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New extended generalized Kudryashov method for solving three nonlinear partial differential equations

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Abstract. New extended generalized Kudryashov method is proposed in this paper for the first time. Many solitons and other solutions of three nonlinear partial differential equations (PDEs), namely, the $(1+1)$-dimensional improved perturbed nonlinear Schrödinger equation with anti-cubic nonlinearity, the $(2+1)$-dimensional Davey–Sterwatson (DS) equation and the $(3+1)$-dimensional modified Zakharov–Kuznetsov (mZK) equation of ion-acoustic waves in a magnetized plasma have been presented. Comparing our new results with the well-known results are given. Our results in this article emphasize that the used method gives a vast applicability for handling other nonlinear partial differential equations in mathematical physics.

Keywords: solitary solutions, a new extended generalized Kudryashov method, the improved perturbed nonlinear Schrödinger equation with anti-cubic nonlinearity, Davey–Sterwatson (DS) equation, the modified Zakharov–Kuznetsov (mZK) equation of ion-acoustic waves in a magnetized plasma.

1 Introduction

It is well known that nonlinear complex physical phenomena are related to nonlinear partial differential equations (NLPDEs), which are implicated in many fields from physics to biology, chemistry, mechanics, engineering, etc. As mathematical models of the phenomena, the investigations of exact solutions of NLPDEs will help one to understand these phenomena better. In the past several decades, many significant methods for obtaining exact solutions of NLPDEs have been showed, such as the sine-cosine method [3, 5, 18, 44], the modified simple equation method [2, 13, 27, 28, 40, 43], the soliton ansatz method [6–8, 15, 16, 24, 32], the (G'/G)-expansion method [4, 12, 20, 21, 42], the generalized Kudryashov method [25, 30, 41], the modified transformed rational function method [38], the Lie symmetry method [29, 34], the travelling wave hypothesis [14, 33, 39], the extended trial equation method [9–11, 22, 23, 31] and so on.

The objective of this article is to use a new extended generalized Kudryashov method, for the first time, to construct new exact solutions of the following three nonlinear partial differential equations (PDEs).

(I) The $(1+1)$-dimensional improved perturbed nonlinear Schrödinger equation with anti-cubic nonlinearity [17]:

$$\begin{aligned}&\mathrm{i}E_t + aE_{xt} + bE_{xx} + \left(\frac{b_1}{|E|^4} + b_2|E|^2 + b_3|E|^4\right)E \\ &\quad = \mathrm{i}\big[\alpha E_x + \lambda(|E|^2E)_x + \upsilon_1\big(|E|^2\big)_x E\big],\end{aligned} \tag{1}$$

where $\mathrm{i} = \sqrt{-1}$, a, b, b_1, b_2, b_3, α, λ and υ_1 are real constants. The independent variables x and t represent spatial and temporal variables, respectively. The dependent variable $E(x,t)$ is the complex valued wave profile for the $(1+1)$-dimensional improved perturbed nonlinear Schrödinger equation with anti-cubic nonlinearity. Here the coefficients a and b represent the improved term that introduces stability to the NLS equation and the usual group velocity dispersion (GVD), respectively. The nonlinearities stem out from the coefficients of b_j for $j = 1,2,3$, where b_1 gives the effect of anti-cubic nonlinearity, b_2 is the coefficient of Kerr law nonlinearity, and b_3 is the coefficient of pseudo-quintic nonlinearity, respectively. The parameters α and λ represent the intermodal dispersion and the self-steepening perturbation term, respectively. Finally, υ_1 is the nonlinear dispersion coefficient. If $b_1 = 0$, there is no anti-cubic nonlinearity. which has been discussed in [17] using the soliton ansatz method.

(II) The $(2+1)$-dimensional Davey–Sterwatson (DS) equation [19, 26, 35, 45]:

$$\begin{aligned}&\mathrm{i}u_t + \frac{1}{2}\sigma^2\big(u_{xx} + \sigma^2u_{yy}\big) + \lambda|u|^2u - uv_x = 0, \\ &v_{xx} - \sigma^2v_{yy} - 2\lambda\big(|u|^2\big)_x = 0,\end{aligned} \tag{2}$$

where λ is a real constant. The case $\sigma = 1$ is called the DS-I equation, while $\sigma = \mathrm{i}$ is the DS-II equation. The parameter λ characterizes the focusing or defocusing case. The Davey–Stewartson I and II are two well-known examples of integrable equations in two space dimensions, which arise as higher dimensional generalizations of the nonlinear Schrodinger (NLS) equation [19]. They appear in many applications, for example, in the description of gravity–capillarity surface wave packets in the limit of the shallow water. Davey and Stewartson first derived their model in the context of water waves from purely physical considerations. In the context, $u(x,y,t)$ is the amplitude of a surface wave packet, while $v(x,y)$ represents the velocity potential of the mean flow interacting with the surface wave [19]. Equation (2) has been discussed in [35] using the numerical schemes method, in [45] – using the homotopy perturbation method, in [19] – using the multiple scales method and in [26] – using the first-integral method.

(III) The $(3+1)$-dimensional modified ZK equation of ion-acoustic waves in a magnetized plasma [36]:

$$16\left(\frac{\partial q}{\partial t} - c\frac{\partial q}{\partial x}\right) + 30q^{1/2}\frac{\partial q}{\partial x} + \frac{\partial^3 q}{\partial x^3} + \frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)q = 0, \tag{3}$$

where c is a positive real constant. Equation (3) has been discussed by Munro and Parkes [36], where they showed that if the electrons are nonisothermal, then the govering equation of the ZK equation is a modified form refereed to as the mZK equation (3), also they showed that the reductive perturbation method leads to a modified Zakharov–Kuznetsov (mZK) equation.

This article is organised as follows: In Section 2, we give the description of a new extended generalized Kudryashov method for the first time. In Sections 3, 4 and 5, we solve Eqs. (1), (2) and (3) using the proposed method described in Section 2. In Section 6, the graphical representations for some solutions of Eqs. (1), (2) and (3) are plotted. In Sections 7, conclusions are illustrated. To our knowledge, Eqs. (1), (2) and (3) are not discussed before using the proposed method obtained in the next section.

2 Description of a new extended generalized Kudryashov method

Consider the following nonlinear PDE:

$$P(u, u_x, u_y, u_z, u_t, u_{xx}, u_{yy}, u_{zz}, u_{xy}, u_{tt}, \dots) = 0, \tag{4}$$

where P is a polynomial in u and its partial derivatives in which the highest-order derivatives and the nonlinear terms are involved. According to the well-known generalized Kudryashov method [25, 30, 41] and with reference to [38], we can propose the main steps of a new extended generalized Kudryashov method for the first time as follows:

Step 1. We use the traveling wave transformation

$$u(x, y, z, t) = u(\xi), \qquad \xi = l_1 x + l_2 y + l_3 z - l_4 t,$$

where l_1, l_2, l_3 and l_4 are a nonzero constants, to reduce Eq. (4) to the following nonlinear ordinary differential equation (ODE):

$$H(u, u', u'', \dots) = 0, \tag{5}$$

where H is a polynomial in $u(\xi)$ and its total derivatives $u'(\xi)$, $u''(\xi)$ and so on, where $' = \mathrm{d}/\mathrm{d}\xi$.

Step 2. We assume that the formal solution of the ODE (5) can be written in the following rational form:

$$u(\xi) = \frac{\sum_{i=0}^{s} \alpha_i Q^i(\xi)}{\sum_{j=0}^{m} \beta_j Q^j(\xi)} = \frac{A[Q(\xi)]}{B[Q(\xi)]}, \tag{6}$$

where $A[Q(\xi)] = \sum_{i=0}^{s} \alpha_i Q^i(\xi)$ and $B[Q(\xi)] = \sum_{j=0}^{m} \beta_j Q^j(\xi)$ such that α_s and $\beta_m \neq 0$ and

$$Q(\xi) = \left[\frac{1}{1 \pm \exp_a(p\xi)}\right]^{1/p}, \tag{7}$$

where $\exp_a(p\xi) = a^{p\xi}$ and p is a positive integer. The function Q is the solution of the first-order differential equation

$$Q'(\xi) = \left[Q^{p+1}(\xi) - Q(\xi)\right] \ln a, \quad 0 < a \neq 1. \tag{8}$$

From (6) and (8) we have

$$\begin{aligned} u'(\xi) &= Q\left(Q^p - 1\right)\left[\frac{A'B - AB'}{B^2}\right] \ln a, \\ u''(\xi) &= Q\left(Q^p - 1\right)\left[(p+1)Q^p - 1\right]\left[\frac{A'B - AB'}{B^2}\right] \ln^2 a \\ &\quad + Q^2\left(Q^p - 1\right)^2\left[\frac{B(A''B - AB'') - 2A'B'B + 2AB'^2}{B^3}\right] \ln^2 a, \end{aligned} \tag{9}$$

and so on.

Step 3. We determine the positive integers values m and s in (6) by using the homogeneous balance method as follows: If $D(u) = s - m, D(u') = s - m + p, D(u'') = s - m + 2p$, then we have

$$D\left[u^r u^{(q)}\right] = (s - m)(r + 1) + pq. \tag{10}$$

Step 4. We substitute (6), (8) and (9) into Eq. (5) and equate all the coefficients of Q^i $(i = 0, 1, 2, \dots)$ to zero. We obtain a system of algebraic equations, which can be solved using the Maple, to find the α_i $(i = 0, 1, \dots, s)$, β_j $(j = 0, 1, \dots, m)$, l_1, l_2, l_3 and l_4. Consequently, we can get the exact solutions of Eq. (4).

The obtained solutions will be depended on the symmetrical hyperbolic Fibonacci functions given in [1] and [37]. The symmetrical Fibonacci sine, cosine, tangent and cotangent functions are defined as

$$\begin{aligned} \mathrm{sFs}(\xi) &= \frac{a^\xi - a^{-\xi}}{\sqrt{5}}, & \mathrm{cFs}(\xi) &= \frac{a^\xi + a^{-\xi}}{\sqrt{5}}, \\ \mathrm{tanFs}(\xi) &= \frac{a^\xi - a^{-\xi}}{a^\xi + a^{-\xi}}, & \mathrm{cotFs}(\xi) &= \frac{a^\xi + a^{-\xi}}{a^\xi - a^{-\xi}}, \\ \mathrm{sFs}(\xi) &= \frac{2}{\sqrt{5}} \mathrm{sh}[\xi \ln a], & \mathrm{cFs}(\xi) &= \frac{2}{\sqrt{5}} \mathrm{ch}[\xi \ln a], \\ \mathrm{tanFs}(\xi) &= \tanh[\xi \ln a], & \mathrm{cotFs}(\xi) &= \coth[\xi \ln a]. \end{aligned} \tag{11}$$

3 On solving Eq. (1) using the new extended generalized Kudryashov method

In this section, we use the above method describing in Section 2 for solving Eq. (1). To this aim, we assume that Eq. (1) has the formal solution

$$E(x,t) = \psi(\xi) \mathrm{e}^{\mathrm{i}[\chi(\xi) - \omega t]}, \quad \xi = kx - \epsilon t, \tag{12}$$

where $\psi(\xi)$ and $\chi(\xi)$ are real functions of ξ, while ω, k and ϵ are real constants. Substituting (12) into Eq. (1) and separating the real and the imaginary parts, we have the two nonlinear ODEs:

$$(\epsilon + ak\omega + \alpha k)\chi'\psi + \omega\psi + b_1\psi^{-3} + b_2\psi^3 + b_3\psi^5 + k(bk - a\epsilon)\psi'' \\ - k(bk - a\epsilon)\chi'^2\psi + k\lambda\chi'\psi^3 = 0 \tag{13}$$

and

$$-(\epsilon + \alpha k + ak\omega)\psi' + 2k(bk - a\epsilon)\chi'\psi' + k(bk - a\epsilon)\chi''\psi \\ - k[3\lambda + 2\upsilon_1]\psi^2\psi' = 0. \tag{14}$$

To solve the above coupled pair of Eqs. (13) and (14), we introduce the ansatz:

$$\chi'(\xi) = \beta\psi^2(\xi) + \gamma, \tag{15}$$

where β and γ are constants. Inserting (15) into Eq. (14), we obtain

$$\beta = \frac{3\lambda + 2\upsilon_1}{4(kb - a\epsilon)} \quad \text{and} \quad \gamma = \frac{\epsilon + \alpha k + ak\omega}{2k(bk - a\epsilon)}. \tag{16}$$

Substituting (15) along with (16) into Eq. (13), we have the nonlinear ODE

$$\psi^3\psi'' + A_1 + A_2\psi^4 + A_3\psi^6 + A_4\psi^8 = 0, \tag{17}$$

where the coefficients A_1, A_2, A_3 and A_4 are given by

$$A_1 = \frac{b_1}{k(bk - a\epsilon)}, \qquad A_2 = \frac{(\epsilon + \alpha k + ak\omega)^2 + 4k\omega(bk - a\epsilon)}{4k^2(bk - a\epsilon)^2},$$
$$A_3 = \frac{2b_2(bk - a\epsilon) + \lambda(\epsilon + \alpha k + ak\omega)}{2k(bk - a\epsilon)^2},$$
$$A_4 = \frac{16b_3(bk - a\epsilon) - k(3\lambda + 2\upsilon_1)^2 + 4k\lambda(3\lambda + 2\upsilon_1)}{16k(bk - a\epsilon)^2},$$

where $k(bk - a\epsilon) \neq 0$. Setting

$$\psi^2(\xi) = g(\xi), \tag{18}$$

where $g(\xi)$ is a positive function of ξ. Substituting (18) into (17), we have the new equation

$$2gg'' - g'^2 + 4\big(A_1 + A_2g^2 + A_3g^3 + A_4g^4\big) = 0. \tag{19}$$

By balancing gg'' with g^4 in (19), the following formula is obtained:

$$2(s - m) + 2p = 4(s - m) \quad \Longrightarrow \quad s = m + p.$$

Let us now discuss the following cases.

Case 1. If we choose $p = 1$ and $m = 1$, then $s = 2$. Thus, we deduce from (6) that

$$\psi(\xi) = \frac{\alpha_0 + \alpha_1 Q(\xi) + \alpha_2 Q^2(\xi)}{\beta_0 + \beta_1 Q(\xi)}, \tag{20}$$

where α_0, α_1, α_2, β_0 and β_1 are real constants to be determined such that α_2 and $\beta_1 \neq 0$. Substituting (20) along with (8) into Eq. (19), collecting the coefficients of each power of Q^i $(i = 0, 1, \dots, 8)$ and setting each of these coefficients to zero, we obtain a system of algebraic equations, which can be solved using Maple, we obtain the following results.

Result 1.

$$\begin{aligned}
&A_1 = \frac{\alpha_0^2 \ln^2 a\,(\beta_1^2 - 2\beta_0\beta_1 + \beta_0^2)}{4\beta_0^2\beta_1^2}, \qquad A_2 = \frac{-\ln^2 a\,(\beta_1^2 - 6\beta_0\beta_1 + 6\beta_0^2)}{4\beta_1^2},\\
&A_3 = \frac{\beta_0^2 \ln^2 a\,(2\beta_0 - \beta_1)}{\alpha_0\beta_1^2}, \qquad A_4 = \frac{-3\beta_0^4 \ln^2 a}{4\alpha_0^2\beta_1^2},\\
&\beta_0 = \beta_0, \qquad \beta_1 = \beta_1, \qquad \alpha_0 = \alpha_0, \qquad \alpha_1 = 0, \qquad \alpha_2 = \frac{-\beta_1^2\alpha_0}{\beta_0^2}.
\end{aligned} \tag{21}$$

In this case, from (7), (12), (18), (20) and (21) we deduce that Eq. (1) has the solution

$$E(x,t) = \left\{\frac{\alpha_0}{\beta_0} - \frac{\alpha_0\beta_1}{\beta_0^2}\,\frac{1}{1 \pm a^\xi}\right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])}. \tag{22}$$

From (22) we deduce that Eq. (1) has the dark soliton solution

$$E(x,t) = \left\{\frac{\alpha_0}{\beta_0} - \frac{\alpha_0\beta_1}{2\beta_0^2}\left[1 - \tanh\frac{\xi \ln a}{2}\right]\right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])}$$

and the singular soliton solution

$$E(x,t) = \left\{\frac{\alpha_0}{\beta_0} - \frac{\alpha_0\beta_1}{2\beta_0^2}\left[1 - \coth\frac{\xi \ln a}{2}\right]\right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])},$$

provided $\alpha_0\beta_0 > 0$ and $\alpha_0\beta_1 < 0$.

Result 2.

$$\begin{aligned}
&A_1 = \frac{-3[9A_3^4 + 8A_4(2A_4 \ln^2 a + 3A_3^2)\ln^2 a]}{4096A_4^3}, \qquad A_2 = \frac{9A_3^2 + 4A_4 \ln^2 a}{32A_4},\\
&\alpha_0 = 0, \qquad \alpha_1 = \frac{-\beta_1[2\ln a\sqrt{-3A_4} + 3A_3]}{8A_4}, \qquad \alpha_2 = \frac{\beta_1 \ln a\sqrt{-3A_4}}{2A_4},\\
&\beta_0 = 0, \qquad \beta_1 = \beta_1,
\end{aligned} \tag{23}$$

provided $A_4 < 0$. In this case, from (7), (12), (18), (20) and (23) we deduce that Eq. (1) has the solution

$$E(x,t) = \left\{\frac{-3A_3}{8A_4} + \frac{\ln a\sqrt{-3A_4}}{4A_4}\left[\frac{1 \mp a^\xi}{1 \pm a^\xi}\right]\right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])}. \tag{24}$$

From (24) we deduce that Eq. (1) has the dark soliton solution

$$E(x,t)=\left\{-\frac{3A_3}{8A_4}-\frac{\ln a\sqrt{-3A_4}}{4A_4}\tanh\frac{\xi\ln a}{2}\right\}^{1/2}\mathrm{e}^{(\mathrm{i}[\chi(\xi)-\omega t])} \tag{25}$$

and the singular soliton solution

$$E(x,t)=\left\{-\frac{3A_3}{8A_4}-\frac{\ln a\sqrt{-3A_4}}{4A_4}\coth\frac{\xi\ln a}{2}\right\}^{1/2}\mathrm{e}^{(\mathrm{i}[\chi(\xi)-\omega t])},$$

provided $A_4<0$ and $A_3>0$.

Case 2. If we choose $p=2$ and $m=2$, then $s=4$, thus, we deduce from (6) that

$$g(\xi)=\frac{\alpha_0+\alpha_1 Q(\xi)+\alpha_2 Q^2(\xi)+\alpha_3 Q^3(\xi)+\alpha_4 Q^4(\xi)}{\beta_0+\beta_1 Q(\xi)+\beta_2 Q^2(\xi)}, \tag{26}$$

where α_0, α_1, α_2, α_3, α_4, β_0, β_1 and β_2 are real constants to be determined such that α_4 and $\beta_2\neq 0$. Substituting (26) along with (8) into Eq. (19), collecting the coefficients of each power of Q^i $(i=0,1,\dots,16)$ and setting each of these coefficients to zero, we obtain a set of algebraic equations, which can be solved by Maple, to get the following results.

Result 1.

$$\begin{aligned}
&\alpha_0=0,\qquad \alpha_1=\alpha_1,\qquad \alpha_2=\frac{\alpha_1\beta_2}{\beta_1},\qquad \alpha_3=\frac{3\beta_1\ln a}{\sqrt{-3A_4}},\qquad \alpha_4=\frac{3\beta_2\ln(a)}{\sqrt{-3A_4}},\\
&A_1=\frac{-\alpha_1^2[6A_4\alpha_1\beta_1\ln a+[A_4\alpha_1^2-3\beta_1^2\ln^2 a]\sqrt{-3A_4}]}{3\beta_1^4\sqrt{-3A_4}},\\
&A_2=\frac{6A_4\alpha_1\beta_1\ln a+[2A_4\alpha_1^2-\beta_1^2\ln^2 a]\sqrt{-3A_4}}{\beta_1^2\sqrt{-3A_4}},\\
&A_3=\frac{-4A_4[3\beta_1\ln a+2\alpha_1\sqrt{-3A_4}]}{3\beta_1\sqrt{-3A_4}},\qquad \beta_0=0,\qquad \beta_1=\beta_1,\qquad \beta_2=\beta_2,
\end{aligned} \tag{27}$$

provided $A_4<0$. In this case, from (7), (12), (18), (26) and (27) we deduce that Eq. (1) has the solution

$$E(x,t)=\left\{\frac{\alpha_1}{\beta_1}+\frac{3\ln a}{\sqrt{-3A_4}}\frac{1}{1\pm a^{2\xi}}\right\}^{1/2}\mathrm{e}^{(\mathrm{i}[\chi(\xi)-\omega t])}.$$

Equation (1) has the symmetrical Fibonacci cotangent function solutions

$$E(x,t)=\left\{\frac{\alpha_1}{\beta_1}+\frac{3\ln a}{2\sqrt{-3A_4}}\big[1-\mathrm{tanFs}(\xi)\big]\right\}^{1/2}\mathrm{e}^{(\mathrm{i}[\chi(\xi)-\omega t])} \tag{28}$$

and

$$E(x,t)=\left\{\frac{\alpha_1}{\beta_1}+\frac{3\ln a}{2\sqrt{-3A_4}}\big[1-\mathrm{cotFs}(\xi)\big]\right\}^{1/2}\mathrm{e}^{(\mathrm{i}[\chi(\xi)-\omega t])}. \tag{29}$$

From (28) we deduce that Eq. (1) has the dark soliton solution

$$E(x,t) = \left\{ \frac{\alpha_1}{\beta_1} + \frac{3\ln a}{2\sqrt{-3A_4}} \left[1 - \tanh(\xi \ln a)\right] \right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])}, \tag{30}$$

and from (29) we deduce that Eq. (1) has the singular soliton solution

$$E(x,t) = \left\{ \frac{\alpha_1}{\beta_1} + \frac{3\ln a}{2\sqrt{-3A_4}} \left[1 - \coth(\xi \ln a)\right] \right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])}, \tag{31}$$

provided $A_4 < 0$ and $\alpha_1 \beta_1 > 0$.

Result 2.

$$\begin{aligned}
&\alpha_0 = 0, \qquad \alpha_1 = 0, \qquad \alpha_2 = \frac{-\beta_2(4\ln a\sqrt{-3A_4} + 3A_3)}{8A_4}, \qquad \alpha_3 = 0,\\
&\alpha_4 = \frac{\beta_2 \ln a\sqrt{-3A_4}}{A_4}, \qquad \beta_0 = 0, \qquad \beta_1 = 0, \qquad \beta_2 = \beta_2,\\
&A_1 = \frac{-3[256A_4^2\ln^4(a) + 96A_3^2A_4\ln^2(a) + 9A_3^4]}{4096A_4^3},\\
&A_2 = \frac{16A_4\ln^2(a) + 9A_3^2}{32A_4},
\end{aligned} \tag{32}$$

provided $A_4 < 0$. In this case, from (7), (12), (18), (26) and (32) we deduce that Eq. (1) has the solution

$$E(x,t) = \left\{ -\frac{3A_3}{8A_4} + \frac{\ln a}{2}\sqrt{\frac{-3}{A_4}}\,\frac{1 \mp a^{2\xi}}{1 \pm a^{2\xi}} \right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])}.$$

Equation (1) has the symmetrical Fibonacci cotangent function solutions

$$E(x,t) = \left\{ -\frac{3A_3}{8A_4} - \frac{\ln a}{2}\sqrt{\frac{-3}{A_4}}\,\mathrm{tanFs}(\xi) \right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])} \tag{33}$$

and

$$E(x,t) = \left\{ -\frac{3A_3}{8A_4} - \frac{\ln a}{2}\sqrt{\frac{-3}{A_4}}\,\mathrm{cotFs}(\xi) \right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])}. \tag{34}$$

From (33) we deduce that Eq. (1) has the dark soliton solution

$$E(x,t) = \left\{ -\frac{3A_3}{8A_4} - \frac{\ln a}{2}\sqrt{\frac{-3}{A_4}}\,\tanh(\xi \ln a) \right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])},$$

and from (34) we deduce that Eq. (1) has the singular soliton solution

$$E(x,t) = \left\{ -\frac{3A_3}{8A_4} - \frac{\ln a}{2}\sqrt{\frac{-3}{A_4}}\,\coth(\xi \ln a) \right\}^{1/2} \mathrm{e}^{(\mathrm{i}[\chi(\xi) - \omega t])},$$

provided $A_4 < 0$ and $A_3 > 0$. Simliarly, we can find many other solutions by choosing another values for s, m and p.

4 On solving Eq. (2) using the new extended generalized Kudryashov method

In this section, we use the above method describing in Section 2 for solving Eq. (2). To this aim, we assume that Eq. (2) has the formal solution

$$u(x,y,t) = u(\xi)\mathrm{e}^{\mathrm{i}\eta(x,y,t)}, \qquad v(x,y) = v(\xi), \tag{35}$$

and

$$\xi = x - 2\alpha y + \alpha t, \qquad \eta(x,y,t) = \alpha x + y + kt + l, \tag{36}$$

where $u(\xi)$, $\eta(x,y,t)$ and $v(\xi)$ are all real functions, while α, k and l are real constants. Substituting (35) and (36) into Eq. (2) yield the following system of ODEs:

$$\begin{aligned}&\sigma^2\big(1+4\sigma^2\alpha^2\big)u''(\xi) + 2\lambda u^3(\xi) - \big(2k+\sigma^2\alpha^2+\sigma^4\big)u(\xi)\\ &\quad - 2u(\xi)v'(\xi) = 0, \qquad (37)\\ &\big(1-4\sigma^2\alpha^2\big)v''(\xi) = 2\lambda\big[u^2(\xi)\big]'. \qquad (38)\end{aligned}$$

Integrating (38) with respect to ξ, we obtain

$$v'(\xi) = \frac{2\lambda}{(1-4\sigma^2\alpha^2)}u^2(\xi) + \varepsilon, \tag{39}$$

where ε is the constant of integration, and $\alpha^2 \neq \pm 1/4$. Substituting (39) into (37), we have

$$\begin{aligned}&\sigma^2\big(1+4\sigma^2\alpha^2\big)u''(\xi) + 2\lambda\left[1 - \frac{2}{(1-4\sigma^2\alpha^2)}\right]u^3(\xi)\\ &\quad - \big(2k+\sigma^2\alpha^2+\sigma^4+2\varepsilon\big)u(\xi) = 0. \qquad (40)\end{aligned}$$

By balancing u'' with u^3 in (40), the following formula is obtained:

$$(s-m) + 2p = 3(s-m) \quad \Longrightarrow \quad s = m+p.$$

Let us now discuss the following cases.

Case 1. If we choose $p=1$ and $m=1$, then $s=2$. Thus, we deduce from (6) that

$$u(\xi) = \frac{\alpha_0 + \alpha_1 Q(\xi) + \alpha_2 Q^2(\xi)}{\beta_0 + \beta_1 Q(\xi)}, \tag{41}$$

where α_0, α_1, α_2, β_0 and β_1 are real constants to be determined such that α_2 and $\beta_1 \neq 0$. Substituting (41) along with (8) into Eq. (40), collecting the coefficients of each power of Q^i $(i = 0, 1, \dots, 6)$ and setting each of these coefficients to zero, we obtain a system of algebraic equations, which can be solved by Maple, we obtain the following results.

Result 1.

$$
\begin{aligned}
&\varepsilon = -\frac{1}{2}\left[2k + \sigma^2\left(\alpha^2 + \sigma^2\right) - \sigma^2 \ln^2 a \left(1 + 4\sigma^2\alpha^2\right)\right],\\
&\alpha_0 = 0, \qquad \alpha_1 = \alpha_1, \qquad \alpha_2 = -\alpha_1,\\
&\beta_0 = \frac{-\alpha_1}{2\sigma \ln a}\sqrt{\frac{\lambda}{(1 - 4\sigma^2\alpha^2)}}, \qquad \beta_1 = \frac{\alpha_1}{\sigma \ln a}\sqrt{\frac{\lambda}{(1 - 4\sigma^2\alpha^2)}},
\end{aligned}
\tag{42}
$$

provided $(1 - 4\sigma^2\alpha^2)\lambda > 0$. In this case, from (7), (35), (36), (41) and (42) we deduce that Eq. (2) has the solution

$$
u(x, y, t) = \left(\pm 2\sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}}\, \frac{a^\xi}{1 - a^{2\xi}}\right) \mathrm{e}^{\mathrm{i}\eta(x,y,t)}. \tag{43}
$$

From (43) we deduce that Eq. (2) has the singular solitary wave solution

$$
u(x, y, t) = \left(\mp \sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}}\, \operatorname{csch}[\xi \ln a]\right) \mathrm{e}^{\mathrm{i}\eta(x,y,t)}. \tag{44}
$$

Result 2.

$$
\begin{aligned}
&\varepsilon = -\frac{1}{4}\left[4k + 2\sigma^2\left(\alpha^2 + \sigma^2\right) + \sigma^2 \ln^2 a \left(1 + 4\sigma^2\alpha^2\right)\right],\\
&\alpha_0 = 0, \qquad \alpha_1 = \frac{-\beta_1 \sigma \ln a}{2}\sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}},\\
&\alpha_2 = \beta_1 \sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}}, \qquad \beta_0 = 0, \qquad \beta_1 = \beta_1,
\end{aligned}
\tag{45}
$$

provided $(1 - 4\sigma^2\alpha^2)\lambda > 0$. In this case, from (7), (35), (36), (41) and (45) we deduce that Eq. (2) has the solution

$$
u(x, y, t) = \left(\frac{\sigma \ln a}{2}\sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}}\, \frac{1 \mp a^\xi}{1 \pm a^\xi}\right) \mathrm{e}^{\mathrm{i}\eta(x,y,t)}. \tag{46}
$$

From (46) we deduce that Eq. (2) has the shock wave solution

$$
u(x, y, t) = \left(-\frac{\sigma \ln a}{2}\sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}}\, \tanh \frac{\xi \ln a}{2}\right) \mathrm{e}^{\mathrm{i}\eta(x,y,t)}
$$

and the singular solitary wave solution

$$
u(x, y, t) = \left(-\frac{\sigma \ln a}{2}\sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}}\, \coth \frac{\xi \ln a}{2}\right) \mathrm{e}^{\mathrm{i}\eta(x,y,t)}.
$$

Case 2. If we choose $p = 2$ and $m = 2$, then $s = 4$. Thus, we deduce from (6) that

$$
u(\xi) = \frac{\alpha_0 + \alpha_1 Q(\xi) + \alpha_2 Q^2(\xi) + \alpha_3 Q^3(\xi) + \alpha_4 Q^4(\xi)}{\beta_0 + \beta_1 Q(\xi) + \beta_2 Q^2(\xi)}, \tag{47}
$$

where α_0, α_1, α_2, α_3, α_4, β_0, β_1 and β_2 are real constants to be determined such that α_4 and $\beta_2 \neq 0$. Substituting (47) along with (8) into Eq. (40), collecting the coefficients of each power of Q^i ($i = 0, 1, \ldots, 12$) and setting each of these coefficients to zero, we obtain a set of algebraic equations, which can be solved by Maple, to get the following results.

Result 1.

$$\begin{aligned}
&\varepsilon = -\frac{1}{2}\left[2k + \sigma^2\left(\alpha^2 + \sigma^2\right) + 8\sigma^2 \ln^2 a \left(1 + 4\sigma^2\alpha^2\right)\right],\\
&\alpha_0 = \beta_2 \sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}}, \qquad \alpha_1 = 0,\\
&\alpha_2 = -2\beta_2 \sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}},\\
&\alpha_3 = 0, \qquad \alpha_4 = 2\beta_2 \sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}},\\
&\beta_0 = \frac{-1}{2}\beta_2, \qquad \beta_1 = 0, \qquad \beta_2 = \beta_2,
\end{aligned} \tag{48}$$

provided $(1 - 4\sigma^2\alpha^2)\lambda > 0$. In this case, from (7), (35), (36), (47) and (48) we deduce that Eq. (2) has the solution

$$u(x, y, t) = \left(2\sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}}\, \frac{1 + a^{4\xi}}{1 - a^{4\xi}}\right) \mathrm{e}^{\mathrm{i}\eta(x,y,t)}. \tag{49}$$

From (49) we deduce that Eq. (2) has the singular solitary wave solution

$$u(x, y, t) = \left(-2\sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}} \coth[2\xi \ln a]\right) \mathrm{e}^{\mathrm{i}\eta(x,y,t)}. \tag{50}$$

Result 2.

$$\begin{aligned}
&\varepsilon = -\frac{1}{2}\left[2k + \sigma^2\left(\alpha^2 + \sigma^2\right) - 4\sigma^2 \ln^2 a \left(1 + 4\sigma^2\alpha^2\right)\right],\\
&\alpha_0 = 0, \qquad \alpha_1 = 0, \qquad \alpha_2 = -2\beta_2 \sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}},\\
&\alpha_3 = 0, \qquad \alpha_4 = 2\beta_2 \sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}},\\
&\beta_0 = -\frac{1}{2}\beta_2, \qquad \beta_1 = 0, \qquad \beta_2 = \beta_2,
\end{aligned} \tag{51}$$

provided $(1 - 4\sigma^2\alpha^2)\lambda > 0$. In this case, from (7), (35), (36), (47) and (51) we deduce that Eq. (2) has the solution

$$u(x, y, t) = \mp\left(4\sigma \ln a \sqrt{\frac{(1 - 4\sigma^2\alpha^2)}{\lambda}}\, \frac{a^{2\xi}}{1 - a^{4\xi}}\right) \mathrm{e}^{\mathrm{i}\eta(x,y,t)}. \tag{52}$$

From (52) we deduce that Eq. (2) has the singular solitary wave solution

$$u(x,y,t) = \pm\left(2\sigma\ln a\sqrt{\frac{(1-4\sigma^2\alpha^2)}{\lambda}}\operatorname{csch}[2\xi\ln a]\right)\mathrm{e}^{\mathrm{i}\eta(x,y,t)}.$$

Similarly, we can find many other solutions by choosing another values for s, m and p.

5 On solving Eq. (3) using the new extended generalized Kudryashov method

In this section, we use the above method describing in Section 2 for solving Eq. (3). To this aim, we assume that Eq. (3) has the formal solution

$$q(x,y,z,t) = B(\xi), \quad \xi = kx + ly + \rho z - \omega t, \tag{53}$$

where $B(\xi)$ is a real function, while k, l, ρ and ω are real constants, to reduce Eq. (3) into the nonlinear ODE

$$-16(\omega + kc)B'(\xi) + 30kB^{1/2}(\xi)B'(\xi) + k\bigl(k^2 + l^2 + \rho^2\bigr)B'''(\xi) = 0, \tag{54}$$

where $' = \mathrm{d}/\mathrm{d}\xi$. Integrating Eq. (54) once with respect to ξ, we have

$$-16(\omega + kc)B(\xi) + 20kB^{3/2}(\xi) + k\bigl(k^2 + l^2 + \rho^2\bigr)B''(\xi) + \varepsilon = 0,$$

where ε is the constant of integration. Setting

$$B(\xi) = H^2(\xi), \tag{55}$$

we get the equation

$$-16(\omega+kc)H^2(\xi)+20kH^3(\xi)+2k\bigl(k^2+l^2+\rho^2\bigr)\bigl[H'^2(\xi)+H(\xi)H''(\xi)\bigr]+\varepsilon = 0. \tag{56}$$

By balancing HH'' with H^3 in (56), the following formula is obtained:

$$2(s-m) + 2p = 3(s-m) \quad \Longrightarrow \quad s = m + 2p.$$

Let us now discuss the following cases.

Case 1. If we choose $p = 1$ and $m = 1$, then $s = 3$, Thus, we deduce from (6) that

$$H(\xi) = \frac{\alpha_0 + \alpha_1 Q(\xi) + \alpha_2 Q^2(\xi) + \alpha_3 Q^3(\xi)}{\beta_0 + \beta_1 Q(\xi)}, \tag{57}$$

where α_0, α_1, α_2, α_3, β_0 and β_1 are real constants to be determined, such that α_3 and $\beta_1 \neq 0$. Substituting (57) along with (8) into Eq. (56), collecting the coefficients of each power of Q^i ($i = 0, 1, \dots, 10$) and setting each of these coefficients to zero, we obtain a system of algebraic equations, which can be solved by Maple, we obtain the following results.

Result 1.

$$\alpha_0 = 0, \quad \alpha_1 = \frac{-\beta_1(k^2+l^2+\rho^2)\ln^2 a}{6}, \qquad \alpha_2 = \beta_1(k^2+l^2+\rho^2)\ln^2 a,$$
$$\alpha_3 = -\beta_1(k^2+l^2+\rho^2)\ln^2 a, \qquad \omega = \frac{-k[(k^2+l^2+\rho^2)\ln^2 a + 4c]}{4}, \tag{58}$$
$$\varepsilon = \frac{-k[(k^2+l^2+\rho^2)\ln^2 a]^3}{54}, \qquad \beta_0 = 0, \qquad \beta_1 = \beta_1.$$

In this case, from (7), (53), (55), (57) and (58) we deduce that Eq. (3) has the solution

$$B(\xi) = \left[\frac{-(k^2+l^2+\rho^2)\ln^2 a}{6}\left(1 \mp \frac{6a^\xi}{(1\pm a^\xi)^2}\right)\right]^2. \tag{59}$$

From (59) we deduce that Eq. (3) has the solitary wave solution

$$B(\xi) = \frac{(k^2+l^2+\rho^2)^2\ln^4(a)}{36}\left[1 - \frac{3}{2}\operatorname{sech}^2\frac{\xi\ln a}{2}\right]^2 \tag{60}$$

and the singular solitary wave solution

$$B(\xi) = \frac{(k^2+l^2+\rho^2)^2\ln^4 a}{36}\left[1 + \frac{3}{2}\operatorname{csch}^2\frac{\xi\ln a}{2}\right]^2,$$

where $\xi = kx + ly + \rho z + (k/4)[(k^2+l^2+\rho^2)\ln^2 a + 4c]t$.

Result 2.

$$\alpha_0 = 0, \qquad \alpha_1 = 0, \qquad \alpha_2 = \beta_1(k^2+l^2+\rho^2)\ln^2 a,$$
$$\alpha_3 = -\beta_1\left(k^2+l^2+\rho^2\right)\ln^2 a, \qquad \omega = \frac{k[(k^2+l^2+\rho^2)\ln^2 a - 4c]}{4}, \tag{61}$$
$$\varepsilon = 0, \qquad \beta_0 = 0, \qquad \beta_1 = \beta_1.$$

In this case, from (7), (53), (55), (57) and (61) we deduce that Eq. (3) has the solution

$$B(\xi) = \left(k^2+l^2+\rho^2\right)^2\ln^4 a\left(\frac{a^\xi}{(1\pm a^\xi)^2}\right)^2. \tag{62}$$

From (62) we deduce that Eq. (3) has the solitary wave solution

$$B(\xi) = \frac{(k^2+l^2+\rho^2)^2\ln^4 a}{16}\operatorname{sech}^4\frac{\xi\ln a}{2}$$

and the singular solitary wave solution

$$B(\xi) = \frac{(k^2+l^2+\rho^2)^2\ln^4 a}{16}\operatorname{csch}^4\frac{\xi\ln a}{2},$$

where $\xi = kx + ly + \rho z - (k/4)[(k^2+l^2+\rho^2)\ln^2 a - 4c]t$.

Case 2. If we choose $p = 2$ and $m = 1$, then $s = 5$. Thus, we deduce from (6) that

$$H(\xi) = \frac{\alpha_0 + \alpha_1 Q(\xi) + \alpha_2 Q^2(\xi) + \alpha_3 Q^3(\xi) + \alpha_4 Q^4(\xi) + \alpha_5 Q^5(\xi)}{\beta_0 + \beta_1 Q(\xi)}, \tag{63}$$

where α_0, α_1, α_2, α_3, α_4, α_5, β_0 and β_1 are real constants to be determined, such that α_5 and $\beta_1 \neq 0$. Substituting (63) along with (8) into Eq. (56), collecting the coefficients of each power of Q^i ($i = 0, 1, \dots, 16$) and setting each of these coefficients to zero, we obtain a set of algebraic equations, which can be solved by Maple, to get the following results.

Result 1.

$$\begin{gathered}
\alpha_0 = -\frac{2}{3}\beta_0\big(k^2+l^2+\rho^2\big)\ln^2 a, \qquad \alpha_1 = -\frac{2}{3}\beta_1(k^2+l^2+\rho^2\big)\ln^2 a, \qquad \beta_0 = \beta_0, \\
\beta_1 = \beta_1, \qquad \alpha_2 = 4\beta_0\big(k^2+l^2+\rho^2\big)\ln^2 a, \qquad \alpha_3 = 4\beta_1\big(k^2+l^2+\rho^2\big)\ln^2 a, \\
\alpha_4 = -4\beta_0\big(k^2+l^2+\rho^2\big)\ln^2 a, \qquad \alpha_5 = -4\beta_1\big(k^2+l^2+\rho^2\big)\ln^2 a, \\
\omega = -k\big[\big(k^2+l^2+\rho^2\big)\ln^2 a + c\big], \qquad \varepsilon = \frac{-32}{27}k\big[\big(k^2+l^2+\rho^2\big)\ln^2 a\big]^3.
\end{gathered} \tag{64}$$

In this case, (7), (53), (55), (63) and (64) we deduce that Eq. (3) has the solution

$$B(\xi) = \left[\frac{-2}{3}\big(k^2 + l^2 + \rho^2\big)\ln^2 a \left(1 - \frac{6}{1 \pm a^{2\xi}} + \frac{6}{(1 \pm a^{2\xi})^2}\right)\right]^2.$$

With the help of (11), Eq. (3) has the symmetrical Fibonacci cotangent function solutions

$$B(\xi) = \frac{1}{9}\big(k^2 + l^2 + \rho^2\big)^2 \ln^4 a \left[1 - 3\operatorname{tanFs}^2(\xi)\right]^2, \tag{65}$$

and

$$B(\xi) = \frac{1}{9}\big(k^2 + l^2 + \rho^2\big)^2 \ln^4 a \left[1 - 3\operatorname{cotFs}^2(\xi)\right]^2. \tag{66}$$

From (65) we deduce that Eq. (3) has the shock wave solution

$$B(\xi) = \frac{1}{9}\big(k^2 + l^2 + \rho^2\big)^2 \ln^4 a \left[1 - 3\tanh^2(\xi \ln a)\right]^2,$$

and from (66) we deduce that Eq. (3) has the singular solitary wave solution

$$B(\xi) = \frac{1}{9}(k^2 + l^2 + \rho^2)^2 \ln^4 a \left[1 - 3\coth^2(\xi \ln a)\right]^2,$$

where $\xi = kx + ly + \rho z + k[(k^2 + l^2 + \rho^2)\ln^2 a + c]t$.

Result 2.

$$\begin{aligned} &\alpha_0 = 0, \qquad \alpha_1 = 0, \qquad \beta_0 = \beta_0, \qquad \beta_1 = \beta_1, \\ &\alpha_2 = 4\beta_0\left(k^2 + l^2 + \rho^2\right)\ln^2 a, \qquad \alpha_3 = 4\beta_1\left(k^2 + l^2 + \rho^2\right)\ln^2 a, \\ &\alpha_4 = -4\beta_0\left(k^2 + l^2 + \rho^2\right)\ln^2 a, \qquad \alpha_5 = -4\beta_1\left(k^2 + l^2 + \rho^2\right)\ln^2 a, \\ &\varepsilon = 0, \qquad \omega = k\left[\left(k^2 + l^2 + \rho^2\right)\ln^2 a - c\right]. \end{aligned} \tag{67}$$

In this case, from (7), (53), (55), (63) and (67) we deduce that Eq. (3) has the solution

$$B(\xi) = 16\left(k^2 + l^2 + \rho^2\right)^2 \ln^4 a \left(\frac{a^{2\xi}}{(1 \pm a^{2\xi})^2}\right)^2.$$

With the help of (11), Eq. (3) has the symmetrical Fibonacci cotangent function solutions

$$B(\xi) = \left(k^2 + l^2 + \rho^2\right)^2 \ln^4 a \left[1 - \mathrm{tanFs}^2(\xi)\right]^2 \tag{68}$$

and

$$B(\xi) = \left(k^2 + l^2 + \rho^2\right)^2 \ln^4 a \left[1 - \mathrm{cotFs}^2(\xi)\right]^2. \tag{69}$$

From (68) we deduce that Eq. (3) has the solitary wave solution

$$B(\xi) = \left(k^2 + l^2 + \rho^2\right)^2 \ln^4 a \ \mathrm{sech}^4[\xi \ln a], \tag{70}$$

and from (69) we deduce that Eq. (3) has the singular solitary wave solution

$$B(\xi) = \left(k^2 + l^2 + \rho^2\right)^2 \ln^4 a \,\mathrm{csch}^4[\xi \ln a],$$

where $\xi = kx + ly + \rho z - k[(k^2 + l^2 + \rho^2)\ln^2 a - c]t$.

6 Some graphical representations of some solutions

In this section, we present the graphs of some solutions for Eqs. (1), (2) and (3). Let us now examine Figs. 1–6. as it illustrates some of our solutions obtained in this paper. To this aim, we select some special values of the obtained parameters: $a = 4$, $b_2 = 1/2$, $b_3 = 1/4$, $k = 1$, $b = 2$, $\upsilon = 3$, $\lambda = 2$, $\alpha = 2$, $\omega = 1/2$ and $-10 \leqslant x, t \leqslant 10$ in Fig. 1; $a = 4$, $\alpha_1 = 2$, $\beta_1 = 1/4$, $b_3 = 4$, $k = 2$, $b = -1/2$, $\upsilon = 1/3$, $\lambda = 2$, $\alpha = 2$, $\psi = 1/12$ and $-10 \leqslant x, t \leqslant 10$ in Fig. 2; $a = 4$, $\lambda = -3$, $\alpha = 2$, $t = 1/2$, $\sigma = 1$ and $-10 \leqslant x, y \leqslant 10$ in Fig. 3; $a = 5/2$, $\lambda = 2$, $\alpha = 1/4$, $y = 0$, $\sigma = -1$ and $-10 \leqslant x, t \leqslant 10$ in Fig. 4; $a = 2$, $k = 2$, $l = 3$, $\rho = 1$, $c = 1/4$, $z = 1$, $t = 4$, $\sigma = 1$ and $-10 \leqslant x, y \leqslant 10$ in Fig. 5; $a = 5$, $k = 1/2$, $l = 1/2$, $\rho = -1/\sqrt{2}$, $c = 1/2$, $x = 2$, $y = -2$ and $-10 \leqslant z, t \leqslant 10$ in Fig 6.

From Figs. 1–6 one can see that the obtained solutions possess the dark soliton solution, the singular soliton solution, the singular solitary wave solution, the solitary wave solution. Also, these figures express the behaviour of these solutions, which give some perspective readers how the behaviour solutions are produced.

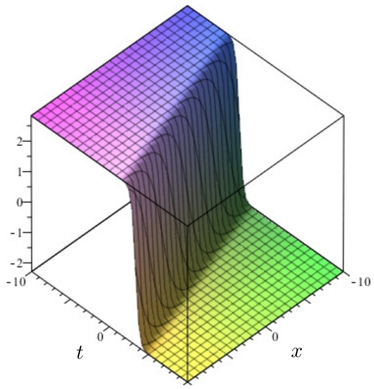

Figure 1. Dark soliton solution $\Psi(\xi)$ of (25).

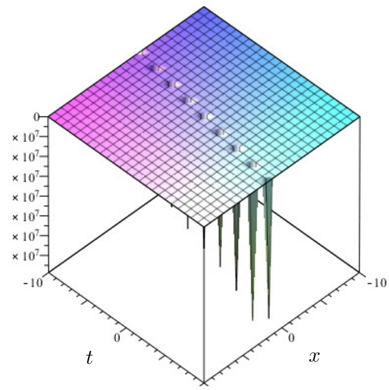

Figure 2. Singular soliton solution $\Psi(\xi)$ of (31).

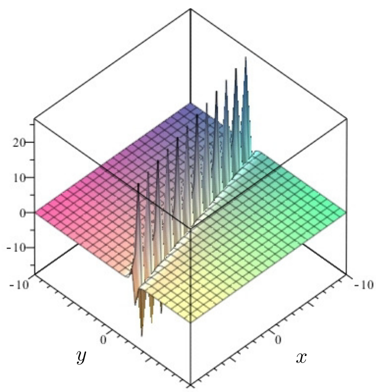

Figure 3. Singular solitary wave solution $u(\xi)$ of (44).

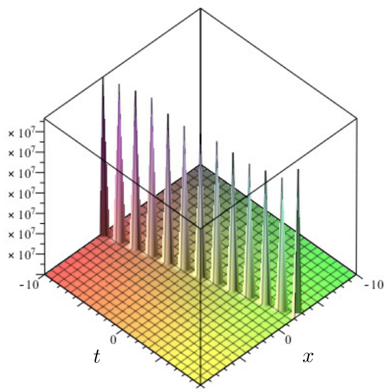

Figure 4. Singular solitary wave solution $u(\xi)$ of (50).

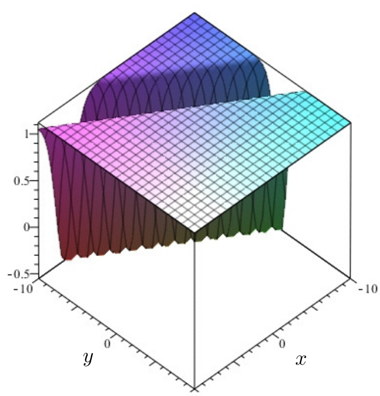

Figure 5. Solitary wave solution $H(\xi)$ of (60).

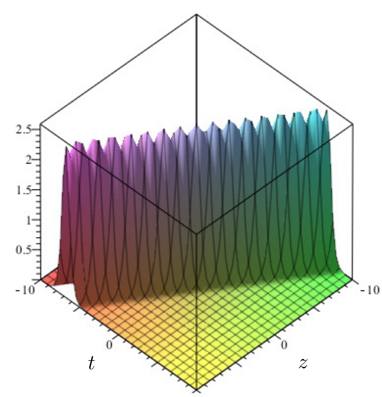

Figure 6. Solitary wave solution $H(\xi)$ of (70).

7 Conclusions

For the first time, we have derived many new exact solutions of the three nonlinear partial differential equations (PDEs), namely, the $(1+1)$-dimensional improved perturbed nonlinear Schrödinger equation with anti-cubic nonlinearity, the $(2+1)$-dimensional Davey–Sterwatson (DS) equation and the $(3+1)$-dimensional modified Zakharov–Kuznetsov (mZK) equation of ion-acoustic waves in a magnetized plasma using the new extended generalized Kudryashov method. The obtained solutions will be depended on the symmetrical hyperbolic Fibonacci functions. Equation (1) is nonlinear optics, where its solutions in Section 3 are called bright soliton solutions, dark soliton solutions, singular soliton solutions and trigonometric function solutions, while Eq. (2) is fluid dynamics, and Eq. (3) is plasma physics, where their solutions in Sections 4 and 5 are called solitary wave, shock wave and singular solitary waves. All the solutions obtained in Sections 3–5 will be a good guide line and great help for a large family of scientists. On comparing our solutions of these equations with that obtained in [17, 19, 26, 35, 36, 45], we deduce that our solutions are new and not reported previously in the literature. Finally, our results in this article have been checked using the Maple by putting them back into the original equations (1), (2) and (3).

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