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Exponential state estimation for competitive neural network via stochastic sampled-data control with packet losses*

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Abstract. This paper investigates the exponential state estimation problem for competitive neural networks via stochastic sampled-data control with packet losses. Based on this strategy, a switched system model is used to describe packet dropouts for the error system. In addition, transmittal delays between neurons are also considered. Instead of the continuous measurement, the sampled measurement is used to estimate the neuron states, and a sampled-data estimator with probabilistic sampling in two sampling periods is proposed. Then the estimator is designed in terms of the solution to a set of linear matrix inequalities (LMIs), which can be solved by using available software. When the missing of control packet occurs, some sufficient conditions are obtained to guarantee that the exponentially stable of the error system by means of constructing an appropriate Lyapunov function and using the average dwell-time technique. Finally, a numerical example is given to show the effectiveness of the proposed method.

Keywords: exponential state estimation, competitive neural network, sampled-data control, packet losses.

1 Introduction

During the last few decades, neural networks have become increasingly popular due to their wide application prospects, such as image processing, associative memory, pattern

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recognition and optimization problems [6, 19, 23]. Recently, many important results have been reported on various aspects of neural networks [3, 10, 16, 24]. The competitive neural network model was first proposed by Cohen and Grossberg in 1983 [5], which has two types of state variables: the short-term memory (STM) variable and the long-term memory (LTM) variable. They describe the fast neural activity and the slow unsupervised synaptic modifications, respectively. Competitive neural networks contain two time scales: the one dealing with the fast change of the state, and the other one with the slow change of the synapse by external stimulation. Currently, the dynamic behavior of competitive neural networks with different time scales has been considered in [7, 9]. Especially, analysis problems of stability and synchronization for the competitive neural networks have aroused the interest of a large number of research scholars [31, 34].

In many applications, the neuron states need to be known to achieve certain practical performance. In practice, it is sometimes the case that only partial information about the neuron states is available in the network output [32]. Therefore, it is of great significance to estimate the neuron states through available output measurements of the networks, and the state estimation problem for neural networks has received increasing research attention. Recently, some profound results of the state estimation problem have been established [2, 22, 30]. For instance, in [1], the sampled-data state estimation problem has been studied for genetic regulatory networks with time-varying delays. In [14], the authors studied the state estimation problem for delayed recurrent neural networks with sampled-data.

As we all know, time delay is an important factor that widely exists in practical systems due to the transmission congestion in networks [4, 8, 11, 28], it can cause many uncertain complex dynamic behaviors, for instance, oscillation divergence and instability. Recently, there are many results that consider the impact of time-varying delay on system stability. The state estimation problem for delayed neural networks with time-varying delays has been considered in [13]. On the other hand, transmittal delays between neurons are often neglected, which are important for the analysis of practical systems. As the transmittal delay varies from neuron to neuron, a common buffer is employed to make all controllers operate at the same time. In [12], the authors studied a leader-following consensus problem for nonlinear multi-agent systems with stochastic sampling, and the transmittal delay from the sensor to the controller is considered.

Note that there exist information exchanges among the interconnected neurons. When the network scale is large, it is easy to make the channel block for the continuous information transmission. So, if unnecessary information transmission can be reduced, it will improve the operational efficiency of the networks. However, most existed controllers focus on using continuous-time control [17, 20, 29]. With the rapid development of high-speed computers, sampled-data control theory has gained considerable attention in control area [15, 33]. In a general way, many researchers have analyzed sampled-data control systems with constant sampling period. As the large sampling interval means that signals are sampled during a relatively long time period and less energy is consumed, therefore, the sampling scheme with fewer signals sampled is more efficient. Stochastic sampling [25], a further extension which allows the sampling period to switch among different values, has received more and more attention. The synchronization problem for a chaotic

Lur'e system via stochastic sampled-data control has been studied in [26]. So far, there are few results about exponential state estimation for the competitive neural networks via stochastic sampled-data control. Moreover, developing a practical controller with stochastic sampling for the competitive neural networks and further investigating the effects of transmittal delays on state estimation problem is the motivation of this paper.

It is worth noting that all of the above mentioned work assumed that the control packet from the controller to the actuator is transmitted in a perfect way, that is, there is no loss in the control information. However, this assumption may fail in many practical situations due to actuator failure, communication interference or congestion, intermittent unavailability of controllers and so on. When the control packet from the controller to the actuator is lost, the actuator input to the plant will be zero, it will lead to many difficulties in the study of the stability of the system. Thus, it is necessary to consider the influence of the control packet loss. At present, synchronization of neural networks with control packet losses has been investigated in [21]. In [35], the authors studied the sampled-data consensus problem for linear multi-agent systems with packet losses. Actually, until now, the exponential state estimation problem for competitive neural networks via stochastic sampled-data control with packet losses has not been resolved.

Motivated by the above discussion, the exponential state estimation problem is investigated for competitive neural networks via stochastic sampled-data control with packet losses, transmittal delays between neurons are also considered. The sampling period is assumed to be time-varying that switches between two different values in a random way with the given probability. Furthermore, exponential stability criteria for the error systems are derived by constructing an appropriate Lyapunov function and designing stochastic sampled-data controllers. The solvability of derived conditions depends on not only the size of the delay, but also the probability of taking values of the sampling period. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

The remainder of this paper is organized as follows. In Section 2, the model formulation and some preliminaries are briefly outlined. The main results will be obtained in the form of LMIs in Section 3. An example is given to show the effectiveness of our results in Section 4. Finally, conclusions are made in Section 5.

Notations. Throughout this paper, $\mathbb{R}^n$ denotes n-dimensional real numbers set, $\mathbb{N}$ denotes natural numbers set. For symmetric matrices X and Y, the notation $X < Y$ means that the matrix $X - Y$ is negative definite. $A_{m\times n}$ and I_n refer to $m \times n$ matrix and $n \times n$ identity matrix, respectively. The superscript "T" denotes vector transposition. "$*$" denotes the symmetric terms in a symmetric matrix. $\mathbf{P}\{\pi\}$ is the occurrence probability of an event π. If not explicitly stated, matrices are assumed to have compatible dimensions.

2 Preliminaries

In this section, a competitive neural network model is first proposed. Furthermore, some preliminaries including definition, assumptions, and lemmas are given.

Considering the following competitive neural networks with time-varying delays:

$$\begin{aligned} \text{STM:}\quad & \dot{x}(t) = -Ax(t) + Bf\bigl(x(t)\bigr) + \bar{B}f\bigl(x\bigl(t-\tau(t)\bigr)\bigr) + DS(t) + I, \\ \text{LTM:}\quad & \dot{S}(t) = -CS(t) + Mf\bigl(x(t)\bigr), \end{aligned} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector of the competitive neural networks, $S(t) = (S_1(t), S_2(t), \dots, S_n(t))^{\mathrm{T}}$ is the dynamic variable about synaptic efficiency; $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t)))^{\mathrm{T}} \in \mathbb{R}^n$ denotes the neuron activation function, $\tau(t)$ is the time-varying delay and satisfies $0 < \tau(t) \leqslant \bar{\tau}$, $\dot{\tau}(t) \leqslant \mu < 1$, where $\bar{\tau}$ and μ are known constants; $A = \operatorname{diag}(a_1, a_2, \dots, a_n)$ is a diagonal matrix with $a_i > 0$ for $i = 1, 2, \dots, n$ represents the time constant of the neuron, $B = (b_{ij})_{n\times n}$ and $\bar{B} = (\bar{b}_{ij})_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively; $D = (d_{ij})_{n\times n}$ is the strength of the external stimulus, $I = (I_1, I_2, \dots, I_n)^{\mathrm{T}} \in \mathbb{R}^n$ is an external input vector; $C = \operatorname{diag}(c_1, c_2, \dots, c_n)$ with $c_i > 0$ represents disposable scaling constant, $M = (m_{ij})_{n\times n}$ is the constant external stimulus.

There are generally two kinds of state variables in the practical biological neural networks model: STM and LTM variables. STM variables represent the instantaneous change of the dynamic behavior of neurons, while the corresponding LTM variables represent the slow change of the dynamic behavior of neurons. Thus, for this type of neural network model, there are two kinds of time scale, the first is a kind of instant changes, and the second one is of slow action.

Define the network measurements as follows:

$$y_x(t) = Ex(t), \qquad y_S(t) = GS(t), \tag{2}$$

where $y_x(t) \in \mathbb{R}^m$, $y_S(t) \in \mathbb{R}^m$ are the measurement outputs, and $E, G \in \mathbb{R}^{m\times n}$ are known constant matrices.

Assumption 1. (See [27].) The activation function $f_i(\cdot)$ satisfies the following inequality:

$$0 \leqslant \frac{f_i(a) - f_i(b)}{a - b} \leqslant \varrho_i \quad \text{for every } a, b \in \mathbb{R},\ a \neq b,$$

where $\varrho_i > 0$, $i = 1, 2, \dots, n$, is known constant, and $\Lambda = \operatorname{diag}(\varrho_1, \varrho_2, \dots, \varrho_n) > 0$.

The purpose of this paper is to present an efficient estimation scheme to observe the neuron states from the available network output. For this reason, the following full-order state estimator for the competitive neural networks (1) is proposed:

$$\begin{aligned} \text{STM:}\quad & \dot{\hat{x}}(t) = -A\hat{x}(t) + Bf\bigl(\hat{x}(t)\bigr) + \bar{B}f\bigl(\hat{x}\bigl(t-\tau(t)\bigr)\bigr) + D\hat{S}(t) + I + U_1(t), \\ \text{LTM:}\quad & \dot{\hat{S}}(t) = -C\hat{S}(t) + Mf\bigl(\hat{x}(t)\bigr) + U_2(t). \end{aligned} \tag{3}$$

Then, the estimation output vector is as follows:

$$\hat{y}_x(t) = E\hat{x}(t), \qquad \hat{y}_S(t) = G\hat{S}(t), \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$, $\hat{S}(t) \in \mathbb{R}^n$ are the estimation of neuron state variables $x(t)$ and $S(t)$, respectively. $U_1(t) \in \mathbb{R}^n$, $U_2(t) \in \mathbb{R}^n$ are the control input. $\hat{y}_x(t) \in \mathbb{R}^n$, $\hat{y}_S(t) \in \mathbb{R}^m$ are the estimated output vector.

Define the error vector by $e_x(t) = x(t) - \hat{x}(t)$, $e_S(t) = S(t) - \hat{S}(t)$. The error dynamical system is expressed from (1) and (4) as follows:

$$\begin{aligned} \dot{e}_x(t) &= -Ae_x(t) + BF\big(e_x(t)\big) + \bar{B}F\big(e_x\big(t-\tau(t)\big)\big) + De_S(t) - U_1(t), \\ \dot{e}_S(t) &= -Ce_S(t) + MF\big(e_x(t)\big) - U_2(t), \end{aligned} \tag{5}$$

where $F(e_x(t)) = f(x(t)) - f(\hat{x}(t))$, $F(e_x(t-\tau(t))) = f(x(t-\tau(t))) - f(\hat{x}(t-\tau(t)))$.

In this paper, the controller was assumed to use sampled-data control with stochastic sampling, at the same time, the transmittal delay from the sensor to the controller is considered. Let $\tau_{ik}, i = 1, 2, \dots, n$, be the communication delay between the sensor i and the buffer. Thus, the delay from the sensor i to the controller i can be defined as $\tau_k = \max\{\tau_{ik},\, i = 1, 2, \dots, n\}$. The controller i updates its input and sends its output to the actuator with zero-order hold (ZOH). The function of ZOH is to keep the control input constant from $t = t_k + \tau_k$ to $t = t_{k+1} + \tau_{k+1}$. Then, the sampled-data controller can be described as

$$\begin{aligned} U_1(t) &= K\big(y_x(t_k) - \hat{y}_x(t_k)\big) = KEe_x(t_k), \\ U_2(t) &= \bar{K}\big(y_S(t_k) - \hat{y}_S(t_k)\big) = \bar{K}Ge_S(t_k), \end{aligned} \tag{6}$$

where $t \in [t_k + \tau_k,\, t_{k+1} + \tau_{k+1})$, K and $\bar{K}$ are the gain matrix of the feedback controller to be determined later, and t_k denotes the sampling instant satisfying: $0 = t_0 < t_1 < \dots < t_k < \cdots$, $lim_{k\to\infty} t_k = \infty$. Let $h(t) = t - t_k$ for $t \in [t_k + \tau_k,\, t_{k+1} + \tau_{k+1})$, as $t_k = t - h(t)$, then controller (6) can be written as

$$U_1(t) = KEe_x\big(t - h(t)\big) \qquad U_2(t) = \bar{K}Ge_S\big(t - h(t)\big) \tag{7}$$

with $\tau_k \leqslant h(t) \leqslant t_{k+1} - t_k + \tau_{k+1}$. The sampling period $\{t_{k+1} - t_k\}$ is allowed to randomly switch between two different values h_1 and h_2, where h_1 and h_2 are known constants satisfies $0 < h_1 < h_2$. The probabilities are $\mathbf{P}\{h = h_1\} = \rho$ and $\mathbf{P}\{h = h_2\} = 1 - \rho$, where $\rho \in [0, 1]$ is a given constant. Therefore, the time delay $h(t)$ in (7) satisfies

$$\tau_k \leqslant h(t) < h_1 + \tau_{k+1} \quad \text{or} \quad \tau_k \leqslant h(t) < h_2 + \tau_{k+1}.$$

Since the sampling period can switch between h_1 and h_2, $h(t)$ is a random variable ranging from τ_k to $h_2 + \tau_{k+1}$. The probability of $h(t)$ can be calculated by

$$\mathbf{P}\big\{\tau_k \leqslant h(t) < h_1 + \tau_{k+1}\big\} = \rho + \left(1 - \frac{h_2 - h_1}{h_2 + \tau_{k+1} - \tau_k}\right)(1 - \rho), \tag{8}$$

$$\mathbf{P}\big\{h_1 + \tau_{k+1} \leqslant h(t) < h_2 + \tau_{k+1}\big\} = \frac{h_2 - h_1}{h_2 + \tau_{k+1} - \tau_k}(1 - \rho), \tag{9}$$

The probability distribution in (8) and (9) depends on transmittal delays. To simplify the analysis, it is assumed that transmittal delays are uniform, that is, $\tau_k = \tau$ for $k = 0, 1, 2, \ldots$. Then (8) and (9) turn to

$$\mathbf{P}\{\tau \leqslant h(t) \leqslant h_1 + \tau\} = \rho + \frac{h_1}{h_2}(1-\rho),$$
$$\mathbf{P}\{h_1 + \tau \leqslant h(t) \leqslant h_2 + \tau\} = \left(1 - \frac{h_1}{h_2}\right)(1-\rho).$$

By introducing a new random variable

$$\alpha(t) = \begin{cases} 1, & \tau \leqslant h(t) < h_1 + \tau, \\ 0, & h_1 + \tau \leqslant h(t) < h_2 + \tau. \end{cases}$$

Accordingly, $\mathbf{P}\{\alpha(t) = 1\} = \alpha$ and $\mathbf{P}\{\alpha(t) = 0\} = 1-\alpha$ with $\alpha = \rho + (h_1/h_2)(1-\rho)$, where α is known constant satisfies $\alpha \in [0, 1]$. $\alpha(t)$ satisfies a Bernoulli distribution with $\mathbf{E}\{\alpha(t)\} = \alpha$ and $\mathbf{E}\{(\alpha(t)-\alpha)^2\} = \alpha(1-\alpha)$. Then, the controller (7) can be converted into

$$\begin{aligned} U_1(t) &= \alpha(t)KEe_x\big(t - h_1(t)\big) + \big(1-\alpha(t)\big)KEe_x\big(t - h_2(t)\big), \\ U_2(t) &= \alpha(t)\bar{K}Ge_S\big(t - h_1(t)\big) + (1-\alpha(t))\bar{K}Ge_S\big(t - h_2(t)\big) \end{aligned}$$

where $h_1(t)$ and $h_2(t)$ are time-varying delays, satisfying $\tau \leqslant h_1(t) < h_1 + \tau$ and $h_1 + \tau \leqslant h_2(t) < h_2 + \tau$. Therefore, system (5) with two sampling intervals can be expressed as follows:

$$\begin{aligned} \dot{e}_x(t) &= -Ae_x(t) + BF\big(e_x(t)\big) + \bar{B}F\big(e_x\big(t-\tau(t)\big)\big) + De_S(t) \\ &\quad - \alpha(t)KEe_x\big(t-h_1(t)\big) - \big(1-\alpha(t)\big)KEe_x\big(t-h_2(t)\big), \\ \dot{e}_S(t) &= -Ce_S(t) + MF\big(e_x(t)\big) - \alpha(t)\bar{K}Ge_S\big(t-h_1(t)\big) \\ &\quad - \big(1-\alpha(t)\big)\bar{K}Ge_S\big(t-h_2(t)\big). \end{aligned} \tag{10}$$

By setting $\eta(t) = [e_x^{\mathrm{T}}(t)\ e_S^{\mathrm{T}}(t)]^{\mathrm{T}}$, the following augmented system can be obtained from (10):

$$\begin{aligned} \dot{\eta}(t) &= \hat{A}\eta(t) + \hat{B}F\big(H\eta(t)\big) + \hat{C}F\big(H\eta\big(t-\tau(t)\big)\big) \\ &\quad - \hat{W}_0\eta\big(t-h_1(t)\big) - \hat{W}_1\eta\big(t-h_2(t)\big), \end{aligned} \tag{11}$$

where

$$\hat{A} = \begin{pmatrix} -A & D \\ 0 & -C \end{pmatrix}, \qquad \hat{B} = \begin{pmatrix} B \\ M \end{pmatrix}, \qquad \hat{C} = \begin{pmatrix} \bar{B} \\ 0 \end{pmatrix}, \qquad H = \begin{pmatrix} I & 0 \end{pmatrix},$$
$$\hat{W}_0 = \begin{pmatrix} \alpha(t)KE & 0 \\ 0 & \alpha(t)\bar{K}G \end{pmatrix}, \qquad \hat{W}_1 = \begin{pmatrix} (1-\alpha(t))KE & 0 \\ 0 & (1-\alpha(t))\bar{K}G \end{pmatrix}.$$

In addition, it cannot be guaranteed that the control packet from the controller to the actuator will always be received completely. So, in this paper, we consider the problem of exponential state estimation for competitive neural network with control packet loss. If the control packet from controller to actuator is lost, then, the actuator does nothing, that is, $U_1(t_k) = 0$ and $U_2(t_k) = 0$, the error system (10) reduces to the following system:

$$\begin{aligned} \dot{e}_x(t) &= -Ae_x(t) + BF\big(e_x(t)\big) + \bar{B}F\big(e_x\big(t-\tau(t)\big)\big) + De_S(t), \\ \dot{e}_S(t) &= -Ce_S(t) + MF\big(e_x(t)\big), \end{aligned} \tag{12}$$

or equivalently,

$$\dot{\eta}(t) = \hat{A}\eta(t) + \hat{B}F\big(H\eta(t)\big) + \hat{C}F\big(H\eta\big(t-\tau(t)\big)\big). \tag{13}$$

In order to describe the control packet loss status of system (10), a piecewise notation function $\sigma(t) : [0, +\infty) \to \{1, 2\}$ is used to estimate the competitive neural network suffering from packet losses or not. When $\sigma(t) = 1$ and $K_1 = K$, $\bar{K}_1 = \bar{K}$. Otherwise, $\sigma(t) = 2$, $K_2 = 0$, $\bar{K}_2 = 0$, we have a control packet loss happens during the interval $[t_k, t_{k+1})$. Then system (10) can be described as the following switched system:

$$\begin{aligned} \dot{e}_x(t) &= -Ae_x(t) + BF\big(e_x(t)\big) + \bar{B}F\big(e_x\big(t-\tau(t)\big)\big) + De_S(t) \\ &\quad - \alpha(t)K_{\sigma(t)}Ee_x\big(t-h_1(t)\big) - \big(1-\alpha(t)\big)K_{\sigma(t)}Ee_x\big(t-h_2(t)\big), \\ \dot{e}_S(t) &= -Ce_S(t) + MF\big(e_x(t)\big) - \alpha(t)\bar{K}_{\sigma(t)}Ge_S\big(t-h_1(t)\big) \\ &\quad - \big(1-\alpha(t)\big)\bar{K}_{\sigma(t)}Ge_S\big(t-h_2(t)\big), \end{aligned} \tag{14}$$

similarly,

$$\begin{aligned} \dot{\eta}(t) = {}& \hat{A}\eta(t) + \hat{B}F\big(H\eta(t)\big) + \hat{C}F\big(H\eta\big(t-\tau(t)\big)\big) \\ & - \hat{\hat{W}}_0\eta\big(t-h_1(t)\big) - \hat{\hat{W}}_1\eta\big(t-h_2(t)\big), \end{aligned} \tag{15}$$

where

$$\hat{\hat{W}}_0 = \begin{pmatrix} \alpha(t)K_{\sigma(t)}E & 0 \\ 0 & \alpha(t)\bar{K}_{\sigma(t)}G \end{pmatrix}, \qquad \hat{\hat{W}}_1 = \begin{pmatrix} (1-\alpha(t))K_{\sigma(t)}E & 0 \\ 0 & (1-\alpha(t))\bar{K}_{\sigma(t)}G \end{pmatrix}.$$

Therefore, $\sigma(t)$ can be referred to a switching signal, the switched system (14) consists of the controlled subsystem (10) and uncontrolled subsystem (12).

Definition 1. (See [14].) The system described by (15) is said to be exponentially mean-square stable if there exist two constants $\epsilon > 0$ and $\delta > 0$ such that

$$\mathbf{E}\big\{\big\|\eta(t)\big\|^2\big\} \leqslant \epsilon \mathrm{e}^{-\delta t} \sup_{-\tau^* \leqslant \theta \leqslant 0} \mathbf{E}\big\{\big\|\phi(\theta)\big\|^2\big\},$$

where $\tau^* = \max\{\bar{\tau}, h_2 + \tau\}$, $\eta(t) = [e_x^{\mathrm{T}}(t)\ e_S^{\mathrm{T}}(t)]^{\mathrm{T}}$ is the error vector in (15), $\phi(\cdot)$ is the initial function of system (15) defined as $\phi(t) = \eta(t)$, $t \in [-\tau^*, 0]$.

Definition 2. (See [18].) If there exist scalars $N_0 > 0$ and $\tau_\alpha > 0$ such that the following inequality holds:

$$N_\sigma(T,t) \leqslant N_0 + \frac{t-T}{\tau_\alpha} \quad \forall t \geqslant T \geqslant 0,$$

where N_0 is the chatter bound, and $N_\sigma(T,t)$ is the switching numbers of $\sigma(t)$ over the interval $[T,t)$, respectively, then the switching signal $\sigma(t)$ is said to have an average dwell time τ_α.

Remark 1. The average dwell time means that the time interval between consecutive switching is at least τ_α on the average. Then, a basic problem for the networks (5) is to specify the minimal τ_α and get the admissible switching signals that the networks (5) are stable.

Lemma 1. (*See* [35].) *For any constant matrix $\Gamma \in \mathbb{R}^{n\times n}$, $\Gamma = \Gamma^{\mathrm{T}} > 0$, scalar $\varpi > 0$, and vector function $\psi : [0,\varpi] \to \mathbb{R}^n$ such that the integration is well defined, the following inequality holds*:

$$\left(\int_0^{\varpi} \psi(s)\,\mathrm{d}s\right)^{\mathrm{T}} \Gamma \left(\int_0^{\varpi} \psi(s)\,\mathrm{d}s\right) \leqslant \varpi \int_0^{\varpi} \psi^{\mathrm{T}}(s)\Gamma\psi(s)\,\mathrm{d}s.$$

3 Main results

In this section, our main aim is to design the sampled-data state estimator to estimate the state of the competitive neural networks (1) such that the estimation error system is exponentially stable. At the same time, several sufficient conditions will be derived for systems (11) by using Lyapunov functional approach.

Theorem 1. *For given scalar $\mu > 0$, $\bar{\tau} > 0$, $\tau > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\gamma_1 > 0$ and matrices $U > 0$, $\Lambda = \operatorname{diag}(\varrho_1, \varrho_2, \dots, \varrho_n) > 0$, the exponential mean square stable of error system* (11) *can be reached if there exist appropriate dimension matrices $P > 0$, $R_i > 0$ $(i = 1,2,3,4,5)$, $Q_i > 0$ $(i = 1,2,3,4)$ such that the following inequality hold*:

$$\Phi = \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} & 0 & \Phi_{15} & -\alpha U\bar{W} & 0 & \Phi_{18} & 0 & U\hat{B} & U\hat{C} \\
* & \Phi_{22} & 0 & 0 & 0 & -\alpha U\bar{W} & 0 & \Phi_{28} & 0 & U\hat{B} & U\hat{C} \\
* & * & \Phi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Phi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Phi_{55} & \Phi_{56} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \Phi_{66} & \Phi_{67} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \Phi_{77} & \Phi_{78} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Phi_{88} & \Phi_{89} & 0 & 0 \\
* & * & * & * & * & * & * & * & \Phi_{99} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 \\
* & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I
\end{bmatrix} < 0, \quad (16)$$

where

$$\begin{aligned}
&\Phi_{11} = 2\gamma_1 P + R_1 + R_2 + R_3 - \mathrm{e}^{-2\gamma_1\bar{\tau}}Q_1 - \mathrm{e}^{-2\gamma_1\tau}Q_2 + \varepsilon_1 H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H + 2U\hat{A},\\
&\Phi_{12} = P - U + U\hat{A}, \qquad \Phi_{13} = \mathrm{e}^{-2\gamma_1\bar{\tau}}Q_1, \qquad \Phi_{15} = \mathrm{e}^{-2\gamma_1\tau}Q_2,\\
&\Phi_{18} = (\alpha-1)U\bar{W}, \qquad \Phi_{22} = W_1 - 2U, \qquad \Phi_{28} = (\alpha-1)U\bar{W},\\
&\Phi_{33} = -\mathrm{e}^{-2\gamma_1\bar{\tau}}R_2 - \mathrm{e}^{-2\gamma_1\bar{\tau}}Q_1, \qquad \Phi_{44} = (\mu-1)\mathrm{e}^{-2\gamma_1\bar{\tau}}R_1 + \varepsilon_2 H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H,\\
&\Phi_{55} = \mathrm{e}^{-2\gamma_1\tau}(R_4 - R_3) - \mathrm{e}^{-2\gamma_1\tau}Q_2 - \mathrm{e}^{-2\gamma_1(h_1+\tau)}Q_3,\\
&\Phi_{56} = \mathrm{e}^{-2\gamma_1(h_1+\tau)}Q_3, \qquad \Phi_{66} = -2\mathrm{e}^{-2\gamma_1(h_1+\tau)}Q_3, \qquad \Phi_{67} = \mathrm{e}^{-2\gamma_1(h_1+\tau)}Q_3,\\
&\Phi_{77} = -\mathrm{e}^{-2\gamma_1(h_1+\tau)}Q_3 - \mathrm{e}^{-2\gamma_1(h_2+\tau)}Q_4 + \mathrm{e}^{-2\gamma_1(h_1+\tau)}(R_5 - R_4),\\
&\Phi_{78} = \mathrm{e}^{-2\gamma_1(h_2+\tau)}Q_4, \qquad \Phi_{88} = -2\mathrm{e}^{-2\gamma_1(h_2+\tau)}Q_4, \qquad \Phi_{89} = \mathrm{e}^{-2\gamma_1(h_2+\tau)}Q_4,\\
&\Phi_{99} = -\mathrm{e}^{-2\gamma_1(h_2+\tau)}(R_5 + Q_4),\\
&\bar{W} = \begin{pmatrix} KE & 0\\ 0 & \bar{K}G\end{pmatrix}, \qquad W_1 = \bar{\tau}^2 Q_1 + \tau^2 Q_2 + h_1^2 Q_3 + (h_2 - h_1)^2 Q_4.
\end{aligned}$$

Proof. Constructing the following Lyapunov functional:

$$V_1(t) = V_{11}(t) + V_{12}(t) + V_{13}(t), \tag{17}$$

where

$$\begin{aligned}
V_{11}(t) &= \eta^{\mathrm{T}}(t)P\eta(t),\\
V_{12}(t) &= \int_{t-\tau(t)}^{t} \mathrm{e}^{2\gamma_1(s-t)}\eta^{\mathrm{T}}(s)R_1\eta(s)\,\mathrm{d}s + \int_{t-\bar{\tau}}^{t} \mathrm{e}^{2\gamma_1(s-t)}\eta^{\mathrm{T}}(s)R_2\eta(s)\,\mathrm{d}s\\
&\quad + \int_{t-\tau}^{t} \mathrm{e}^{2\gamma_1(s-t)}\eta^{\mathrm{T}}(s)R_3\eta(s)\,\mathrm{d}s + \int_{t-h_1-\tau}^{t-\tau} \mathrm{e}^{2\gamma_1(s-t)}\eta^{\mathrm{T}}(s)R_4\eta(s)\,\mathrm{d}s\\
&\quad + \int_{t-h_2-\tau}^{t-h_1-\tau} \mathrm{e}^{2\gamma_1(s-t)}\eta^{\mathrm{T}}(s)R_5\eta(s)\,\mathrm{d}s,\\
V_{13}(t) &= \bar{\tau}\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} \mathrm{e}^{2\gamma_1(s-t)}\big(\varphi^{\mathrm{T}}(s)Q_1\varphi(s) + \bar{\varphi}^{\mathrm{T}}(s)Q_1\bar{\varphi}(s)\big)\,\mathrm{d}s\,\mathrm{d}\theta\\
&\quad + \tau\int_{-\tau}^{0}\int_{t+\theta}^{t} \mathrm{e}^{2\gamma_1(s-t)}\big(\varphi^{\mathrm{T}}(s)Q_2\varphi(s) + \bar{\varphi}^{\mathrm{T}}(s)Q_2\bar{\varphi}(s)\big)\,\mathrm{d}s\,\mathrm{d}\theta
\end{aligned}$$

$$
\begin{aligned}
&+ h_1 \int_{-h_1-\tau}^{-\tau} \int_{t+\theta}^{t} \mathrm{e}^{2\gamma_1(s-t)} \big(\varphi^{\mathrm{T}}(s) Q_3 \varphi(s) + \bar{\varphi}^{\mathrm{T}}(s) Q_3 \bar{\varphi}(s)\big)\, \mathrm{d}s\, \mathrm{d}\theta \\
&+ (h_2 - h_1) \int_{-h_2-\tau}^{-h_1-\tau} \int_{t+\theta}^{t} \mathrm{e}^{2\gamma_1(s-t)} \big(\varphi^{\mathrm{T}}(s) Q_4 \varphi(s) + \bar{\varphi}^{\mathrm{T}}(s) Q_4 \bar{\varphi}(s)\big)\, \mathrm{d}s\, \mathrm{d}\theta,
\end{aligned}
$$

$$
\begin{aligned}
\varphi(t) &= \hat{A}\eta(t) + \hat{B}F\big(H\eta(t)\big) + \hat{C}F\big(H\eta\big(t-\tau(t)\big)\big) - \alpha\bar{W}\eta\big(t-h_1(t)\big) \\
&\quad - (1-\alpha)\bar{W}\eta\big(t-h_2(t)\big), \\
\bar{\varphi}(t) &= \bar{W_2}\big(\eta\big(t-h_1(t)\big) - \eta\big(t-h_2(t)\big)\big), \\
\bar{W_2} &= \begin{pmatrix} \sqrt{\alpha(1-\alpha)}KE & 0 \\ 0 & \sqrt{\alpha(1-\alpha)}\bar{K}G \end{pmatrix}.
\end{aligned}
$$

Let $LV(t) = \lim_{\Delta\to 0^+} \Delta^{-1}[\mathbf{E}\{V(t+\Delta) \mid r(t)\} - V(t)]$ be the infinitesimal operator of $V(t)$. It follows from (17) that

$$
LV_1(t) = LV_{11}(t) + LV_{12}(t) + LV_{13}(t). \tag{18}
$$

Then the time derivative of (17) along the solution of system (11) can be calculated as follows:

$$
\begin{aligned}
&LV_{11}(t) + 2\gamma_1 V_{11}(t) = 2\eta^{\mathrm{T}}(t)P\dot{\eta}(t) + 2\gamma_1\eta^{\mathrm{T}}(t)P\eta(t), \\
&LV_{12}(t) + 2\gamma_1 V_{12}(t) \\
&\quad \leqslant \eta^{\mathrm{T}}(t)(R_1 + R_2 + R_3)\eta(t) - (1-\mu)\mathrm{e}^{-2\gamma_1\bar{\tau}}\eta^{\mathrm{T}}\big(t-\tau(t)\big)R_1\eta\big(t-\tau(t)\big) \\
&\qquad - \mathrm{e}^{-2\gamma_1\bar{\tau}}\eta^{\mathrm{T}}\big(t-\bar{\tau}\big)R_2\eta\big(t-\bar{\tau}\big) + \mathrm{e}^{-2\gamma_1\tau}\eta^{\mathrm{T}}(t-\tau)(R_4 - R_3)\eta(t-\tau) \\
&\qquad + \mathrm{e}^{-2\gamma_1(h_1+\tau)}\eta^{\mathrm{T}}(t-h_1-\tau)(R_5 - R_4)\eta(t-h_1-\tau) \\
&\qquad - \mathrm{e}^{-2\gamma_1(h_2+\tau)}\eta^{\mathrm{T}}(t-h_2-\tau)R_5\eta(t-h_2-\tau), \\
&LV_{13}(t) + 2\gamma_1 V_{13}(t) \\
&\quad \leqslant \varphi^{\mathrm{T}}(t)W_1\varphi(t) + \bar{\varphi}^{\mathrm{T}}(t)W_1\bar{\varphi}(t) - \mathrm{e}^{-2\gamma_1\bar{\tau}}\bar{\tau} \int_{t-\bar{\tau}}^{t} \varPi_1(s)\, \mathrm{d}s \\
&\qquad - \mathrm{e}^{-2\gamma_1\tau}\tau \int_{t-\tau}^{t} \varPi_2(s)\, \mathrm{d}s - \mathrm{e}^{-2\gamma_1(h_1+\tau)}h_1 \int_{t-h_1-\tau}^{t-\tau} \varPi_3(s)\, \mathrm{d}s \\
&\qquad - (h_2 - h_1)\mathrm{e}^{-2\gamma_1(h_2+\tau)} \int_{t-h_2-\tau}^{t-h_1-\tau} \varPi_4(s)\, \mathrm{d}s,
\end{aligned} \tag{19}
$$

where

$$
\varPi_i(s) = \varphi^{\mathrm{T}}(s)Q_i\varphi(s) + \bar{\varphi}^{\mathrm{T}}(s)Q_i\bar{\varphi}(s) \quad (i = 1,2,3,4).
$$

By simple calculation, it is easy to obtain that

$$\mathbf{E}\{\varphi^{\mathrm{T}}(t)W_1\varphi(t)+\bar{\varphi}^{\mathrm{T}}(t)W_1\bar{\varphi}(t)\}=\mathbf{E}\{\dot{\eta}^{\mathrm{T}}(t)W_1\dot{\eta}(t)\}, \tag{20}$$

on the other hand,

$$\begin{aligned}
&-\mathrm{e}^{-2\gamma_1\bar{\tau}}\bar{\tau}\mathbf{E}\left\{\int_{t-\bar{\tau}}^{t}\Pi_1(s)\,\mathrm{d}s\right\}=-\mathrm{e}^{-2\gamma_1\bar{\tau}}\bar{\tau}\mathbf{E}\left\{\int_{t-\bar{\tau}}^{t}\dot{\eta}^{\mathrm{T}}(s)Q_1\dot{\eta}(s)\,\mathrm{d}s\right\}\\
&\quad\leqslant-\mathrm{e}^{-2\gamma_1\bar{\tau}}\mathbf{E}\left\{\left(\int_{t-\bar{\tau}}^{t}\dot{\eta}(s)\,\mathrm{d}s\right)^{\mathrm{T}}Q_1\left(\int_{t-\bar{\tau}}^{t}\dot{\eta}(s)\,\mathrm{d}s\right)\right\}\\
&\quad=\mathrm{e}^{-2\gamma_1\bar{\tau}}\mathbf{E}\left\{\left(\eta^{\mathrm{T}}(t),\eta^{\mathrm{T}}(t-\bar{\tau})\right)\Gamma_1\begin{pmatrix}\eta(t)\\ \eta(t-\bar{\tau})\end{pmatrix}\right\},
\end{aligned}\tag{21}$$

similarly, one has

$$\begin{aligned}
&-\mathrm{e}^{-2\gamma_1\tau}\tau\mathbf{E}\left\{\int_{t-\tau}^{t}\Pi_2(s)\,\mathrm{d}s\right\}\\
&\quad\leqslant\mathrm{e}^{-2\gamma_1\tau}\mathbf{E}\left\{\left(\eta^{\mathrm{T}}(t),\eta^{\mathrm{T}}(t-\tau)\right)\Gamma_2\begin{pmatrix}\eta(t)\\ \eta(t-\tau)\end{pmatrix}\right\},\\
&\quad-\mathrm{e}^{-2\gamma_1(h_1+\tau)}h_1\mathbf{E}\left\{\int_{t-h_1-\tau}^{t-\tau}\Pi_3(s)\,\mathrm{d}s\right\}\\
&\quad\leqslant\mathrm{e}^{-2\gamma_1(h_1+\tau)}\mathbf{E}\left\{-\left(t-\tau-\left(t-h_1(t)\right)\right)\int_{t-h_1(t)}^{t-\tau}\dot{\eta}^{\mathrm{T}}(s)Q_3\dot{\eta}(s)\,\mathrm{d}s\right.\\
&\quad\left.-\left(\left(t-h_1(t)\right)-(t-h_1-\tau)\right)\int_{t-h_1-\tau}^{t-h_1(t)}\dot{\eta}^{\mathrm{T}}(s)Q_3\dot{\eta}(s)\,\mathrm{d}s\right\}\\
&\quad\leqslant\mathrm{e}^{-2\gamma_1(h_1+\tau)}\mathbf{E}\left\{\left(\eta^{\mathrm{T}}(t-\tau),\eta^{\mathrm{T}}\left(t-h_1(t)\right)\right)\Gamma_3\begin{pmatrix}\eta(t-\tau)\\ \eta(t-h_1(t))\end{pmatrix}\right\}\\
&\quad+\mathrm{e}^{-2\gamma_1(h_1+\tau)}\mathbf{E}\left\{\left(\eta^{\mathrm{T}}\left(t-h_1(t)\right),\eta^{\mathrm{T}}\left(t-h_1-\tau\right)\right)\Gamma_3\begin{pmatrix}\eta(t-h_1(t))\\ \eta(t-h_1-\tau)\end{pmatrix}\right\},\\
&\quad-\mathrm{e}^{-2\gamma_1(h_2+\tau)}(h_2-h_1)\mathbf{E}\left\{\int_{t-h_2-\tau}^{t-h_1-\tau}\Pi_4(s)\,\mathrm{d}s\right\}
\end{aligned}$$

$$\leqslant e^{-2\gamma_1(h_2+\tau)}\mathbf{E}\left\{\left(\eta^{\mathrm{T}}(t-h_1-\tau),\eta^{\mathrm{T}}\big(t-h_2(t)\big)\right)\Gamma_4\begin{pmatrix}\eta(t-h_1-\tau)\\ \eta(t-h_2(t))\end{pmatrix}\right\}$$
$$+ e^{-2\gamma_1(h_2+\tau)}\mathbf{E}\left\{\left(\eta^{\mathrm{T}}\big(t-h_2(t)\big),\eta^{\mathrm{T}}(t-h_2-\tau)\right)\Gamma_4\begin{pmatrix}\eta(t-h_2(t))\\ \eta(t-h_2-\tau)\end{pmatrix}\right\}, \quad (22)$$

where

$$\Gamma_i = \begin{pmatrix} -Q_i & Q_i \\ Q_i & -Q_i \end{pmatrix}, \quad i = 1,2,3,4.$$

From Assumption 1, for a diagonal matrix $\Lambda = \mathrm{diag}(\varrho_1, \varrho_2, \dots, \varrho_n) > 0$, one can obtain the following inequalities:

$$\begin{aligned} &F^{\mathrm{T}}\big(H\eta(t)\big)F\big(H\eta(t)\big) - \eta^{\mathrm{T}}(t)H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H\eta(t) \leqslant 0,\\ &F^{\mathrm{T}}\big(H\eta\big(t-\tau(t)\big)\big)F\big(H\eta\big(t-\tau(t)\big)\big)\\ &\quad - \eta^{\mathrm{T}}\big(t-\tau(t)\big)H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H\eta\big(t-\tau(t)\big) \leqslant 0. \end{aligned} \quad (23)$$

Noting that, for any positive scalars $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, there exist:

$$\begin{aligned} &\varepsilon_1\big[\eta^{\mathrm{T}}(t)H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H\eta(t) - F^{\mathrm{T}}(H\eta(t))F\big(H\eta(t)\big)\big] \geqslant 0,\\ &\varepsilon_2\big[\eta^{\mathrm{T}}\big(t-\tau(t)\big)H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H\eta\big(t-\tau(t)\big)\\ &\quad - F^{\mathrm{T}}\big(H\eta\big(t-\tau(t)\big)\big)F\big(H\eta\big(t-\tau(t)\big)\big)\big] \geqslant 0. \end{aligned} \quad (24)$$

Then, for any appropriately dimensioned matrix U, the following equation holds:

$$\begin{aligned} &\mathbf{E}\big\{2\big[\eta^{\mathrm{T}}(t)U + \dot\eta^{\mathrm{T}}(t)U\big]\big[-\dot\eta(t) + \hat A\eta(t) + \hat BF\big(H\eta(t)\big)\\ &\quad + \hat CF\big(H\eta\big(t-\tau(t)\big)\big) - \hat W_0\eta\big(t-h_1(t)\big) - \hat W_1\eta\big(t-h_2(t)\big)\big]\big\} = 0. \end{aligned} \quad (25)$$

From (25) one has

$$\begin{aligned} &-2\eta^{\mathrm{T}}(t)U\dot\eta(t) - 2\dot\eta^{\mathrm{T}}(t)U\dot\eta(t) + 2\eta^{\mathrm{T}}(t)U\hat A\eta(t) + 2\dot\eta^{\mathrm{T}}(t)U\hat A\eta(t)\\ &\quad + 2\eta^{\mathrm{T}}(t)U\hat BF\big(H\eta(t)\big) + 2\dot\eta^{\mathrm{T}}(t)U\hat BF\big(H\eta(t)\big)\\ &\quad + 2\eta^{\mathrm{T}}(t)U\hat CF\big(H\eta\big(t-\tau(t)\big)\big)\\ &\quad + 2\dot\eta^{\mathrm{T}}(t)U\hat CF\big(H\eta\big(t-\tau(t)\big)\big) - 2\alpha\eta^{\mathrm{T}}(t)U\bar W\eta\big(t-h_1(t)\big)\\ &\quad - 2\alpha\dot\eta^{\mathrm{T}}(t)U\bar W\eta\big(t-h_1(t)\big) - 2(1-\alpha)\eta^{\mathrm{T}}(t)U\bar W\eta\big(t-h_2(t)\big)\\ &\quad - 2(1-\alpha)\dot\eta^{\mathrm{T}}(t)U\bar W\eta\big(t-h_2(t)\big) = 0. \end{aligned} \quad (26)$$

From (17)–(26) one can obtain that

$$LV_1(t) + 2\gamma_1V_1(t)\} \leqslant \mathbf{E}\big\{\xi^{\mathrm{T}}(t)\Phi\xi(t)\big\} \leqslant 0, \quad (27)$$

where Φ is defined in (16), and

$$\xi(t) = \big[\eta^{\mathrm{T}}(t) \;\; \dot{\eta}^{\mathrm{T}}(t) \;\; \eta^{\mathrm{T}}(t-\bar{\tau}) \;\; \eta^{\mathrm{T}}\big(t-\tau(t)\big) \;\; \eta^{\mathrm{T}}(t-\tau) \\ \eta^{\mathrm{T}}\big(t-h_1(t)\big) \;\; \eta^{\mathrm{T}}(t-h_1-\tau) \;\; \eta^{\mathrm{T}}\big(t-h_2(t)\big) \\ \eta^{\mathrm{T}}(t-h_2-\tau) \;\; F^{\mathrm{T}}\big(H\eta(t)\big) \;\; F^{\mathrm{T}}\big(H\eta\big(t-\tau(t)\big)\big)\big]^{\mathrm{T}}.$$

From (27) one has

$$V_1(t) \leqslant \mathrm{e}^{-2\gamma_1(t-t_k)}V_1(t_k) \leqslant \mathrm{e}^{-2\gamma_1(t-t_{k-1})}V_1(t_{k-1}) \leqslant \cdots \leqslant \mathrm{e}^{-2\gamma_1 t}V_1(0). \tag{28}$$

Moreover, from the definition of $V_1(t)$ we have

$$\mathrm{e}^{2\gamma_1 t}\mathbf{E}\big\{\big\|\eta(t)\big\|^2\big\}\lambda_{\min}(P) \leqslant \mathrm{e}^{2\gamma_1 t}\mathbf{E}\big\{V_1(t)\big\} \leqslant \mathbf{E}\big\{V_1(0)\big\}. \tag{29}$$

On the other hand, let

$$\zeta_1 = \frac{1-\mathrm{e}^{-2\gamma_1\bar{\tau}}}{2\gamma_1}, \qquad \zeta_2 = \frac{1-\mathrm{e}^{-2\gamma_1\tau}}{2\gamma_1},$$
$$\zeta_3 = \bar{\tau}\left(\frac{\bar{\tau}}{2\gamma_1} - \frac{1-\mathrm{e}^{-2\gamma_1\bar{\tau}}}{4\gamma_1^2}\right), \qquad \zeta_4 = \tau\left(\frac{\tau}{2\gamma_1} - \frac{1-\mathrm{e}^{-2\gamma_1\tau}}{4\gamma_1^2}\right).$$

Then from (16) one can obtain that

$$\mathbf{E}\big\{V_1(0)\big\} \leqslant \chi \sup_{-\tau^* \leqslant \theta \leqslant 0} \mathbf{E}\big\{\big\|\phi(\theta)\big\|^2\big\} \tag{30}$$

with

$$\chi = \lambda_{\max}(P) + \zeta_1\big(\lambda_{\max}(R_1) + \lambda_{\max}(R_2)\big) \\ + \zeta_2\lambda_{\max}(R_3) + \zeta_3\lambda_{\max}(Q_1) + \zeta_4\lambda_{\max}(Q_2).$$

From (28)–(30) we have

$$\mathbf{E}\big\{\big\|\eta(t)\big\|^2\big\} \leqslant \chi\frac{\mathrm{e}^{-2\gamma_1 t}}{\lambda_{\min}(P)} \sup_{-\tau^* \leqslant \theta \leqslant 0} \mathbf{E}\big\{\big\|\phi(\theta)\big\|^2\big\} \tag{31}$$

Therefore, according to Definition 1, the exponential mean square stable of error system (11) can be reached. This completes the proof. □

Remark 2. For any $t > s$, let $T_s(s,t)$ (resp., $T_u(s,t)$) denote the total activation time of the controlled subsystem (resp., uncontrolled subsystem) during (s,t). The packet loss rate over the time interval (s,t) is defined by $\theta = T_u(s,t)/(s-t)$. Obviously, $T_s(s,t)$ and $T_u(s,t)$ satisfy $T_s(s,t) + T_u(s,t) = t - T$. The main purpose of this paper is to estimate the upper bound θ^* of θ such that the exponential mean square stable of error system (15) can be reached for any admissible switching signal $\sigma(t)$ satisfying $\theta \leqslant \theta^*$.

In the next theorem, system (15) with deterministic packet losses will be studied. Next, construct the following Lyapunov functional for system (13):

$$V_2(t) = V_{21}(t) + V_{22}(t) + V_{23}(t),$$

where

$$\begin{aligned}
V_{21}(t) &= \eta^{\mathrm{T}}(t)\widetilde{P}\eta(t),\\
V_{22}(t) &= \int\limits_{t-\tau(t)}^{t} \mathrm{e}^{2\gamma_2(s-t)}\eta^{\mathrm{T}}(s)\widetilde{R}_1\eta(s)\,\mathrm{d}s + \int\limits_{t-\bar{\tau}}^{t} \mathrm{e}^{2\gamma_2(s-t)}\eta^{\mathrm{T}}(s)\widetilde{R}_2\eta(s)\,\mathrm{d}s\\
&\quad + \int\limits_{t-\tau}^{t} \mathrm{e}^{2\gamma_2(s-t)}\eta^{\mathrm{T}}(s)\widetilde{R}_3\eta(s)\,\mathrm{d}s,\\
V_{23}(t) &= \bar{\tau}\int\limits_{-\bar{\tau}}^{0}\int\limits_{t+\theta}^{t} \mathrm{e}^{2\gamma_2(s-t)}\dot{\eta}^{\mathrm{T}}(t)\widetilde{Q}_1\dot{\eta}(t)\,\mathrm{d}s\,\mathrm{d}\theta\\
&\quad + \tau\int\limits_{-\tau}^{0}\int\limits_{t+\theta}^{t} \mathrm{e}^{2\gamma_2(s-t)}\dot{\eta}^{\mathrm{T}}(t)\widetilde{Q}_2\dot{\eta}(t)\,\mathrm{d}s\,\mathrm{d}\theta.
\end{aligned}$$

Then the following theorem can be developed.

Theorem 2. *For given scalar $\mu > 0$, $\bar{\tau} > 0$, $\tau > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > 0$, $\gamma_2 < 0$ and matrices $U > 0$, $\Lambda = \mathrm{diag}(\varrho_1, \varrho_2, \dots, \varrho_n) > 0$ if there exist appropriate dimension matrices $\widetilde{P} > 0$, $\widetilde{R}_i > 0$ $(i = 1, 2, 3)$, $\widetilde{Q}_1 > 0$, $\widetilde{Q}_2 > 0$, such that the following inequality hold*:

$$\bar{\Phi} = \begin{bmatrix}
\bar{\Phi}_{11} & \widetilde{P}-U+U\hat{A} & \mathrm{e}^{-2\gamma_2\bar{\tau}}\widetilde{Q}_1 & 0 & \mathrm{e}^{-2\gamma_2\tau}\widetilde{Q}_2 & U\hat{B} & U\hat{C}\\
* & \bar{\tau}^2\widetilde{Q}_1+\tau^2\widetilde{Q}_2-2U & 0 & 0 & 0 & U\hat{B} & U\hat{C}\\
* & * & \bar{\Phi}_{33} & 0 & 0 & 0 & 0\\
* & * & * & \bar{\Phi}_{44} & 0 & 0 & 0\\
* & * & * & * & \bar{\Phi}_{55} & 0 & 0\\
* & * & * & * & * & -\varepsilon_3 I & 0\\
* & * & * & * & * & * & -\varepsilon_4 I
\end{bmatrix} < 0, \quad (32)$$

where

$$\begin{aligned}
\bar{\Phi}_{11} &= 2\gamma_2\widetilde{P} + \widetilde{R}_1 + \widetilde{R}_2 + \widetilde{R}_3 - \mathrm{e}^{-2\gamma_2\bar{\tau}}\widetilde{Q}_1 - \mathrm{e}^{-2\gamma_2\tau}\widetilde{Q}_2 + \varepsilon_3 H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H + 2U\hat{A},\\
\bar{\Phi}_{33} &= -\mathrm{e}^{-2\gamma_2\bar{\tau}}(\widetilde{R}_2 + \widetilde{Q}_1), \qquad \bar{\Phi}_{44} = (\mu - 1)\mathrm{e}^{-2\gamma_2\bar{\tau}}\widetilde{R}_1 + \varepsilon_4 H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H,\\
\bar{\Phi}_{55} &= -\mathrm{e}^{-2\gamma_2\tau}(\widetilde{R}_3 + \widetilde{Q}_2).
\end{aligned}$$

Then, the Lyapunov functional (30) *satisfies*

$$V_2(t) \leqslant \mathrm{e}^{-2\gamma_2(t-t_k)}V_2(t_k), \quad t \in [t_k, t_{k+1}).$$

Proof. Proceeding with the same procedure in Theorem 1, for any positive scalars $\varepsilon_3 > 0$ and $\varepsilon_4 > 0$, add the left-hand side of the inequalities

$$\begin{aligned}
&\varepsilon_3\big[\eta^{\mathrm{T}}(t)H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H\eta(t) - F^{\mathrm{T}}\big(H\eta(t)\big)F\big(H\eta(t)\big)\big] \geqslant 0,\\
&\varepsilon_4\big[\eta^{\mathrm{T}}\big(t-\tau(t)\big)H^{\mathrm{T}}\Lambda^{\mathrm{T}}\Lambda H\eta\big(t-\tau(t)\big)\\
&\quad - F^{\mathrm{T}}\big(H\eta\big(t-\tau(t)\big)\big)F\big(H\eta\big(t-\tau(t)\big)\big)\big] \geqslant 0.
\end{aligned}$$

According to (31), one has

$$LV_2(t) + 2\gamma_2 V_2(t) \leqslant 0,$$

and therefore

$$V_2(t) \leqslant \mathrm{e}^{-2\gamma_2(t-t_k)}V_2(t_k), \quad t \in [t_k, t_{k+1}).$$

This completes the proof. □

In the following, based on Theorems 1 and 2, we will propose a condition to guarantee that the exponential mean square stable of error system (15) with control packet loss can be reached.

Theorem 3. *For given scalar $\mu > 0$, $\bar{\tau} > 0$, $\tau > 0$, $\varepsilon_i > 0$ $(i=1,2,3,4)$, $\gamma_1 > 0$, $\gamma_2 < 0$, $\upsilon_1 \geqslant 1$, $\upsilon_2 \geqslant 1$ and matrices $\Lambda = \operatorname{diag}(\varrho_1, \varrho_2, \dots, \varrho_n) > 0$, the exponential mean square stable of error system* (15) *can be reached if there exist appropriate dimension matrices $P > 0$, $\widetilde{P} > 0$, $R_i > 0$ $(i = 1,2,3,4,5)$, $\widetilde{R}_i > 0$ $(i = 1,2,3)$, $Q_i > 0$ $(i = 1,2,3,4)$, $\widetilde{Q}_i > 0$ $(i = 1,2)$ and any matrix $U > 0$ such conditions* (16) ,(31) *and the following LMIs hold*:

$$\begin{array}{llll}
P \leqslant \upsilon_1\widetilde{P}, & \widetilde{P} \leqslant \upsilon_2 P, & R_1 \leqslant \upsilon_1\widetilde{R}_1, & \widetilde{R}_1 \leqslant \upsilon_2 R_1,\\
R_2 \leqslant \upsilon_1\widetilde{R}_2, & \widetilde{R}_2 \leqslant \upsilon_2 R_2, & R_3 \leqslant \upsilon_1\widetilde{R}_3, & \widetilde{R}_3 \leqslant \upsilon_2 R_3,\\
Q_1 \leqslant \upsilon_1\widetilde{Q}_1, & \widetilde{Q}_1 \leqslant \upsilon_2 Q_1, & Q_2 \leqslant \upsilon_1\widetilde{Q}_2, & \widetilde{Q}_2 \leqslant \upsilon_2 Q_2,
\end{array}$$

and if the switching signal $\sigma(t)$ has an average dwell time τ_α satisfying

$$\tau_\alpha > \tau_\alpha^* = \frac{\ln(\upsilon_1\upsilon_2)}{2(\gamma_1 - (\gamma_1 - \gamma_2)\theta)}, \quad \theta \leqslant \theta^* = \frac{\gamma_1}{\gamma_1 - \gamma_2}. \tag{33}$$

Proof. Considering the following Lyapunov functional:

$$V(t) = V_{\sigma(t)}(t),$$

where $V_1(t)$ and $V_2(t)$ are given as in (18) and (30), respectively. When $\sigma(t) = 1$, one can obtain that

$$V(t) \leqslant \mathrm{e}^{-2\gamma_1(t-t_k)}V(t_k), \quad t \in [t_k, t_{k+1}),$$

and

$$V_1(t_k) \leqslant \upsilon_1 V_2(t_k^-).$$

Similarly, if $\sigma(t) = 2$, then

$$V(t) \leqslant \mathrm{e}^{-2\gamma_2(t-t_k)} V(t_k), \quad t \in [t_k, t_{k+1}),$$

and

$$V_2(t_k) \leqslant \upsilon_2 V_1(t_k^-).$$

For any $t \geqslant 0$, there exists a scalar $k \geqslant 0$ such that $t \in [t_k, t_{k+1})$. Let $T_1, T_2, \dots, T_j$ be the switching instants of $\sigma(t)$ on the interval $[0, t)$ and assume $0 < T_1 < T_2 < \dots < T_j$. Then, for each T_i, there exists $l \in \{1, 2, \dots, k\}$ such that $T_i = t_l$. Thus, for any $t \in [t_k, t_{k+1})$,

$$\begin{aligned}
V(t) &\leqslant \mathrm{e}^{-2\gamma_{\sigma(T_j)}(t-T_j)} V_{\sigma(T_j)}(T_j) \\
&\leqslant \mathrm{e}^{-2\gamma_{\sigma(T_j)}(t-T_j)} (\upsilon_1\upsilon_2) V_{\sigma(T_j^-)}(T_j^-) \\
&\leqslant \cdots \\
&\leqslant \mathrm{e}^{-2\gamma_1 T_s(0,t) - 2\gamma_2 T_\mu(0,t)} (\upsilon_1\upsilon_2)^{N_\sigma(0,t)} V_{\sigma(0)}(0) \\
&\leqslant \mathrm{e}^{-2\gamma_1(1-\theta)t - 2\gamma_2\theta t} \mathrm{e}^{(N_0 + t/\tau_\alpha)\ln(\upsilon_1\upsilon_2)} V_{\sigma(0)}(0) \\
&= \mathrm{e}^{N_0 \ln(\upsilon_1\upsilon_2)} \mathrm{e}^{-2\gamma t} V_{\sigma(0)}(0),
\end{aligned}$$

where $T_s(0,t)$ and $T_\mu(0,t)$ are defined in Remark 2, and

$$\gamma = \gamma_1 - (\gamma_1 - \gamma_2)\theta - \frac{\ln(\upsilon_1\upsilon_2)}{2\tau_\alpha} \in \big(0,\ \gamma_1 - (\gamma_1 - \gamma_2)\theta\big]. \tag{34}$$

From the definition of $V(t)$, let $c_1 = \min\{\lambda_{\min}(P), \lambda_{\min}(\widetilde{P})\}$, one can deduce that

$$c_1 \mathbf{E}\big\{\|\eta(t)\|^2\big\} \leqslant \mathbf{E}\big\{V(t)\big\} \leqslant \mathrm{e}^{N_0 \ln(\upsilon_1\upsilon_2)} \mathrm{e}^{-2\gamma t} \mathbf{E}\big\{V_{\sigma(0)}(0)\big\}.$$

Next, similar to the same procedure in Theorem 1, we have

$$\mathbf{E}\{\|\eta(t)\|^2\} \leqslant c_2 \frac{\mathrm{e}^{N_0 \ln(\upsilon_1\upsilon_2)}}{c_1} \mathrm{e}^{-2\gamma t} \sup_{-\tau^* \leqslant \theta \leqslant 0} \mathbf{E}\{\|\phi(\theta)\|^2\},$$

where

$$\begin{aligned}
c_2 = {} & \lambda_{\max}(P) + \zeta_1\big(\lambda_{\max}(R_1) + \lambda_{\max}(R_2)\big) \\
&+ \zeta_2\lambda_{\max}(R_3) + \zeta_3\lambda_{\max}(Q_1) + \zeta_4\lambda_{\max}(Q_2), \\
&+ \lambda_{\max}(\widetilde{P}) + \widetilde{\zeta}_1\big(\lambda_{\max}(\widetilde{R}_1) + \lambda_{\max}(\widetilde{R}_2)\big) \\
&+ \widetilde{\zeta}_2\lambda_{\max}(\widetilde{R}_3) + \widetilde{\zeta}_3\lambda_{\max}(\widetilde{Q}_1) + \widetilde{\zeta}_4\lambda_{\max}(\widetilde{Q}_2)
\end{aligned}$$

and

$$\widetilde{\zeta}_1 = \frac{1 - \mathrm{e}^{-2\gamma_2\bar{\tau}}}{2\gamma_2}, \qquad \widetilde{\zeta}_2 = \frac{1 - \mathrm{e}^{-2\gamma_2\tau}}{2\gamma_2},$$

$$\widetilde{\zeta}_3 = \bar{\tau}\left(\frac{\bar{\tau}}{2\gamma_2} - \frac{1 - \mathrm{e}^{-2\gamma_2\bar{\tau}}}{4\gamma_2^2}\right), \qquad \widetilde{\zeta}_4 = \tau\left(\frac{\tau}{2\gamma_2} - \frac{1 - \mathrm{e}^{-2\gamma_2\tau}}{4\gamma_2^2}\right).$$

Thus, according to Definition 1, for any switching signal $\sigma(t)$ with the average dwell time satisfying $\tau_\alpha > \ln(\upsilon_1\upsilon_2)/(2(\gamma_1 - (\gamma_1 - \gamma_2)\theta))$, the exponential mean square stable of error system (15) with control packet loss can be reached. This completes the proof. □

Remark 3. Compared with the reference [14] and [15], the controller we designed is a stochastic sampling controller, which can save control cost and more realistic. Compared with the reference [13] and [25], we consider the loss of control packets. Therefore, our results enrich and expand the results of the above work.

Remark 4. It is worth noting that the matrix inequalities (16) and (32) in Theorem 3 need to be satisfied, which narrows the scope of the solution to a certain extent. We need to solve them by means of LMI in the MATLAB toolbox, and we still found a feasible solution. Thus, for any $\theta \leqslant \theta^*$, the exponential state estimation problem can be solved. In the existing literature, there were only a very few works based on the exponential state estimation problem for competitive neural network. Therefore, the main contribution of this paper is to deal with the exponential state estimation problem for competitive neural network under a sampled-data controller with stochastically varying sampling periods and control packet loss. Thus, the theoretical results proposed enrich the study on exponential state estimation problem.

Remark 5. It can be viewed from (49) that the exponential decay rate γ of the error system (5) depends on θ and τ_α, it is easy to see that a smaller τ_α and a larger θ lead to a smaller exponential decay rate γ. Thus, it means that the loss of more control packet and frequent switching between the cases of packet and nonpacket-missing will degrade the exponential stability of system (5).

Remark 6. It is worth pointing out that, from (33), the upper bound of the control packet loss rate θ^* not only depends on the convergence rate γ_1 but also on the divergence rate γ_2. On the other hand, the lower bound of average dwell-time τ_α^* not only depends on the packet loss rate θ but also on the convergence rate γ_1 and the divergence rate γ_2. It also reflects the admissible switching frequency between the cases of nonpacket-missing and packet-missing.

4 Numerical simulations

In this section, a numerical example with simulation results is given to show the effectiveness of the proposed method.

Example 1. Consider a competitive neural network (1) and the measurement equation (2), the parameters are given as $\varepsilon_1 = 1.1$, $\varepsilon_2 = 1.2$, $\varepsilon_3 = 1.3$, $\varepsilon_4 = 1.4$, $\gamma_1 = 0.54$, $\gamma_2 = -1.50$, $\upsilon_1 = 1$, $\upsilon_2 = 1.1$, $h_1 = 0.2$, $h_2 = 0.3$, $\rho = 0.6$, $\tau = 0.016$, and

$$A = \begin{pmatrix} 0.3 & 0 \\ 0 & -0.2 \end{pmatrix}, \qquad B = \begin{pmatrix} 2.55 & -0.1 \\ -0.16 & 3.5 \end{pmatrix}, \qquad \bar{B} = \begin{pmatrix} -2.1 & -0.52 \\ -0.3 & -2 \end{pmatrix},$$

$$D = \begin{pmatrix} 1.2 & -0.1 \\ 0.2 & 0.3 \end{pmatrix}, \qquad C = \begin{pmatrix} 1.28 & 0 \\ 0 & 2 \end{pmatrix}, \qquad M = \begin{pmatrix} 2 & -0.16 \\ -0.1 & 1.5 \end{pmatrix}.$$

The nonlinear function $f(x) = \tanh(x)$, $\tau(t) = 0.2(1 + \sin(t))$, $\mu = 0.2$, $\bar{\tau} = 0.4$. Let $W = U\bar{W}$, from the parameters above, according to solve the LMI conditions (16) and (32), the following feasible solution can be obtained:

$$U = \begin{pmatrix} 0.0377 & 0.0013 & 0 & 0 \\ 0.0013 & 0.0359 & 0 & 0 \\ 0 & 0 & 0.2009 & 0.0076 \\ 0 & 0 & 0.0076 & 0.1690 \end{pmatrix},$$

$$W = \begin{pmatrix} 0.0736 & 0.0011 & 0 & 0 \\ 0.0011 & 0.0792 & 0 & 0 \\ 0 & 0 & 0.2042 & 0.0424 \\ 0 & 0 & 0.0424 & 0.1081 \end{pmatrix},$$

$$\bar{W} = U^{-1}W = \begin{pmatrix} 1.9531 & -0.0455 & 0 & 0 \\ -0.0389 & 2.2072 & 0 & 0 \\ 0 & 0 & 1.0088 & 0.1874 \\ 0 & 0 & 0.2058 & 0.6312 \end{pmatrix}.$$

We take $E = G = I$, the gain matrix can be obtained:

$$K = \begin{pmatrix} 1.9531 & -0.0455 \\ -0.0389 & 2.2072 \end{pmatrix}, \qquad \bar{K} = \begin{matrix} 1.0088 & 0.1874 \\ 0.2058 & 0.6312 \end{matrix}.$$

In this simulation, let $h_1 = 0.2$, $h_2 = 0.3$, $\gamma_1 = 0.54$, $\gamma_2 = -1.5$, $\upsilon_1 = 1$, $\upsilon_2 = 1.1$, the average dwell time $\tau_\alpha = 0.36$, and the control packet loss rate as $\theta = 0.15$. From these values one has $\tau_\alpha > \tau_\alpha^* = 0.2037$, $\theta < \theta^* = 0.2647$ and the exponential convergence rate $\gamma = 0.1149$. In addition, we take $h_1 = 0.2$, $\gamma_2 = -1.5$, $\upsilon_1 = 1$, $\upsilon_2 = 1.1$, for a different sampling interval h_2, Table 1 lists the admissible convergence rate γ_1, the upper bound θ^* of control packet missing rate, the lower bound τ_α^* of average dwell time, and the convergence rate γ. It can be seen from Table 1 that the sampling interval h_2 has an important influence on the above-mentioned parameters. When the sampling interval h_2 becomes larger, it will reduce the control cost, but the convergence rate of system (5) will also slow down. Furthermore, for a fixed control packet missing rate $\theta < \theta^*$, a larger sampling interval h_2 corresponds to a larger τ_α^*. At the same time, if the average dwell time $\tau_\alpha > \tau_\alpha^*$ is fixed, the convergence rate of system (5) is faster when the sampling interval h_2 becomes smaller. Therefore, there exists a tradeoff between convergence speed and control cost.

Table 1. γ_1, θ^*, τ_α^* and γ for different values of h_2.

h_2	0.25	0.3	0.35	0.4
γ_1	0.58	0.54	0.51	0.48
θ^*	0.2788	0.2647	0.2537	0.2424
τ_α^* $(\theta = 0.15)$	0.1778	0.2037	0.2286	0.2604
γ $(\theta = 0.15, \tau_\alpha = 0.36)$	0.1356	0.1016	0.0761	0.0506

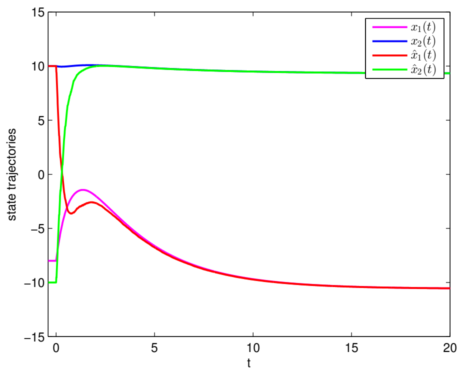

Figure 1. The true state $x(t)$ and its estimated state $\hat{x}(t)$.

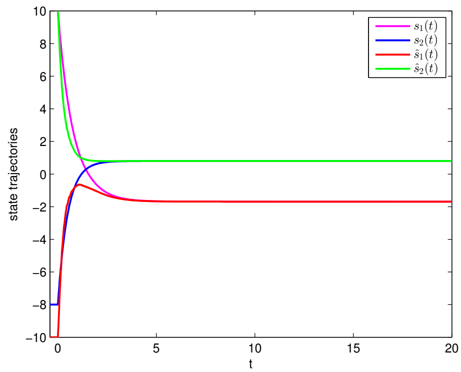

Figure 2. The true state $S(t)$ and its estimated state $\hat{S}(t)$.

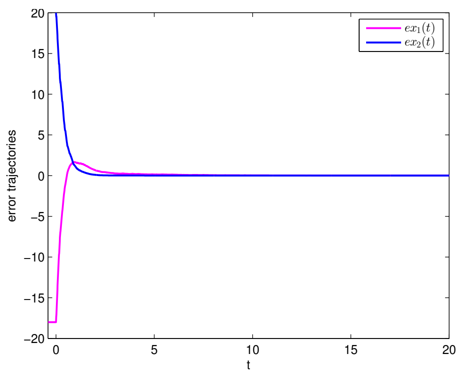

Figure 3. The error of true state $x(t)$ and its estimated state $\hat{x}(t)$.

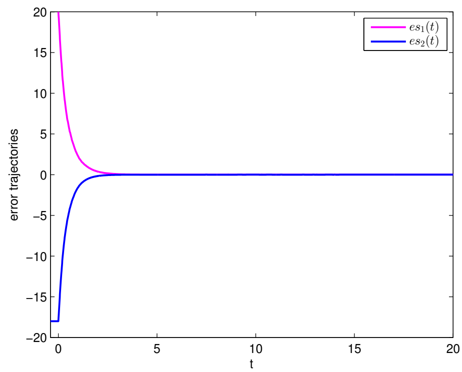

Figure 4. The error of true state $S(t)$ and its estimated state $\hat{S}(t)$.

For the networks (1) and (3), the true state trajectories $x(t)$, $S(t)$ and its estimated state trajectories $\hat{x}(t)$, $\hat{S}(t)$ are described in Figs. 1, 2. Figures 3, 4 display the error of true state $x(t)$, $S(t)$ and its estimated state $\hat{x}(t)$, $\hat{S}(t)$. The sampling intervals are provided in Fig. 5, and each stem shows the sampling time t_k, $k = 0, 1, 2, 3 \ldots$. The value of each stem represents the length of the time period $t_{k+1} - t_k$. Finally, the switching signal $\sigma(t)$ is shown in Fig. 6.

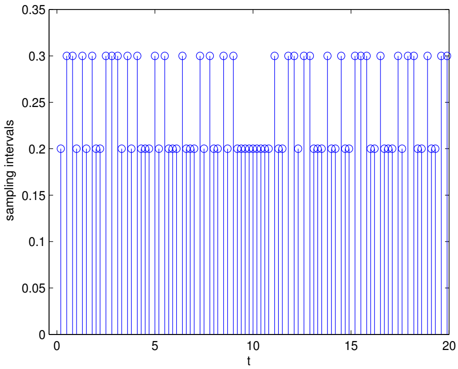

Figure 5. Sampling intervals.

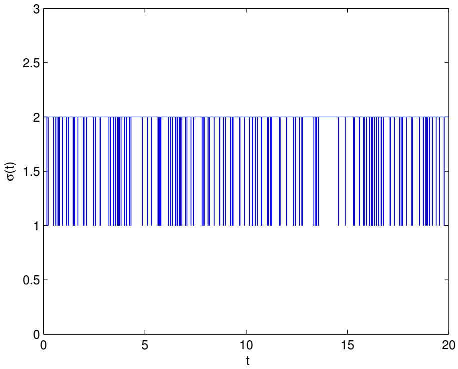

Figure 6. Switching signal $\sigma(t)$.

Remark 7. Compared with the reference [21], we have a larger sampling interval, which saves control costs. From Table 1 it can be seen that the sampling interval h_2 has significant effects on the above-mentioned parameters, with the increase of sampling interval h_2, which means the reduction in control cost, system (5) has slower convergence speed. Furthermore, for a fixed control packet missing rate $\theta < \theta^*$, a larger sampling interval h_2 corresponds to a larger τ_α^*. At the same time, when the average dwell time $\tau_\alpha > \tau_\alpha^*$ is fixed, a smaller sampling interval h_2, speeds up the convergence of system (5). Therefore, there exists a tradeoff between convergence speed and control cost.

5 Conclusion

In this paper, the exponential state estimation problem has been studied for competitive neural networks via stochastic sampled-data control with packet losses. First, transmittal delays between neurons are considered to reflect more realistic dynamical behaviors of competitive neural networks. Second, a sampled-data controller involving two sampling periods is designed to ensure that the error system can achieve exponential stability and the corresponding stability conditions are obtained in terms of LMIs. Third, by utilizing an input delay approach, the probabilistic sampling state estimator is transformed into a continuous time-delay system with stochastic delays and a piecewise constant function is used to specify deterministic packet losses. Then, by constructing an appropriate Lyapunov function and using some basic inequality techniques, the exponential stability problem can be solved with some appropriate feedback gains and sampling intervals. Finally, the rightness of the proposed criteria is demonstrated by a numerical example. It is possible to extend the study by using the stochastic sampled-data with actuator saturation approach in the future.

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