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Lie symmetry analysis, conservation laws and analytical solutions for chiral nonlinear Schrödinger equation in $(2+1)$-dimensions*

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Abstract. In this work, we consider the chiral nonlinear Schrödinger equation in $(2+1)$-dimensions, which describes the envelope of amplitude in many physical media. We employ the Lie symmetry analysis method to study the vector field and the optimal system of the equation. The similarity reductions are analyzed by considering the optimal system. Furthermore, we find the power series solution of the equation with convergence analysis. Based on a new conservation law, we construct the conservation laws of the equation by using the resulting symmetries.

Keywords: the chiral nonlinear Schrödinger equation in $(2+1)$-dimensions, Lie symmetry analysis, symmetry reductions, the power series solutions, conservation laws.

1 Introduction

The nonlinear Schrödinger equation, which plays a very important role in nonlinear evolution equations (NLEEs), has been fully applied in many phenomena, such as fluid dynamics, nonlinear fiber, molecular biology, quantum mechanics, deep water modeling, etc. [9, 10, 30]. Up to now, finding for the exact solution of NLEEs still plays a very important role in the study dynamics of nonlinear phenomena. In the last few decades, the exact solutions of NLEEs have been extensively studied. The main methods used are Darboux transformation, the inverse scattering method, Hirota bilinear method, Lie

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symmetry group method [1, 4, 11, 19, 23, 26]. Among them, the Lie symmetry group method can simultaneously obtain the symmetry, exact solutions and conservation laws of NLEEs through some effective calculations [3, 7, 12, 16, 29, 38–45, 49].

In the past few decades, the conservation laws has played an increasingly important role in the research of NLEEs. At the same time, various methods for solving conservation law of the NLEE are also produced, such as Noether's theorem, characteristic method, variational approach, conservation theorem [5, 8, 15, 24, 25, 48, 50], etc. The famous Noether's theorem establishes the connection between symmetries of NLEEs and conservation laws. But the disadvantage of the Noether's theorem is that it is not suitable for solving NLEE without Lagrangian. In order to solve this NLEE without Lagrangian, Ibragimov entered a new method for solving the conservation law in 2007. This new method relies on the notion of Lie symmetry generators, the adjoint equation and formal Lagrangians of NLEEs. Therefore, this new conservation law will also play an important role in solving the conservation laws of NLEEs.

In this work, we mainly study the chiral nonlinear Schrödinger (NLS) equation in $(2+1)$-dimensions

$$\mathrm{i}q_t + a(q_{xx} + q_{yy}) + \mathrm{i}\big[b_1(qq_x^* - q^*q_x) + b_2(qq_y^* - q^*q_y)\big]q = 0, \tag{1}$$

where $q = q(x, y, t)$, a means the coefficient of dispersion term, and b_1, b_2 are the coefficients of nonlinear coupling terms. In [6], the bright and dark soliton solution of the chiral NLS equation in $(2+1)$-dimensions (1) is obtained using the constant coefficient method. In [14], the singular periodic solution of the chiral NLS equation in $(2+1)$-dimensions (1) is obtained by using the trial solution method. As we all know, the Lie symmetry and conservation laws of equation (1) have not been studied. Therefore, in this work, we will mainly study the Lie symmetry and conservation laws of equation (1).

The rest of the paper is structured as follows. In Section 2, we first transform equation (1) into a form of equations, and then vector field and optimal system are constructed by using the Lie symmetry analysis method. In Section 3, the symmetry reductions of equation (1) is obtained by using the optimal system. In Section 4, we obtain the power series solution of the system by using the power series method. In Section 5, the conservation law of the equation is obtained by the new conservation law. In Section 6, we give some summaries and discussions.

2 Lie symmetries analysis

In this section, Lie symmetry analysis will be performed on the chiral NLS equation in $(2+1)$-dimensions (1). Firstly, we consider the complex-valued function $q(x, y, t)$ in the following form:

$$q(x, y, t) = u(x, y, t) + \mathrm{i}v(x, y, t), \tag{2}$$

where $u(x, y, t)$ and $v(x, y, t)$ are real-valued functions, and q^* represents the conjugate of q. Substituting equation (2) into equation (1) and equating the real and imaginary parts, we can obtain

$$
\begin{aligned}
&u_t + a(v_{xx} + v_{yy}) + 2b_1\big(uvv_x - v^2u_x\big) + 2b_2(uvv_y - v^2u_y) = 0,\\
&-v_t + a(u_{xx} + u_{yy}) + 2b_1\big(u^2v_x - uvu_x\big) + 2b_2\big(u^2v_y - uvu_y\big) = 0.
\end{aligned} \tag{3}
$$

To construct the point symmetry of equation (1), we first introduce a Lie group with a one-parameter Lie transformation group

$$
\begin{aligned}
x &\to x + \varepsilon\xi^1(x,y,t,u,v) + O\big(\varepsilon^2\big),\\
y &\to y + \varepsilon\xi^2(x,y,t,u,v) + O\big(\varepsilon^2\big),\\
t &\to t + \varepsilon\xi^3(x,y,t,u,v) + O\big(\varepsilon^2\big),\\
u &\to u + \varepsilon\eta^1(x,y,t,u,v) + O\big(\varepsilon^2\big),\\
v &\to v + \varepsilon\eta^2(x,y,t,u,v) + O\big(\varepsilon^2\big),
\end{aligned}
$$

where $\varepsilon \ll 1$ means a group parameter, and ξ^1, ξ^2, ξ^3, η^1 and η^2 are the infinitesimal generators. The vector field corresponding to the above group of transformation as

$$
\begin{aligned}
V = {}&\xi^1(x,y,t,u,v)\frac{\partial}{\partial x} + \xi^2(x,y,t,u,v)\frac{\partial}{\partial y} + \xi^3(x,y,t,u,v)\frac{\partial}{\partial t}\\
&+ \eta^1(x,y,t,u,v)\frac{\partial}{\partial u} + \eta^2(x,y,t,u,v)\frac{\partial}{\partial v},
\end{aligned} \tag{4}
$$

where $\xi^1(x,y,t,u,v)$, $\xi^2(x,y,t,u,v)$, $\xi^3(x,y,t,u,v)$, $\eta^1(x,y,t,u,v)$ and $\eta^2(x,y,t,u,v)$ are functions of coefficient to be determined. For system (3), pr^2 will be the second prolongation, then its invariance condition is

$$
\mathrm{pr}^2 V(\varDelta_1)\big|_{\varDelta_1=0} = 0, \qquad \mathrm{pr}^2 V(\varDelta_2)\big|_{\varDelta_2=0} = 0, \tag{5}
$$

where

$$
\begin{aligned}
\varDelta_1 &= u_t + a(v_{xx} + v_{yy}) + 2b_1\big(uvv_x - v^2u_x\big) + 2b_2\big(uvv_y - v^2u_y\big),\\
\varDelta_2 &= -v_t + a(u_{xx} + u_{yy}) + 2b_1\big(u^2v_x - uvu_x\big) + 2b_2\big(u^2v_y - uvu_y\big).
\end{aligned}
$$

On the basis of Lie's theory, the second prolongation of equation (4) can be written as

$$
\begin{aligned}
\mathrm{pr}^2 V = {}&\eta^1\frac{\partial}{\partial u} + \eta^2\frac{\partial}{\partial v} + \eta^{1t}\frac{\partial}{\partial u_t} + \eta^{1x}\frac{\partial}{\partial u_x} + \eta^{1y}\frac{\partial}{\partial u_y}\\
&+ \eta^{2x}\frac{\partial}{\partial v_x} + \eta^{2y}\frac{\partial}{\partial v_y} + \eta^{2xx}\frac{\partial}{\partial v_{xx}} + \eta^{2yy}\frac{\partial}{\partial v_{yy}},\\
\mathrm{pr}^2 V = {}&\eta^1\frac{\partial}{\partial u} + \eta^2\frac{\partial}{\partial v} + \eta^{2t}\frac{\partial}{\partial v_t} + \eta^{1x}\frac{\partial}{\partial u_x} + \eta^{1y}\frac{\partial}{\partial u_y}\\
&+ \eta^{2x}\frac{\partial}{\partial v_x} + \eta^{2y}\frac{\partial}{\partial v_y} + \eta^{1xx}\frac{\partial}{\partial u_{xx}} + \eta^{1yy}\frac{\partial}{\partial u_{yy}},
\end{aligned}
$$

where the coefficient functions are shown as

$$\begin{aligned}
\eta^{1x} &= D_x\big(\eta^1 - \xi^1 u_x - \xi^2 u_y - \xi^3 u_t\big) + \xi^1 u_{xx} + \xi^2 u_{xy} + \xi^3 u_{xt},\\
\eta^{1y} &= D_y\big(\eta^1 - \xi^1 u_x - \xi^2 u_y - \xi^3 u_t\big) + \xi^1 u_{xy} + \xi^2 u_{yy} + \xi^3 u_{yt},\\
\eta^{1t} &= D_t\big(\eta^1 - \xi^1 u_x - \xi^2 u_y - \xi^3 u_t\big) + \xi^1 u_{xt} + \xi^2 u_{yt} + \xi^3 u_{tt},\\
\eta^{2x} &= D_x\big(\eta^1 - \xi^1 v_x - \xi^2 v_y - \xi^3 v_t\big) + \xi^1 v_{xx} + \xi^2 v_{xy} + \xi^3 v_{xt},\\
\eta^{2y} &= D_y\big(\eta^1 - \xi^1 v_x - \xi^2 v_y - \xi^3 v_t\big) + \xi^1 v_{xy} + \xi^2 v_{yy} + \xi^3 v_{yt},\\
\eta^{2t} &= D_t\big(\eta^1 - \xi^1 v_x - \xi^2 v_y - \xi^3 v_t\big) + \xi^1 v_{xt} + \xi^2 v_{yt} + \xi^3 v_{tt},\\
\eta^{1xx} &= D_{xx}\big(\eta^1 - \xi^1 u_x - \xi^2 u_y - \xi^3 u_t\big) + \xi^1 u_{xxx} + \xi^2 u_{xxy} + \xi^3 u_{xxt},\\
\eta^{1yy} &= D_{yy}\big(\eta^1 - \xi^1 u_x - \xi^2 u_y - \xi^3 u_t\big) + \xi^1 u_{xyy} + \xi^2 u_{yyy} + \xi^3 u_{yyt},\\
\eta^{2xx} &= D_{xx}\big(\eta^1 - \xi^1 v_x - \xi^2 v_y - \xi^3 v_t\big) + \xi^1 v_{xxx} + \xi^2 v_{xxy} + \xi^3 v_{xxt},\\
\eta^{2yy} &= D_{yy}\big(\eta^1 - \xi^1 v_x - \xi^2 v_y - \xi^3 v_t\big) + \xi^1 v_{xyy} + \xi^2 v_{yyy} + \xi^3 v_{yyt}.
\end{aligned} \tag{6}$$

Combining (5) and (6), we can get an equivalent condition of (5) as

$$\begin{aligned}
&(2b_1 v v_x + 2b_2 v v_y)\eta^1 + (2b_1 u v_x - 4b_1 v u_x + 2b_2 u v_y - 4b_2 v u_y)\eta^2 + \eta^{1t}\\
&\quad - 2b_1 v^2 \eta^{1x} - 2b_2 v^2 \eta^{1y} + 2b_1 u v \eta^{2x} + 2b_2 u v \eta^{2y} + a\eta^{2xx} + a\eta^{2yy} = 0,\\
&(4b_1 u v_x - 2b_1 v u_x + 4b_2 u v_y - 2b_2 v u_y)\eta^1 + (-2b_1 u u_x - 2b_2 u u_y)\eta^2 - \eta^{2t}\\
&\quad - 2b_1 u v \eta^{1x} - 2b_2 u v \eta^{1y} + 2b_1 u^2 \eta^{2x} + 2b_2 u^2 \eta^{2y} + a\eta^{1xx} + a\eta^{1yy} = 0.
\end{aligned} \tag{7}$$

Substituting (6) into (7) and then simplifying, we can get the determining equations of system (3) as

$$\begin{gathered}
\xi^3_{tt} = 0, \qquad \xi^3_u = 0, \qquad \xi^3_v = 0, \qquad \xi^3_x = 0, \qquad \xi^3_y = \frac{1}{2}\xi^1_t,\\
\xi^2_y = 0, \qquad \xi^2_u = 0, \qquad \xi^2_v = 0, \qquad \xi^2_x = \frac{1}{2}\xi^1_t, \qquad \xi^2_t = -\frac{b_2}{b_1}\xi^3_t,\\
\xi^1_u = 0 \qquad \xi^1_v = 0, \qquad \xi^1_x = 0, \qquad \xi^1_y = 0, \qquad \xi^1_{tt} = 0,\\
\eta^1_t = 0, \qquad \eta^1_x = \frac{b_2 v \xi^3_t}{2ab_1}, \qquad \eta^1_y = -\frac{v\xi^3_t}{2a}, \qquad \eta^1_u = -\frac{1}{4}\xi^1_t,\\
\eta^1_v = \frac{\xi^1_t u + 4\eta^1}{4v}, \qquad \eta^2 = \frac{1}{4v}\big(-\xi^1_t u^2 - \xi^1_t v^2 - 4\eta^1 u\big).
\end{gathered}$$

Then we can obtain a very important theorem through further calculations as follows.

Theorem 1. *The Lie algebra of infinitesimal symmetry of equation* (1) *are spanned by the following six linear independent operators*:

$$\begin{aligned}
V_1 &= \frac{\partial}{\partial t}, \qquad V_2 = \frac{\partial}{\partial x}, \qquad V_3 = \frac{\partial}{\partial y}, \qquad V_4 = v\frac{\partial}{\partial u} - u\frac{\partial}{\partial v},\\
V_5 &= \frac{1}{2}x\frac{\partial}{\partial x} + \frac{1}{2}y\frac{\partial}{\partial y} + t\frac{\partial}{\partial t} - \frac{1}{4}u\frac{\partial}{\partial u} - \frac{1}{4}v\frac{\partial}{\partial v},\\
V_6 &= t\frac{\partial}{\partial x} - \frac{b_1 t}{b_2}\frac{\partial}{\partial y} + \frac{v(b_1 y - b_2 x)}{2ab_2}\frac{\partial}{\partial u} - \frac{u(b_1 y - b_2 x)}{2ab_2}\frac{\partial}{\partial v}.
\end{aligned} \tag{8}$$

Table 1. Lie bracket of system (3).

Lie	V_1	V_2	V_3	V_4	V_5	V_6
V_1	0	0	0	0	V_1	$V_2 - \frac{b_1}{b_2}V_3$
V_2	0	0	0	0	$\frac{1}{2}V_2$	$-\frac{1}{2a}V_4$
V_3	0	0	0	0	$\frac{1}{2}V_3$	$\frac{b_1}{2ab_2}V_4$
V_4	0	0	0	0	0	0
V_5	$-V_1$	$-\frac{1}{2}V_2$	$-\frac{1}{2}V_3$	0	0	$\frac{1}{2}V_6$
V_6	$\frac{b_1}{b_2}V_3 - V_2$	$\frac{1}{2a}V_4$	$-\frac{b_1}{2ab_2}V_4$	0	$-\frac{1}{2}V_6$	0

Based on the commutator operator $[V_k, V_j] = V_kV_j - V_jV_k$, *we can get the commutator table of system* (3) (*see Table* 1).

Based on the commutator relations in Table 1, we want to get the adjoint representations of the vector fields by using the following Lie series:

$$Ad\big(\exp(\epsilon V_k)\big)V_j = V_j - \epsilon\big[V_k, V_j\big] + \frac{1}{2}\epsilon\big[V_k, [V_k, V_j]\big] - \cdots .$$

Then we have

$$Ad\big(\exp(\epsilon V_k)\big)V_k = 0, \quad k = 1,2,3,4,5,6, \qquad Ad\big(\exp(\epsilon V_1)\big)V_2 = V_2,$$
$$Ad\big(\exp(\epsilon V_1)\big)V_3 = V_3, \qquad Ad\big(\exp(\epsilon V_1)\big)V_4 = V_4, \qquad Ad\big(\exp(\epsilon V_2)\big)V_1 = V_1,$$
$$Ad\big(\exp(\epsilon V_2)\big)V_3 = V_3, \qquad Ad\big(\exp(\epsilon V_2)\big)V_4 = V_4, \qquad Ad\big(\exp(\epsilon V_3)\big)V_2 = V_2,$$
$$Ad\big(\exp(\epsilon V_3)\big)V_1 = V_1, \qquad Ad\big(\exp(\epsilon V_4)\big)V_6 = V_6, \qquad Ad\big(\exp(\epsilon V_3)\big)V_4 = V_4,$$
$$Ad\big(\exp(\epsilon V_4)\big)V_1 = V_1, \qquad Ad\big(\exp(\epsilon V_4)\big)V_2 = V_2, \qquad Ad\big(\exp(\epsilon V_4)\big)V_3 = V_3,$$
$$Ad\big(\exp(\epsilon V_4)\big)V_5 = V_5, \qquad Ad\big(\exp(\epsilon V_5)\big)V_4 = V_4, \qquad Ad\big(\exp(\epsilon V_6)\big)V_4 = V_4,$$

$$Ad\big(\exp(\epsilon V_1)\big)V_6 = V_6 - \epsilon\left(V_2 - \frac{b_1}{b_2}V_3\right), \qquad Ad\big(\exp(\epsilon V_1)\big)V_5 = V_5 - \epsilon V_1,$$
$$Ad\big(\exp(\epsilon V_6)\big)V_5 = V_5 - \frac{b_1}{2}\epsilon V_6, \qquad Ad\big(\exp(\epsilon V_6)\big)V_1 = V_1 + \epsilon\left(V_2 - \frac{b_1}{b_2}V_3\right),$$
$$Ad\big(\exp(\epsilon V_3)\big)V_5 = V_5 - \frac{1}{2}\epsilon V_3, \qquad Ad\big(\exp(\epsilon V_3)\big)V_6 = V_6 - \frac{b_1}{2ab_2}\epsilon V_4,$$
$$Ad\big(\exp(\epsilon V_2)\big)V_6 = V_6 + \frac{1}{2a}\epsilon V_4, \qquad Ad\big(\exp(\epsilon V_2)\big)V_5 = V_5 - \frac{1}{2}\epsilon V_2,$$
$$Ad\big(\exp(\epsilon V_5)\big)V_2 = \left(1 + \frac{1}{2}\epsilon\right)V_2, \qquad Ad\big(\exp(\epsilon V_5)\big)V_1 = (1 - \epsilon)V_1,$$
$$Ad\big(\exp(\epsilon V_5)\big)V_3 = \left(1 + \frac{1}{2}\epsilon\right)V_3, \qquad Ad\big(\exp(\epsilon V_5)\big)V_6 = (1 - \frac{1}{2}\epsilon)V_6,$$
$$Ad\big(\exp(\epsilon V_6)\big)V_3 = V_3 + \frac{b_1}{2ab_2}\epsilon V_4, \qquad Ad\big(\exp(\epsilon V_6)\big)V_2 = V_2 - \frac{1}{2a}\epsilon V_4.$$

Therefore, we can get the optimal system of (3) according to the adjoint representation of the vector field (8), then have get V_1, V_2, V_3, $V_1 + hV_2$, $V_1 + hV_3$, $V_2 + hV_3$, $V_5 - yV_3/2 - tV_1$, $V_5 - xV_2/2 - tV_1$, where h is arbitrary constant.

3 Symmetry reductions

In the previous section, we mainly obtained the vector field and the optimal system of equation (1). Therefore, in this section, we will mainly do the symmetry reduction of these optimal systems.

3.1 The generator V_1

For the generator V_1, we can get

$$u(x,y,t) = F(\xi), \qquad v(x,y,t) = G(\tau), \tag{9}$$

where $\xi = x$ and $\tau = y$. Inserting (9) into (3), we can get system (3) of ordinary differential equations (ODEs) in which F and G satisfy

$$\begin{aligned} &aG'' - 2b_1G^2F' + 2b_2FGG' = 0,\\ &aF'' - 2b_1FGF' + 2b_2F^2G' = 0. \end{aligned} \tag{10}$$

3.2 The generator V_2

For the generator V_2, we can obtain

$$u(x,y,t) = F(\xi), \qquad v(x,y,t) = G(\tau), \tag{11}$$

where $\xi = y$ and $\tau = t$. Inserting (11) into (3), we can get system (3) of ODEs in which F and G satisfy

$$-2b_2G^2F' = 0, \qquad -G' + aF'' - 2b_2FGF' = 0, \tag{12}$$

where $F' = \mathrm{d}F/\mathrm{d}\xi$, $F'' = \mathrm{d}^2F/\mathrm{d}\xi^2$ and $G' = \mathrm{d}G/\mathrm{d}\tau$. Solving system (12), we can get $F(\xi) = c_1$, $G(\tau) = c_2$ or $F(\xi) = c_1\xi + c_2$, $G(\tau) = 0$. Therefore, we get the solution of equation (1) as

$$q(x,y,t) = c_1 + \mathrm{i}c_2 \quad \text{or} \quad q(x,y,t) = c_1y + c_2,$$

where c_1 and c_2 are arbitrary functions.

3.3 The generator V_3

For the generator V_3, we can obtain

$$u(x,y,t) = F(\xi), \qquad v(x,y,t) = G(\tau), \tag{13}$$

where $\xi = x$ and $\tau = t$. Inserting (13) into (3), we can get system (3) of ODEs in which F and G satisfy

$$-2b_1G^2F' = 0, \qquad -G' + aF'' - 2b_1FGF' = 0, \tag{14}$$

where $F' = \mathrm{d}F/\mathrm{d}\xi$, $F'' = \mathrm{d}^2F/\mathrm{d}\xi^2$ and $G' = \mathrm{d}G/\mathrm{d}\tau$. Solving system (14), we can get $F(\xi) = c_1$, $G(\tau) = c_2$ or $F(\xi) = c_1\xi + c_2$, $G(\tau) = 0$. Therefore, we get the solution of equation (1) as

$$q(x, y, t) = c_1 + \mathrm{i}c_2 \quad \text{or} \quad q(x, y, t) = c_1 y + c_2,$$

where c_1 and c_2 are arbitrary functions.

3.4 The generator $V_1 + hV_2$

For the generator $V_1 + hV_2$, we can get

$$u(x, y, t) = F(\xi), \qquad v(x, y, t) = G(\tau), \tag{15}$$

where $\xi = y$ and $\tau = t - x/h$. Inserting (15) into (3), we can get system (3) of ODEs in which F and G satisfy

$$\begin{aligned}
&\frac{a}{h^2}G'' - \frac{2b_1}{h}FGG' - 2b_2G^2F' = 0,\\
&-G' + aF'' - \frac{2b_1}{h}F^2G' - 2b_2FGF' = 0.
\end{aligned}$$

3.5 The generator $V_1 + hV_3$

For the generator $V_1 + hV_3$, we can get

$$u(x, y, t) = F(\xi), \qquad v(x, y, t) = G(\tau), \tag{16}$$

where $\xi = x$ and $\tau = t - y/h$. Inserting (16) into (3), we can get system (3) of ODEs in which F and G satisfy

$$\begin{aligned}
&\frac{a}{h^2}G'' - 2b_1G^2F' - \frac{2b_2}{h}FGG' = 0,\\
&-G' + aF'' - 2b_1FGF' - \frac{2b_2}{h}F^2G' = 0.
\end{aligned}$$

3.6 The generator $V_2 + hV_3$

For the generator $V_2 + hV_3$, we can get

$$u(x, y, t) = F(\xi), \qquad v(x, y, t) = G(\tau), \tag{17}$$

where $\xi = -hx + y$ and $\tau = t$. Inserting (17) into (3), we can get system (3) of ODEs in which F and G satisfy

$$\begin{aligned}
&2b_1hG^2F' - 2b_2G^2F' = 0,\\
&-G' + ah^2F'' + aF'' + 2b_1hFGF' - 2b_2FGF' = 0.
\end{aligned} \tag{18}$$

where $F' = \mathrm{d}F/\mathrm{d}\xi$, $F'' = \mathrm{d}^2F/\mathrm{d}\xi^2$ and $G' = \mathrm{d}G/\mathrm{d}\tau$. Solving system (18), we can get $F(\xi) = c_1\xi + c_2$, $G(\tau) = 0$ or $F(\xi) = c_1$, $G(\tau) = c_2$. Therefore, we get the solution of equation (1) as

$$q(x,y,t) = c_1(hx+y) + c_2 \quad \text{or} \quad q(x,y,t) = c_1 + \mathrm{i}c_2,$$

where c_1 and c_2 are arbitrary functions.

3.7 The generator $V_5 - yV_3/2 - tV_1$

For the generator $V_5 - (1/2)yV_3 - tV_1$, we can obtain

$$u(x,y,t) = \frac{F(\xi)}{x^{1/2}}, \qquad v(x,y,t) = \frac{G(\tau)}{x^{1/2}}, \tag{19}$$

where $\xi = y$ and $\tau = t$. Inserting (19) into (3), we can get system (3) of ODEs in which F and G satisfy

$$\begin{aligned} &\frac{3}{4}ax^{-1} - 2b_2GF' = 0, \\ &-G' + \frac{3}{4}ax^{-2}F + aF'' - 2b_2x^{-1}FGF' = 0, \end{aligned} \tag{20}$$

where $F' = \mathrm{d}F/\mathrm{d}\xi$, $F'' = \mathrm{d}^2F/\mathrm{d}\xi^2$ and $G' = \mathrm{d}G/\mathrm{d}\tau$. Solving system (20), we can get $F(\xi) = c_1\xi + c_2$, $G(\tau) = 3a/(8b_2c_1x)$. Therefore, we get the solution of equation (1) as

$$q(x,y,t) = \frac{c_1y + c_2}{x^{1/2}} + \frac{3ia}{8b_2c_1x^{3/2}}, \tag{21}$$

where c_1 and c_2 are arbitrary functions.

3.8 The generator $V_5 - xV_2/2 - tV_1$

For the generator $V_5 - (1/2)xV_2 - tV_1$, we can obtain

$$u(x,y,t) = \frac{F(\xi)}{y^{1/2}}, \qquad v(x,y,t) = \frac{G(\tau)}{y^{1/2}}, \tag{22}$$

where $\xi = x$ and $\tau = t$. Inserting (22) into (3), we can get system (3) of ODEs in which F and G satisfy

$$\begin{aligned} &\frac{3}{4}ay^{-1} - 2b_1GF' = 0, \\ &-G' + \frac{3}{4}ay^{-2}F + aF'' - 2b_1y^{-1}FGF' = 0, \end{aligned}$$

where $F' = \mathrm{d}F/\mathrm{d}\xi$, $F'' = \mathrm{d}^2F/\mathrm{d}\xi^2$ and $G' = \mathrm{d}G/\mathrm{d}\tau$. Solving system (20), we can get $F(\xi) = c_1\xi + c_2$, $G(\tau) = 3a/(8b_1c_1y)$. Therefore, we get the solution of equation (1) as

$$q(x,y,t) = \frac{c_1y + c_2}{y^{1/2}} + \frac{3\mathrm{i}a}{8b_1c_1y^{3/2}},$$

where c_1 and c_2 are arbitrary functions.

4 The power series solutions

Based on the symbolic calculation methods [13, 17, 18, 22, 27, 28, 31–37, 46, 47], we study the analytical solution of ODE by through the power series method. When we get the analytical solution of ODE, we can easily obtain the power series solutions of the original partial differential equation.

According to (10), we can get

$$\begin{aligned} &aG'' - 2b_1G^2F' + 2b_2FGG' = 0, \\ &aF'' - 2b_1FGF' + 2b_2F^2G' = 0. \end{aligned} \tag{23}$$

Below we will use the following hypothetical form to calculate the solution of (23)

$$F(\xi) = \sum_{n=0}^{\infty} P_n \xi^n, \qquad G(\tau) = \sum_{n=0}^{\infty} Q_n \tau^n, \tag{24}$$

where the coefficients P_n and Q_n $(n = 0, 1, \dots)$ are all constants.

Inserting (24) into (23) yields

$$\begin{aligned} &a\sum_{n=0}^{\infty}(n+1)(n+2)Q_{n+2}\tau^n - 2b_1\left(\sum_{n=0}^{\infty}Q_n\tau^n\right)^2\sum_{n=0}^{\infty}(n+1)P_{n+1}\xi^n \\ &\quad + 2b_2\sum_{n=0}^{\infty}Q_n\tau^n\sum_{n=0}^{\infty}P_n\xi^n\sum_{n=0}^{\infty}(n+1)Q_{n+1}\tau^n = 0, \\ &a\sum_{n=0}^{\infty}(n+1)(n+2)P_{n+2}\xi^n - 2b_1\sum_{n=0}^{\infty}Q_n\tau^n\sum_{n=0}^{\infty}P_n\xi^n\sum_{n=0}^{\infty}(n+1)P_{n+1}\xi^n \\ &\quad + 2b_2\left(\sum_{n=0}^{\infty}P_n\xi^n\right)^2\sum_{n=0}^{\infty}(n+1)Q_{n+1}\tau^n = 0. \end{aligned}$$

When $n = 0$, we compare coefficients of ξ to get

$$Q_2 = \frac{1}{a}\left(b_1Q_0^2P_1 - b_2Q_0P_0Q_1\right), \qquad P_2 = \frac{1}{a}\left(b_1Q_0P_0P_1 - b_2P_0^2Q_1\right). \tag{25}$$

Generally, when $n \geqslant 1$, we can obtain

$$Q_{n+2} = \frac{M_Q}{a(n+1)(n+2)}, \qquad P_{n+2} = \frac{M_P}{a(n+1)(n+2)} \tag{26}$$

with

$$\begin{aligned} M_Q = {} & 2b_1\sum_{k=0}^{n}\sum_{j=0}^{k}(n-k+1)Q_jQ_{k-j}P_{n-k+1} \\ & - 2b_2\sum_{k=0}^{n}\sum_{j=0}^{k}(n-k+1)Q_jP_{k-j}Q_{n-k+1}, \end{aligned}$$

$$M_P = 2b_1 \sum_{k=0}^{n} \sum_{j=0}^{k} (n-k+1) Q_j P_{k-j} P_{n-k+1} - 2b_2 \sum_{k=0}^{n} \sum_{j=0}^{k} (n-k+1) P_j P_{k-j} Q_{n-k+1}].$$

Then we can get the following results:

when $n = 1$,

$$Q_3 = \frac{1}{3a}\bigl(2b_1 Q_0^2 P_2 + 2b_1 Q_0 Q_1 P_1 - 2b_2 Q_0 P_0 Q_2 - b_2 Q_0 P_1 Q_1 - b_2 Q_1^2 P_0\bigr),$$
$$P_3 = \frac{1}{3a}\bigl(2b_1 Q_0 P_0 P_2 + b_1 Q_1 P_0 P_1 + b_1 Q_0 P_1^2 - 2b_2 P_0^2 Q_2 - 2b_2 P_0 P_1 Q_1\bigr),$$

when $n = 2$,

$$Q_4 = \frac{1}{6a}\bigl(3b_1 Q_0^2 P_3 + 4b_1 Q_0 Q_1 P_2 + 2b_1 Q_0 Q_2 P_1 + b_1 Q_1^2 P_1 - 3b_2 Q_0 Q_3 P_0 - 2b_2 Q_0 P_1 Q_2 - 3b_2 Q_1 P_0 Q_2 - b_2 Q_0 P_2 Q_1 - b_2 Q_1^2 P_1\bigr),$$
$$P_4 = \frac{1}{6a}\bigl(3b_1 Q_0 P_0 P_3 + 3b_1 Q_0 P_1 P_2 + 2b_1 Q_1 P_0 P_2 + b_1 Q_1 P_1^2 + b_1 Q_2 P_0 P_1 - 3b_2 P_0^2 Q_3 - 4b_2 P_0 P_1 Q_2 - 2b_2 P_0 P_2 Q_1 - b_2 P_1^2 Q_1\bigr).$$

From the above derivation we can see that all the coefficients (P_n, Q_n) in the power series solution of (24) can be represented by $a, b_1, b_2, Q_0, Q_1, P_0, P_1$, where a, b_1, b_2, Q_0, Q_1, P_0, P_1 are arbitrary constants. Besides, on the basis of [2, 21], we can also prove the convergence of the coefficients determined by (25)–(26). Thus, we obtain that a power series solution (24) is the power series solution of (23). Then a power series solution of (24) can be rewritten into

$$F(\xi) = P_0 + P_1\xi + \frac{1}{a}\bigl(b_1 Q_0 P_0 P_1 - b_2 P_0^2 Q_1\bigr)\xi^2 + \frac{M_P}{a(n+1)(n+2)}\xi^{n+2} + \frac{1}{3a}\bigl(2b_1 Q_0 P_0 P_2 + b_1 Q_1 P_0 P_1 + b_1 Q_0 P_1^2 - 2b_2 P_0^2 Q_2 - 2b_2 P_0 P_1 Q_1\bigr)\xi^3 + \cdots,$$

$$G(\tau) = Q_0 + Q_1\tau + \frac{1}{a}\bigl(b_1 Q_0^2 P_1 - b_2 Q_0 P_0 Q_1\bigr)\tau^2 + \frac{M_Q}{a(n+1)(n+2)}\tau^{n+2} + \frac{1}{3a}\bigl(2b_1 Q_0^2 P_2 + 2b_1 Q_0 Q_1 P_1 - 2b_2 Q_0 P_0 Q_2 - b_2 Q_0 P_1 Q_1 - b_2 Q_1^2 P_0\bigr)\tau^3 + \cdots.$$

Then the power series solution of equation (1) is

$$\begin{aligned} q(x,y,t) &= \biggl[P_0 + P_1 x + \frac{1}{a}\bigl(b_1 Q_0 P_0 P_1 - b_2 P_0^2 Q_1\bigr)x^2 \\ &\quad + \frac{M_P}{a(n+1)(n+2)}x^{n+2} + \frac{1}{3a}\bigl(2b_1 Q_0 P_0 P_2 + b_1 Q_1 P_0 P_1 \\ &\quad + b_1 Q_0 P_1^2 - 2b_2 P_0^2 Q_2 - 2b_2 P_0 P_1 Q_1\bigr)x^3 + \cdots\biggr] \\ &\quad + \mathrm{i}\biggl[Q_0 + Q_1 y + \frac{1}{a}\bigl(b_1 Q_0^2 P_1 - b_2 Q_0 P_0 Q_1\bigr)y^2 \\ &\quad + \frac{M_Q}{a(n+1)(n+2)}y^{n+2} + \frac{1}{3a}\bigl(2b_1 Q_0^2 P_2 + 2b_1 Q_0 Q_1 P_1 \\ &\quad - 2b_2 Q_0 P_0 Q_2 - b_2 Q_0 P_1 Q_1 - b_2 Q_1^2 P_0\bigr)y^3 + \cdots\biggr], \end{aligned}$$

where a, b_1, b_2, Q_0, Q_1, P_0, P_1 are arbitrary constants, and other coefficients determined by (25)–(26).

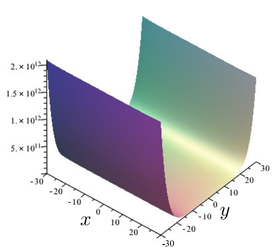
Based on the previous detailed derivation, we can obtain the following theorem.

Theorem 2. *The chiral NLS equation in* $(2+1)$*-dimensions* (1) *has the following power series solution*:

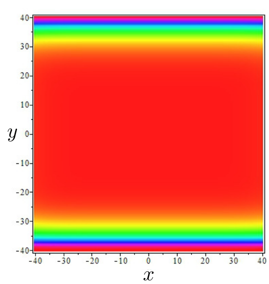
$$q(x,y,t) = \sum_{n=0}^{\infty} P_n x^n + \mathrm{i}\sum_{n=0}^{\infty} Q_n y^n, \tag{27}$$

where a, b_1, b_2, Q_0, Q_1, P_0, P_1 *are arbitrary constants, and other coefficients determined by* (25)–(26).

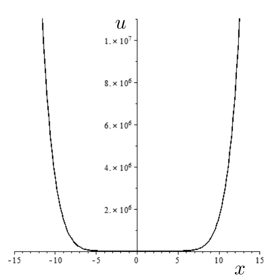
Next, by choosing the appropriate parameters, we draw the graph of the power series solution and thus illustrate its properties (see Figs. 1, 2).



(a)



(b)



(c)

Figure 1. The power series solution of (1) by choosing suitable parameters: (a) perspective view of the real part of power series solutions ($n = 4$); (b) the overhead view of the solutions, (c) the wave propagation pattern of the wave along the x-axis. Here $a = 1$, $b_1 = 1$, $b_2 = 1$, $P_0 = 1$, $P_1 = 1$, $Q_0 = 10$, $Q_1 = 11$. (Online version in color.)

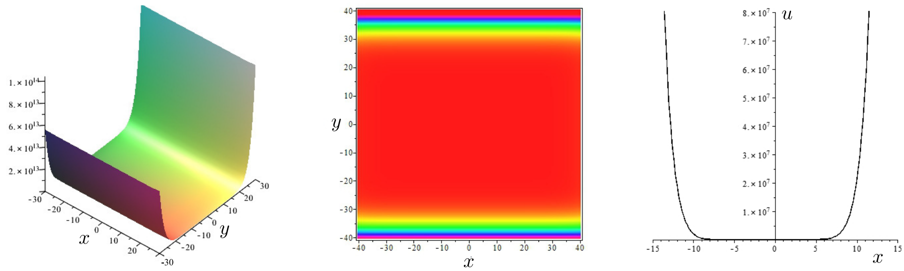


Figure 2. The power series solution of (1) by choosing suitable parameters: (a) perspective view of the real part of power series solutions ($n = 5$); (b) the overhead view of the solutions; (c) the wave propagation pattern of the wave along the x-axis. Here $a = 1$, $b_1 = 1$, $b_2 = 1$, $P_0 = 1$, $P_1 = 1$, $Q_0 = 10$, $Q_1 = 11$. (Online version in color.)

5 Conservation laws

In this section, if we want to derived the conservation law of equation (1), it is necessary to first find the conservation law of system (3). Therefore, we will use Lie point symmetry (8) to construct the conservation law of system (3).

A vector $C = (C^t, C^x, C^y)$ is called a conserved vector for equation (1) if it satisfy the following conservation equations:

$$D_t\big(C^t\big) + D_x\big(C^x\big) + D_y\big(C^y\big) = 0.$$

In [20], Ibragimov proposes a new conservation theorem, that is, constructing a conservation law without a Lagrangian quantity in a differential equation. Then on the basis of [20], the Lagrangian of system (3) can be written as follows:

$$\begin{aligned} L = {}& \phi(x,y,t)\big[-v_t + a(u_{xx}+u_{yy}) + 2b_1\big(u^2v_x - uvu_x\big) + 2b_2\big(u^2v_y - uvu_y\big)\big] \\ & + \psi(x,y,t)\big[u_t + a(v_{xx}+v_{yy}) + 2b_1\big(uvv_x - v^2u_x\big) + 2b_2\big(uvv_y - v^2u_y\big)\big]. \end{aligned} \quad (28)$$

where $\phi(x,y,t)$ and $\psi(x,y,t)$ are two new dependent variables. The adjoint equations of system (3) can be written as following form:

$$F^* = \frac{\delta L}{\delta u} = 0, \qquad G^* = \frac{\delta L}{\delta v} = 0 \quad (29)$$

with

$$\begin{aligned} \frac{\delta L}{\delta u} &= \frac{\partial L}{\partial u} - D_t\frac{\partial L}{\partial u_t} - D_x\frac{\partial L}{\partial u_x} - D_y\frac{\partial L}{\partial u_y} + D_x^2\frac{\partial L}{\partial u_{xx}} + D_y^2\frac{\partial L}{\partial u_{yy}}, \\ \frac{\delta L}{\delta v} &= \frac{\partial L}{\partial v} - D_t\frac{\partial L}{\partial v_t} - D_x\frac{\partial L}{\partial v_x} - D_y\frac{\partial L}{\partial v_y} + D_x^2\frac{\partial L}{\partial v_{xx}} + D_y^2\frac{\partial L}{\partial v_{yy}}. \end{aligned}$$

Combining with (28) and adjoint equations (29), we can obtain

$$\begin{aligned} F^* &= 6b_1uv_x\phi + 6b_2uv_y\phi + 6b_1vv_x\psi + 6b_2vv_y\psi + 2b_1uv\phi_x + 2b_1v^2\psi_x \\ &\quad + 2b_2uv\phi_y + 2b_2v^2\psi_y - \psi_t + a\phi_{xx} + a\phi_{yy}, \\ G^* &= -6b_1uu_x\phi - 6b_2uu_y\phi - 6b_1vu_x\psi - 6b_2vu_y\psi - 2b_1uv\psi_x - 2b_1u^2\phi_x \\ &\quad - 2b_2uv\psi_y - 2b_2u^2\phi_y + \phi_t + a\psi_{xx} + a\psi_{yy}. \end{aligned} \tag{30}$$

In the above system (30), if we substitute v instead of ϕ and u instead of $-\psi$, we can get system (3). In [20], we know that the conservation vector $C = (C^1, C^2, C^3, \dots)$ has the following form:

$$\begin{aligned} C^n &= \xi^n L + W^\alpha \biggl[\frac{\partial L}{\partial u_n^\alpha} - D_j\biggl(\frac{\partial L}{\partial u_{nj}^\alpha}\biggr) + D_jD_k\biggl(\frac{\partial L}{\partial u_{njk}^\alpha}\biggr) - \cdots\biggr] \\ &\quad + D_j\bigl(W^\alpha\bigr)\biggl[\frac{\partial L}{\partial u_{nj}^\alpha} - D_k\biggl(\frac{\partial L}{\partial u_{njk}^\alpha}\biggr) + \cdots\biggr] + D_jD_k\bigl(W^\alpha\bigr)\biggl[\frac{\partial L}{\partial u_{njk}^\alpha} - \cdots\biggr], \end{aligned}$$

where $W^\alpha = \eta^\alpha - \xi^j u_j^\alpha$ ($\alpha = 1, 2, \dots, m$) are shown in [20].

Using the above formula, we can further write about the conservation vector of (28) as

$$\begin{aligned} C^t &= \xi^t L + W^u\frac{\partial L}{\partial u_t} + W^v\frac{\partial L}{\partial v_t}, \\ C^x &= \xi^x L + W^u\biggl(\frac{\partial L}{\partial u_x} - D_x\frac{\partial L}{\partial u_{xx}}\biggr) + D_x\bigl(W^u\bigr)\frac{\partial L}{\partial u_{xx}} \\ &\quad + W^v\biggl(\frac{\partial L}{\partial v_x} - D_x\frac{\partial L}{\partial v_{xx}}\biggr) + D_x\bigl(W^v\bigr)\frac{\partial L}{\partial v_{xx}}, \\ C^y &= \xi^y L + W^u\biggl(\frac{\partial L}{\partial u_y} - D_y\frac{\partial L}{\partial u_{yy}}\biggr) + D_y\bigl(W^u\bigr)\frac{\partial L}{\partial u_{yy}} \\ &\quad + W^v\biggl(\frac{\partial L}{\partial v_y} - D_y\frac{\partial L}{\partial v_{yy}}\biggr) + D_y\bigl(W^v\bigr)\frac{\partial L}{\partial v_{yy}} \end{aligned} \tag{31}$$

in which W^u and W^v are the Lie characteristic functions.

In order to obtain the conservation vector of system (3), we can use the symmetry generators V_1, V_2, V_3, V_4, V_5 and V_6 as an example to illustrate.

Case 1. For the generator $V_1 = \partial/\partial t$, we can get the following Lie characteristic functions:

$$W^u = -u_t, \qquad W^v = -v_t. \tag{32}$$

Inserting (32) into (31), we can get the following conserved vector:

$$\begin{aligned} C_1^t &= avu_{xx} + avu_{yy} - auv_{xx} - auv_{yy}, \\ C_1^x &= au_tv_x - avu_{tx} - av_tu_x + auv_{tx}, \\ C_1^y &= au_tv_y - avu_{ty} - au_yv_t + auv_{ty}. \end{aligned} \tag{33}$$

After calculation, we can find the following equation:

$$D_t(C_1^t) + D_x(C_1^x) + D_y(C_1^y) = 0.$$

Thus, we know that (33) is a conservation law of system (3). Inserting $u = (q + q^*)/2$ and $v = (q - q^*)/(2\mathrm{i})$ into (33), we can obtain conservation laws of equation (1) as

$$\begin{aligned} T_1^t &= \frac{a}{2\mathrm{i}}(qq_{xx}^* + qq_{yy}^* - q^*q_{xx} - q^*q_{yy}), \\ T_1^x &= \frac{a}{2\mathrm{i}}(q_xq_t^* + q^*q_{tx} - q_tq_x^* - qq_{tx}^*), \\ T_1^y &= \frac{a}{2\mathrm{i}}(q_yq_t^* + q^*q_{ty} - q_tq_y^* - qq_{ty}^*). \end{aligned}$$

Case 2. For the generator $V_2 = \partial/\partial x$, we can get the following Lie characteristic functions:

$$W^u = -u_x, \qquad W^v = -v_x. \tag{34}$$

Inserting (34) into (31), we can get the following conserved vector:

$$\begin{aligned} &C_2^t = uu_x + vv_x, \qquad C_2^x = -vv_t - uu_t + avu_{yy} - auv_{yy}, \\ &C_2^y = au_xv_y - avu_{xy} - au_yv_x + auv_{xy}. \end{aligned} \tag{35}$$

After calculation, we can find the following equation:

$$D_t(C_2^t) + D_x(C_2^x) + D_y(C_2^y) = 0.$$

Thus, we know that (35) is a conservation law of system (3). Inserting $u = (q + q^*)/2$ and $v = (q - q^*)/(2\mathrm{i})$ into (35), we can obtain conservation laws of equation (1) as

$$\begin{aligned} &T_2^t = \frac{1}{2}(qq_x^* + q^*q_x), \qquad T_2^x = -\frac{1}{2}(qq_t^* + q^*q_t) + \frac{a}{2\mathrm{i}}(qq_{yy}^* - q^*q_{yy}), \\ &T_2^y = \frac{a}{2\mathrm{i}}(q_yq_x^* + q^*q_{xy} - q_xq_y^* - qq_{xy}^*). \end{aligned}$$

Case 3. For the generator $V_3 = \partial/\partial y$, we can get the following Lie characteristic functions:

$$W^u = -u_y, \qquad W^v = -v_y. \tag{36}$$

Inserting (36) into (31), we can get the following conserved vector:

$$\begin{aligned} &C_3^t = uu_y + vv_y, \qquad C_3^x = au_yv_x - avu_{xy} - au_xv_y + auv_{xy}, \\ &C_3^y = -vv_t - uu_t + avu_{xx} - auv_{xx}. \end{aligned} \tag{37}$$

After calculation, we can find the following equation:

$$D_t(C_3^t) + D_x(C_3^x) + D_y(C_3^y) = 0.$$

Thus, we know that (37) is a conservation law of system (3). Inserting $u = (q + q^*)/2$ and $v = (q - q^*)/(2\mathrm{i})$ into (37), we can obtain conservation laws of equation (1) as

$$T_3^t = \frac{1}{2}(qq_y^* + q^* q_y), \qquad T_3^x = \frac{a}{2i}(q_x q_y^* + q^* q_{xy} - q_y q_x^* - qq_{xy}^*),$$
$$T_3^y = -\frac{1}{2}(qq_t^* + q^* q_t) + \frac{a}{2i}(qq_{xx}^* - q^* q_{xx}).$$

Case 4. For the generator $V_4 = v\partial/\partial u - u\partial/\partial v$, we can get the following Lie characteristic functions:

$$W^u = v, \qquad W^v = -u. \tag{38}$$

Inserting (38) into (31), we can get the conserved vector

$$C_4^t = 0, \qquad C_4^x = 0, \qquad C_4^y = 0. \tag{39}$$

After calculation, we can find the following equation:

$$D_t\big(C_4^t\big) + D_x\big(C_4^x\big) + D_y\big(C_4^y\big) = 0.$$

Thus, we know that (39) is a conservation law of system (3). Inserting $u = (q + q^*)/2$ and $v = (q - q^*)/(2\mathrm{i})$ into (39), we can obtain conservation laws of equation (1) as

$$T_4^t = 0, \qquad T_4^x = 0, \qquad T_4^y = 0.$$

Case 5. For the generator $V_5 = (1/2)x\partial/\partial x + (1/2)y\partial/\partial y + t\partial/\partial t - (1/4)u\partial/\partial u - (1/4)v\partial/\partial v$, we can get the following Lie characteristic functions:

$$W^u = -\frac{1}{2}xu_x - \frac{1}{2}yu_y - tu_t - \frac{1}{4}u, \qquad W^v = -\frac{1}{2}xv_x - \frac{1}{2}yv_y - tv_t - \frac{1}{4}v. \tag{40}$$

Inserting (40) into (31), we can get the following conserved vector:

$$\begin{aligned}
C_5^t &= atvu_{xx} + atvu_{yy} - atuv_{xx} - atuv_{yy} \\
&\quad + \frac{1}{4}u^2 + \frac{1}{2}xuu_x + \frac{1}{2}yuu_y + \frac{1}{4}v^2 + \frac{1}{2}xvv_x + \frac{1}{2}yvv_y, \\
C_5^x &= -\frac{1}{2}xvv_t - \frac{1}{2}xuu_t + \frac{1}{2}axvu_{yy} - \frac{1}{2}axuv_{yy} + \frac{1}{2}ayu_yv_y \\
&\quad + atv_xu_t - avu_x - \frac{1}{2}ayvu_{xy} - atvu_{xt} - \frac{1}{2}ayu_xv_y \\
&\quad - atu_xv_t + auv_x + \frac{1}{2}ayuv_{xy} + atuv_{xt}, \\
C_5^y &= -\frac{1}{2}yvv_t - \frac{1}{2}yuu_t + \frac{1}{2}ayvu_{xx} - \frac{1}{2}ayuv_{xx} + \frac{1}{2}axv_yu_x \\
&\quad + atv_yu_t - avu_y - \frac{1}{2}axvu_{xy} - atvu_{ty} - \frac{1}{2}axu_yv_x \\
&\quad - atu_yv_t + auv_y + \frac{1}{2}axuv_{xy} + atuv_{ty}.
\end{aligned} \tag{41}$$

After calculation, we can find the following equation:

$$D_t\big(C_5^t\big) + D_x\big(C_5^x\big) + D_y\big(C_5^y\big) = 0.$$

Thus, we know that (41) is a conservation law of system (3). Inserting $u = (q + q^*)/2$ and $v = (q - q^*)/(2\mathrm{i})$ into (41), we can obtain conservation laws of equation (1) as

$$\begin{aligned}
T_5^t &= \frac{at}{2\mathrm{i}}\big(qq_{xx}^* - q^*q_{xx} + qq_{yy}^* - q^*q_{yy}\big)\\
&\quad + \frac{1}{4}\big[x\big(qq_x^* + q^*q_x\big) + y\big(qq_y^* + q^*q_y\big) + qq^*\big],\\
T_5^x &= \frac{ax}{4\mathrm{i}}\big(qq_{yy}^* - q^*q_{yy}\big) + \frac{ay}{4\mathrm{i}}\big(q_xq_y^* - q_yq_x^* + q^*q_{xy} - qq_{xy}^*\big)\\
&\quad + \frac{at}{2\mathrm{i}}\big(q_xq_t^* - q_tq_x^* + q^*q_{xt} - qq_{xt}^*\big) - \frac{x}{4}\big(qq_t^* + q^*q_t\big) + \frac{a}{2\mathrm{i}}\big(q^*q_x - qq_x^*\big),\\
T_5^y &= \frac{ay}{4\mathrm{i}}\big(qq_{xx}^* - q^*q_{xx}\big) + \frac{ax}{4\mathrm{i}}\big(q_yq_x^* - q_xq_y^* + q^*q_{xy} - qq_{xy}^*\big)\\
&\quad + \frac{at}{2\mathrm{i}}\big(q_yq_t^* - q_tq_y^* + q^*q_{yt} - qq_{yt}^*\big) - \frac{y}{4}\big(qq_t^* + q^*q_t\big) + \frac{a}{2\mathrm{i}}\big(q^*q_y - qq_y^*\big).
\end{aligned}$$

Case 6. For the generator $t\partial/\partial x - (b_1t/b_2)\partial/\partial y + (v(b_1y - b_2x)/(2ab_2))\partial/\partial u - ((u(b_1y - b_2x))/(2ab_2)\partial/\partial v$, we can get the following Lie characteristic functions:

$$\begin{aligned}
W^u &= -tu_x + \frac{b_1t}{b_2}u_y + \frac{v(b_1y - b_2x)}{2ab_2},\\
W^v &= -tv_x + \frac{b_1t}{b_2}v_y - \frac{u(b_1y - b_2x)}{2ab_2}.
\end{aligned} \tag{42}$$

Inserting (42) into (31), we can get the following conserved vector:

$$\begin{aligned}
C_6^t &= t(uu_x + vv_x) - \frac{b_1t}{b_2}(uu_y + vv_y),\\
C_6^x &= -t(vv_t + uu_t) + at(vu_{yy} - uv_{yy}) - \frac{1}{2}\big(u^2 + v^2\big)\\
&\quad + \frac{ab_1t}{b_2}(u_xv_y - v_xu_y + u_{xy}v - uv_{xy}),\\
C_6^y &= \frac{b_1t}{b_2}(vv_t + uu_t) + \frac{ab_1t}{b_2}(uv_{xx} - u_{xx}v) + \frac{b_1}{2b_2}\big(u^2 + v^2\big)\\
&\quad + at(u_xv_y - v_xu_y + v_{xy}u - vu_{xy}).
\end{aligned} \tag{43}$$

After calculation, we can find the following equation:

$$D_t\big(C_6^t\big) + D_x\big(C_6^x\big) + D_y\big(C_6^y\big) = 0. \tag{44}$$

Thus, we know that (43) is a conservation law of system (3). Inserting $u = (q + q^*)/2$ and $v = (q - q^*)/(2\mathrm{i})$ into (43), we can obtain conservation laws of equation (1) as

$$\begin{aligned}
T_6^t &= \frac{t}{2}(qq_x^* + q^*q_x) - \frac{b_1 t}{2b_2}(qq_y^* + q^*q_y),\\
T_6^x &= -\frac{t}{2}(qq_t^* + q^*q_t) + \frac{at}{2\mathrm{i}}(qq_{yy}^* - q^*q_{yy})\\
&\quad - \frac{1}{2}qq^* + \frac{ab_1 t}{2\mathrm{i}b_2}(q_y q_x^* - q_x q_y^* - q^*q_{xy} + qq_{xy}^*),\\
T_6^y &= \frac{b_1 t}{2b_2}(qq_t^* + q^*q_t) + \frac{ab_1 t}{2ib_2}(qq_{yy}^* - q^*q_{yy})\\
&\quad + \frac{b_1}{2b_2}qq^* + \frac{at}{2\mathrm{i}}(q_y q_x^* - q_x q_y^* + q^*q_{xy} - qq_{xy}^*).
\end{aligned}$$

6 Conclusions and discussions

As we mentioned above, the bright and dark soliton solutions of the chiral NLS equation (1) have been obtained using the constant coefficient method in [6]. The singular periodic solution of equation (1) has been obtained by using the trial solution method in [14]. Compared with previous literatures [6, 14], we have obtained some new results, such as vector field, optimal system, similarity reduction solutions, power series solutions with convergence analysis, and conservation laws of equation (1). Firstly, we have transformed the complex model (1) to the real system (3) by using the transformation $q(x, y, t) = u(x, y, t) + \mathrm{i}v(x, y, t)$. Then, through the Lie symmetry analysis method, we have constructed the optimal systems and symmetry reductions of system (3). In addition, we have also obtained the power series solution of equation (1) by the power series method. In Figs. 1, 2, when $n = 4, 5$, we have obtained a perspective view of the real part of the power series solution and the wave propagation pattern of the wave along the x-axis by selecting the appropriate parameter values. Subsequently, we have obtained the conservation law related to the lie symmetry of equation (1) by using the new conservation law method introduced by Ibragimov in [20]. The new results presented in this work can be used to describe soliton dynamics in nuclear physics and other optical experiments. Therefore, it is hoped that all the research results in this work can be used to enrich the dynamic behavior of nonlinear Schrödinger-type equations in engineering and mathematical physics.

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References

1. M.J. Ablowitz, P.A. Clarkson, *Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform*, Cambridge Univ. Press, Cambridge, 1990.

2. N.H. Asmar, *Partial Differential Equations with Fourier Series and Boundary Value Problems,* 2nd ed., China Machine Press, Beijing, 2005.
3. D. Baleanu, M. Inc, A. Yusuf, A. I. Aliyu, Lie symmetry analysis, exact solutions and conservation laws for the time fractional modified Zakharov–Kuznetsov equation, *Nonlinear Anal. Model. Control*, **22**(6):861–876, 2017.
4. D. Baleanu, A. Yusuf, A.I. Aliyu, Lie symmetry analysis and conservation laws for the time fractional simplified modified Kawahara equation, *Open Phys.*, **16**(1):302–310, 2018.
5. D. Baleanu, A. Yusuf, A.I. Aliyu, Optimal system, nonlinear self-adjointness and conservation laws for generalized shallow water wave equation, *Open Phys.*, **16**(1):364–370, 2018.
6. A. Biswas, Chiral solitons in $1 + 2$ dimensions, *Int. J. Theor. Phys.*, **48**:3403–3409, 2009.
7. A. Biswas, Perturbation of chiral solitons, *Nucl. Phys. B*, **806**(3):457–461, 2009.
8. A. Biswas, Dynamics and conservation laws of generalized chiral solitons, *Open Nucl. Part. Phys. J.*, **4**:21–24, 2011.
9. A. Biswas, A.H. Kara, L. Moraru, A.H. Bokhari, F.D. Zaman, Conservation laws of coupled Klein–Gordon equations with cubic and power law nonlinearities, *Proc. Rom. Acad., A, Math. Phys. Tech. Sci. Inf. Sci.*, **15**(2):123–129, 2014.
10. A. Biswas, M. Song, H. Triki, A.H. Kara, B.S. Ahmed, A. Strong, A. Hama, Solitons, shock waves, conservation laws and bifurcation analysis of Boussinesq equation with power law nonlinearity and dual dispersion, *Appl. Math. Inf. Sci.*, **8**:949–957, 2014.
11. G. Bluman, S. Anco, *Symmetry and Integration Methods for Differential Equations*, Springer, New York, 2002.
12. G. Bluman, Temuerchaolu, Conservation laws for nonlinear telegraph equations, *J. Math. Anal. Appl.*, **310**:459–476, 2005.
13. M.J. Dong, S.F. Tian, X.W. Yan, L. Zou, Solitary waves, homoclinic breather waves and rogue waves of the $(3 + 1)$-dimensional Hirota bilinear equation, *Comput. Math. Appl.*, **75**(3):957–964, 2018.
14. M. Eslami, Trial solution technique to chiral nonlinear Schrödinger's equation in $(1 + 2)$-dimensions, *Nonlinear Dyn.*, **85**:813–816, 2016.
15. E.G. Fan, Two new applications of the homogeneous balance method, *Phys. Lett. A*, **265**:353–357, 2000.
16. L.L. Feng, S.F. Tian, T.T. Zhang, Nonlocal symmetries and consistent Riccati expansions of the $(2 + 1)$-dimensional dispersive long wave equation, *Z. Naturforsch. A*, **72**(5):425–431, 2017.
17. L.L. Feng, S.F. Tian, T.T. Zhang, Solitary wave, breather wave and rogue wave solutions of an inhomogeneous fifth-order nonlinear Schrödinger equation from Heisenberg ferromagnetism, *Rocky Mt. J. Math.*, **49**(1):29–45, 2019.
18. L.L. Feng, T.T. Zhang, Breather wave, rogue wave and solitary wave solutions of a coupled nonlinear Schrödinger equation, *Appl. Math. Lett.*, **78**:133–140, 2018.
19. R. Hirota, *Direct Methods in Soliton Theory*, Springer, Berlin, Heidelberg, 2004.
20. N.H. Ibragimov, A new conservation theorem, *J. Math. Anal. Appl.*, **333**(1):311–328, 2007.
21. H. Liu, J. Li, Lie symmetry analysis and exact solutions for the short pulse equation, *Nonlinear Anal., Theory Methods Appl.*, **71**(5–6):2126–2133, 2009.

22. J.J. Mao, S.F. Tian, L. Zou, T.T. Zhang, Optical solitons, complexitons, Gaussian soliton and power series solutions of a generalized Hirota equation, *Mod. Phys. Lett. B*, **32**(14):1850143, 2018.

23. V.B. Matveev, M.A. Salle, *Darboux Transformations and Solitons*, Springer, Berlin, Heidelberg, 1991.

24. R. Naz, F.M. Mahomed, D.P. Mason, Comparison of different approaches to conservation laws for some partial differential equations in fluid mechanics, *Appl. Math. Comput.*, **205**:212–230, 2008.

25. E. Noether, Invariante variations probleme, *Nachr. Ges. Wiss. Göttingen, Math.-Phys. Kl.*, **1918**:235–257, 1918.

26. P. Olver, *Applications of Lie Groups to Diffierential Equations*, Springer, Berlin, Heidelberg, 1993.

27. W.Q. Peng, S.F. Tian, T.T. Zhang, Dynamics of breather waves and higher-order rogue waves in a coupled nonlinear Schrödinger equation, *Europhys. Lett.*, **123**(5):50005, 2018.

28. C.Y. Qin, S.F. Tian, L. Zou, W.X. Ma, Solitary wave and quasi-periodic wave solutions to a $(3+1)$-dimensional generalized Calogero–Bogoyavlenskii–Schiff equation, *Adv. Appl. Math. Mech.*, **10**(4):948–977, 2018.

29. C.Y. Qin, S.F. Tian, L. Zou, T.T. Zhang, Lie symmetry analysis, conservation laws and exact solutions of fourth-order time fractional Burgers equation, *J. Appl. Anal. Comput.*, **8**(6):1727–1746, 2018.

30. V.N. Serkin, A. Hasegawa, Novel soliton solutions of the nonlinear Schrödinger equation model, *Phys. Rev. Lett.*, **85**(21):4502, 2000.

31. T.T. Zhang S.F. Tian, Long-time asymptotic behavior for the Gerdjikov–Ivanov type of derivative nonlinear Schrödinger equation with time-periodic boundary condition, *Proc. Am. Math. Soc.*, **146**(4):1713–1729, 2018.

32. S.F. Tian, The mixed coupled nonlinear Schrödinger equation on the half-line via the Fokas method, *Proc. R. Soc. Lond., Ser. A*, **472**:20160588, 2016.

33. S.F. Tian, Initial-boundary value problems for the general coupled nonlinear Schrödinger equation on the interval via the Fokas method, *J. Differ. Equ.*, **262**:506, 2017.

34. S.F. Tian, Asymptotic behavior of a weakly dissipative modified two-component Dullin–Gottwald–Holm system, *Appl. Math. Lett.*, **83**:65–72, 2018.

35. S.F. Tian, Initial-boundary value problems for the coupled modified Korteweg–de Vries equation on the interval, *Commun. Pure Appl. Anal.*, **17**(3):923–957, 2018.

36. S.F. Tian, Infinite propagation speed of a weakly dissipative modified two-component Dullin–Gottwald–Holm system, *Appl. Math. Lett.*, **89**:1–7, 2019.

37. S.F. Tian, H.Q. Zhang, On the integrability of a generalized variable-coefficient forced Korteweg–de Vries equation in fluids, *Stud. Appl. Math.*, **132**:212, 2014.

38. S.F. Tian, Y.F. Zhang, B.L. Feng, H.Q. Zhang, On the Lie algebras, generalized symmetries and Darboux transformations of the fifth-order evolution equations in shallow water, *Chin. Ann. Math., Ser. B*, **36**(4):543–560, 2015.

39. S.F. Tian, L. Zou, T.T. Zhang, Lie symmetry analysis, conservation laws and analytical solutions for the constant astigmatism equation, *Chin. J. Phys.*, **55**(5):1938–1952, 2017.

40. G. W. Wang, T.Z. Xu, Invariant analysis and explicit solutions of the time fractional nonlinear perturbed Burgers equation, *Nonlinear Anal. Model. Control*, **20**(4):570–584, 2015.

41. G.W. Wang, A.H. Kara, J. Vega-Guzman, A. Biswas, Group analysis, nonlinear self-adjointness, conservation laws, and soliton solutions for the mKdV systems, *Nonlinear Anal. Model. Control*, **22**(3):334–346, 2017.

42. X.B. Wang, S.F. Tian, Lie symmetry analysis, conservation laws and analytical solutions of the time-fractional thin-film equation, *Comput. Appl. Math.*, **37**(5):6270–6282, 2018.

43. X.B. Wang, S.F. Tian, C.Y. Qin, T.T. Zhang, Lie symmetry analysis, conservation laws and exact solutions of the generalized time-fractional Burgers equation, *Europhys. Lett.*, **114**(2):20003, 2016.

44. X.B. Wang, S.F. Tian, C.Y. Qin, T.T. Zhang, Lie symmetry analysis, analytical solutions, and conservation laws of the generalised Whitham–Broer–Kaup–Like equations, *Z. Naturforsch. A*, **72**(3):269–279, 2017.

45. X.B. Wang, S.F. Tian, C.Y. Qin, T.T. Zhang, Lie symmetry analysis, conservation laws and analytical solutions of a time-fractional generalized KdV-type equation, *J. Nonlinear Math. Phys.*, **24**(4):516–530, 2017.

46. X.B. Wang, S.F. Tian, T.T. Zhang, Characteristics of the breather and rogue waves in a $(2+1)$-dimensional nonlinear Schrödinger equation, *Proc. Am. Math. Soc.*, **146**(8):3353–3365, 2018.

47. X.B. Wang, S.F. Tian, L. Zou, T.T. Zhang, Dynamics of solitary waves and periodic waves in a $(3+1)$-dimensional nonlinear evolution equation, *East Asian J. Appl. Math.*, **8**(3):477–497, 2018.

48. A. Yakut, Conservation laws for partial differential equations, Master thesis, Eskisehir Osmangazi University, 2012.

49. A. Yusuf, A.I. Aliyu, D. Baleanu, Lie symmetry analysis and explicit solutions for the time fractional generalized Burgers–Huxley equation, *Opt. Quantum Electron.*, **50**:94, 2018.

50. A. Yusuf, A.I. Aliyu, M.S. Hashemi, Soliton solutions, stability analysis and conservation laws for the brusselator reaction diffusion model with time-and constant-dependent coefficients, *Eur. Phys. J. Plus*, **133**(5):168, 2018.