \\ -a & \text{if} \quad Y \leq -a \\ a & \text{if} \quad Y \geq a, \end{cases} \tag{5a}$$

b)

$$Y = \begin{cases} \rho(x) & \text{if} \quad \mid Y \mid \leq S(x) \\ S(x) & \text{if} \quad \mid Y \mid > S(x), \end{cases} \tag{5b}$$

where c_1 is a coefficent (3.0), c_2 is a constant (0.65), a is positive value (we took 1.0), $S(x)$ is a bipolar sigmoidal function (we took tanh). Such generalised approximation as a restricted N-shaped current-voltage dendritc membrane relation is presented in Fig. 1.

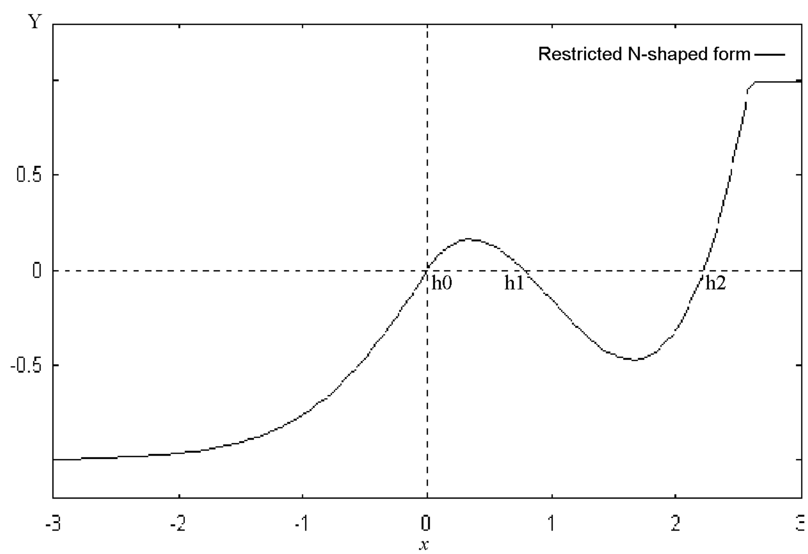

Fig. 1. Restricted N-shaped current-voltage dendritc membrane relation.

The synaptic function $\rho(x)$ reflects the mutual activity between presynaptic and postsynaptic potentials as a result of complex synapse-dendrite activation.

This activation, in most cases, is defined by a restricted N-shaped current-voltage dendritic membrane relation [5] that possesses two stable (Fig. 1, h_0 and h_2 points) and one unstable (Fig. 1, h_1 point) points.

In addition, we would like to note that the production $\mathrm{w_{ij}}\rho_j(x_j(t))$ in (4a) was used for two goals: one is as more realistic to neurophysiological situation described above by (5, 5a or 5b) at $\mathrm{w_{ij}} = 1$, and the other for incorporation of thresholds to improve the neural network learning [17].

Delta equations (2) considered as slow (low frequency) ones were presented in such a way

$$\mathrm{w_{ij}}(\mathrm{k}+1) = \left[(1 - D_{\mathrm{w}})\mathrm{w_{ij}}(\mathrm{k}) + \eta x_i(k)x_j(k)\right] F(x_i(k), x_j(k)), \qquad (6)$$

where k is a recursion step.

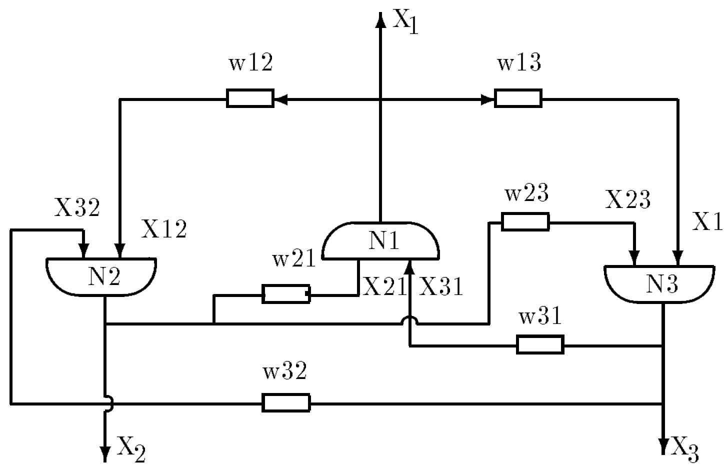

Fig. 2. Neural network with three neuronal elements.

The first modelling experiment has been made on simple cases of a NN architecture: three and five elements of neuronal schema with different nonlinear functions of synaptic connections and a function of neuron activity. The weights among neurons maped by N-shaped current- voltage relation (5) with restrictions (5a) or (5b) were changed to achieve equillibrium or non-equillibrium states in a defined neuronal structure. The model and its developing realisation based on two-mod discrete nonlinear equations (4), (6) were carried out under the origin conditions $I_i(1, -1, 1), \quad i = 1, 2, 3$ and $I_i(1, -1, -1, 1, -1), \quad i = 1, 2, ..., 5$ for three and five neurons with deterministic presentation of I_i as well as synaptic weights. The neuronal schema with three neurons is presented in Fig.2. Here the weights are shown in a simplified form.

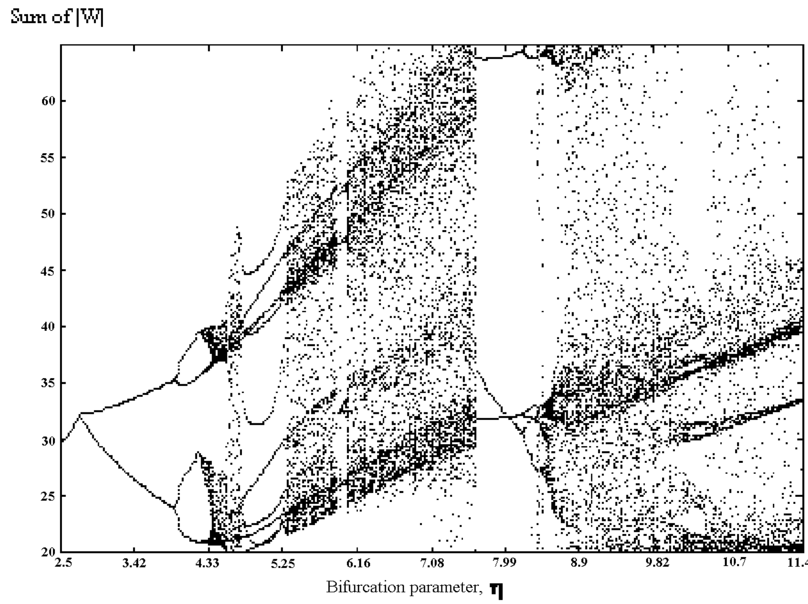

Fig. 3. Bifurcation and chaos diagram for NN with a unipolar sigmoid activity function.There are $\Delta x = 0.1$ and $\Delta \eta = 0.015$.

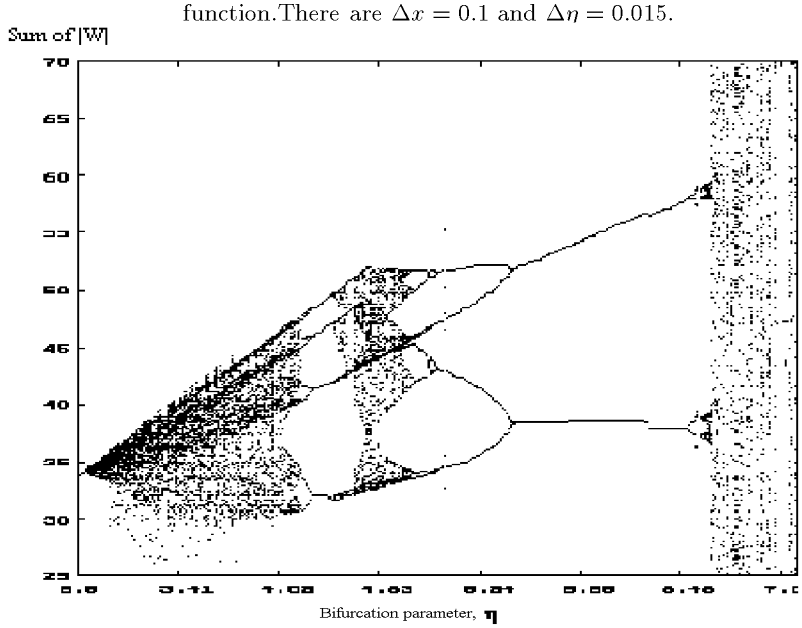

Fig. 4. Bifurcation and chaos diagram for NN with a restricted N-shaped relation.There are $\Delta x = 0.01$ and $\Delta \eta = 0.0199$.

The organisation of iterative calculations has been carried out similarly as in [26]. That is, at the beginning fast equations (10) were simulated in dynamics until the transient process was stabilised then the recursive equations (12) were modelled. Iterative cycles continue the absolute summarised weight ($| W |$) value becomes almost stable. After that a fixed amount (195-250) of iterations is calculated to find the absolute sum of weights versus the learning rate η as a bifurcation parameter in presenting a model.

The evolution of dynamic chaotic processes is well illustrated in Fig. 3-6. As we see it is very surprising and complicated. Even in most simplified case, wherein the weights are presented as only multipliers and the function of neuronal activity is taken as a sigmoidal shape ranging between thresholds. Fig. 3 shows that at the beginning up to $\eta = 2.7$ the evolution process converges to absolute stable points. At $\eta = 2.7$ the bifurcation of solution is caused. For higher values of η a cycled regime and period-doubling processes were reached. Since $\eta = 4.4$ a chaotic phenomenon is cleared at the beginning in a narrow area, then after very complex cycling window a wider area and at more than $\eta = 8.0$ the absolute chaotic regime emerges.

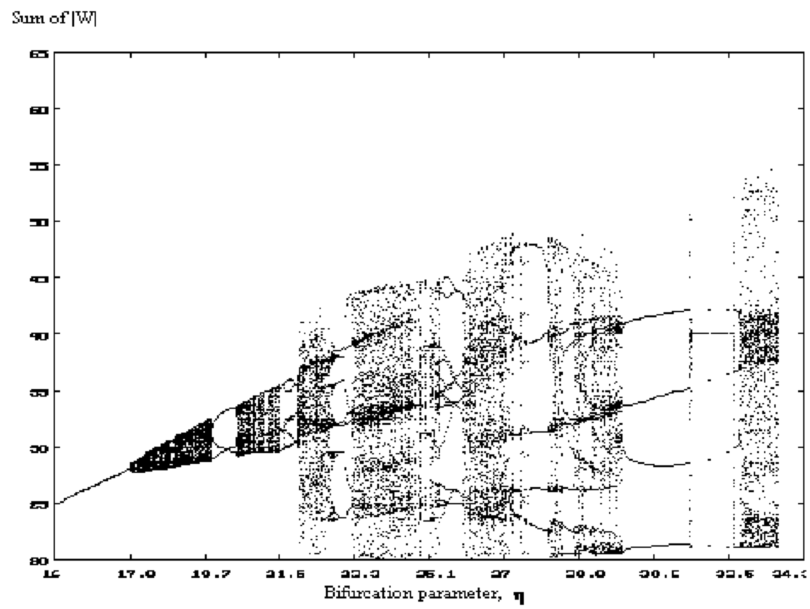

Fig. 5. Bifurcation and chaos diagram for NN with two nonlinear functions: unipolar sigmoidal of a neuron and N-shaped for synapting coupling. There are $\Delta x = 0.0001$ and $\Delta\eta = 0.03$.

Another development of neuronal dynamic processes is presented in Fig. 4. Here due to the N-shaped relation with limitations in the origin a chaotic process appears, then the periodic regime and descreasing period-doubling processes occur.

The diagram in Fig. 4 shows that the range from $\eta = 5.5$ to $\eta = 6.5$ is the area of the two equillibrium points. And only after $\eta = 6.5$ the period-doubling regime appears and absolute chaotic behaviour is arisen.

The results of modelling in more complicated situations are demonstrated in the diagrams in Fig. 5 and 6. Two nonlinear functions are given: one as unipolar sigmoidal (Fig. 5) or bipolar one (Fig. 6) and the second as weight multiplied by N-shaped function. The modelling was caried out with the same parameters as in above. It should be noted: first, the intermittent stable, period- doubling, unstable, and chaotic processes change place from the range with lower values of the bifurcation parameter ($\eta = 2.0 \div 10.0$) to the range with higher ones ($\eta = 12.0 \div 34.0$); second, the processes become richer and more complex. The diagram (Fig. 6) shows that the processes do not possess explicitly distinct period- doubling behaviour but they possess very many windows with stable point areas at η=20.0, 23.0, 25.5, 27.5 and wide stable areas in the range from 30.0 to 32.5. Only at higher values of $\eta = 32.53$ the chaotic regime is continued uninterrupted.

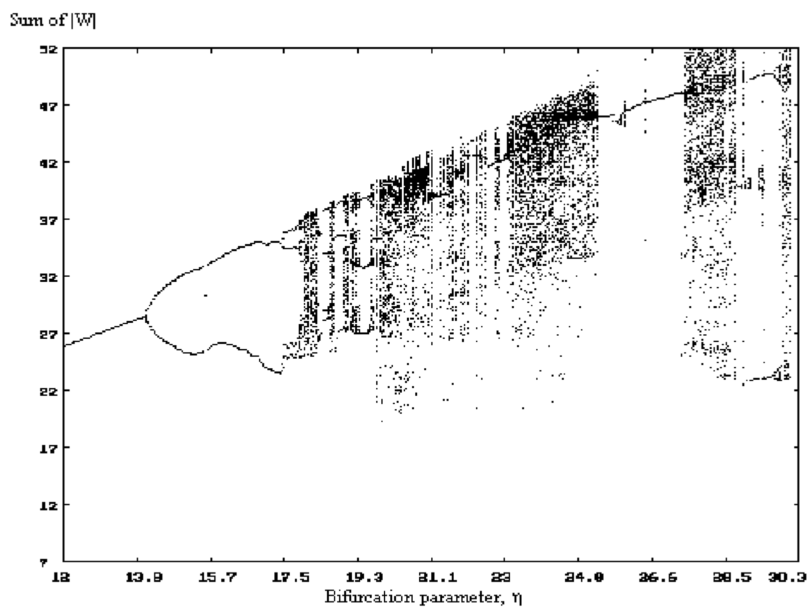

Fig. 6. Bifurcation and chaos diagram for NN with two nonlinear functions:bipolar sigmoidal of a neuron and N-shaped, for synapting coupling. There are $\Delta x = 0.0001$ and $\Delta \eta = 0.03$.

The evolution of dynamic processes, given the bipolar sigmoidal activity function as a hyperbolic tangent and the N-shaped form in neuronal couplings, become less distinct and more chaotic in the general sense. That is shown in the diagram of Fig. 6. Note that for five- element neural network architecture, similar complex processes of solutions take place only they are a little simpler. It is most likely that the more massive neural (and not only neural) system, the more crucial mutual compensation mechanism in competetive systems occurs, the less disorder behaviour survives. Certainly this hypothesis must to be proved.

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INFLUENCE OF SYMMETRY AND ASYMMETRY IN NEURAL NETWORK CHAOS

The second direction of examination of the neural network chaotic paradigm is connected with a simplified mathematical description of NN refusing of two types of equations (4) and (6), the absolute sum of weights as a criterion and complex nonlinear synapse-dendrite couplings among neurons.

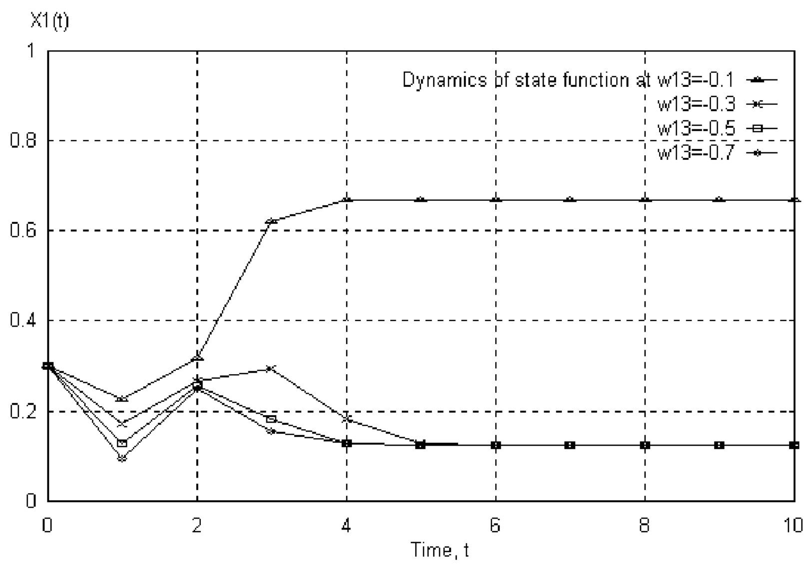

Fig. 7. Dynamics of the state functions $x(t)$ for a symmetric NN versus bifurcation parameters w13 and time, t: (a) stable excited, for $\mathrm{w13} = -0.1$; (b) stable inhibited, for $\mathrm{w13} = -0.3, -0.5, -0.7$.

In order to prove once again that symmetric couplings in a NN cannot degenerate into cyclic or more chaotic behaviour, we took the NN presented in Fig. 2

and simulated in a fixed point dynamics [22].

Diferrent types of neuronal activity functions are given such as simple unipolar sigmoidal, bipolar as a hyperbolic tangent, forsed sigmoidal, even restricted functions $x_1(t)$, $x_2(t)$ and $x_3(t)$ with different bifurcation parameters, weights between the first and the third neurons w_{13} when the matrix W with zero diagonal elements was always stable limited to the stable point position. The dynamics of the stable function $x_1(t)$ in the rate of the bifurcation parameter w_{13} from -0.1 to -0.7 is shown in Fig. 7. The initial condition $x_1(0) = 0.3$ was given. The trajectories were devided into two classes: stable exited, for $\mathrm{w}_{13} = -0.1$, otherwise, stable inhibited.

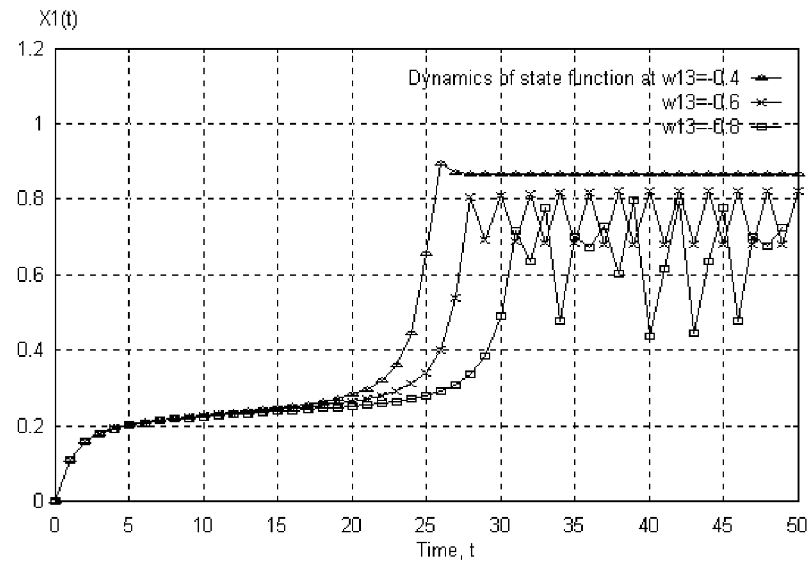

Fig. 8. Dynamics of the state functions $x(t)$ versus t for bifurcation parameters: (a) equillibrium point, for w13 $= -0.4$; (b) stable cycle, for w13 $= -0.6$; (c) chaotic behaviour, for w13 $= -0.8$.

In the asymmetric case of NN, where elements of matrix W $\mathrm{w}_{\mathrm{ij}} \neq \mathrm{w}_{\mathrm{ji}}$, the evolution of the dynamic state with competetive influence is shown in Fig. 8. When the weight w_{21} is positive, the excitatory activation of neuron X_1 is appeared, and when w_{31} is negative, the inhibitory activation is taken place. Here all the trajectories are growing by the exponential law at the beginning of time (up to 5 units of time) then the temporary stable states (plateau at 5–20 units of time) are observed, later the different trajectories appear. The stable point is reached at $\mathrm{w}_{13} = -0.4$ which is the adjusted parameter, the stable cycling – at $\mathrm{w}_{13} = -0.64$ and the chaotic behaviour occurs at $\mathrm{w}_{13} = -0.7$.

Around stable point areas nonmonotonic iteration maps are given in the Fig. 9.

The first stable point area is in the range from 0.07 to 0.2 of $x(t)$ for $w_{13} = -0.4$ and $w_{13} = -0.6$, where $x(t+1) = x(t)$, i.e., the curves intersect the first bisecant. The second area is near $x(t) = 0.85$, where the stable point (Fig. 9. Squares with points) and the stable cycle (Fig. 9. Crosses) as intermittent behaviour occur. The third area is near $x(t) = 0.47$, where the intersection curve slope with the first bisecant is larger than one, which means the existence of unstable (chaotic) dynamics.

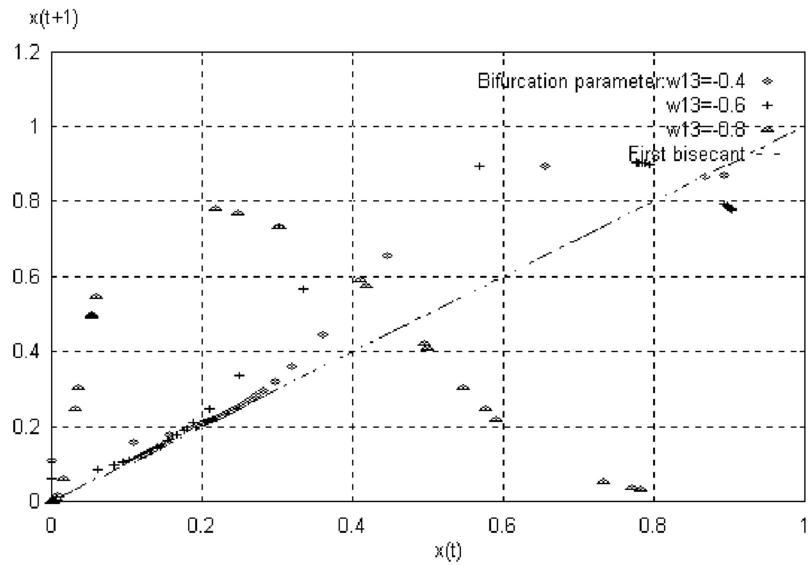

Fig. 9. Nonmonotone iteration maps, for w13 = −0.4, −0.6, −0.8 and the first bisecant.

Thus, an attempt of a better understanding of chaos in general and neural network chaos properties such as symmetry, asymmetry, stronger nonlinearity in particular is emphasized. The simulation of chaotic phenomena in neural networks at different functions of neuronal activity and synaptic couplings in dynamics with a reflection into phase space showed how polygonal this problem is in a reality.

Search of analogy between natural neural network chaos phenomena and some aspect of similar one in applied systems was fruitfull but not so what one was desired. It remains as a stimulus for future investigations.

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