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Quantized passive filtering for switched delayed neural networks*

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Abstract. The issue of quantized passive filtering for switched delayed neural networks with noise interference is studied in this paper. Both arbitrary and semi-Markov switching rules are taken into account. By choosing Lyapunov functionals and applying several inequality techniques, sufficient conditions are proposed to ensure the filter error system to be not only exponentially stable, but also exponentially passive from the noise interference to the output error. The gain matrix for the proposed quantized passive filter is able to be determined through the feasible solution of linear matrix inequalities, which are computationally tractable with the help of some popular convex optimization tools. Finally, two numerical examples are given to illustrate the usefulness of the quantized passive filter design methods.

Keywords: quantization, passive filter, arbitrary switching, semi-Markov switching.

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1 Introduction

In some practical applications of delayed neural networks (DNNs), one needs to estimate the neuron states by available measurements and then employ the estimated values to achieve a certain desired performance [28]. Thus effective algorithms capable of estimating the state of neurons are of both theoretical and practical significance. As we know, the Kalman filtering method is one of the most efficient ways to handle state estimation problems. However, when the external interferences do not have stationary Gaussian noise properties, this scheme will no longer be valid, which limits its application in DNNs and leads to the development of the passive filtering methods. A passive filter consists of resistors, capacitors, and inductors. It has been shown that passive filters are not only suitable for the situation of large current or voltage levels, but also can work well at very high frequencies [14, 17]. In 2010, the problem of passive filtering for DNNs was studied in [1], and a delay-dependent passive filter was proposed for ensuring that the filtering error system is stable as well as passive. Later, robust passive filtering for DNNs with uncertain parameters was considered in [38], where a cone complementarity linearization algorithm was used to calculate the desired filter gain.

Over the last two decades, switched systems, as a particular class of hybrid dynamical systems, have attracted enormous attention owing to their potential applications in the vestibulo–ocular reflex [13], automotive roll dynamics control [25], image encryption [30], and other fields [6, 20, 31]. Generally, switched systems are made up of a set of continuous-time (or discrete-time) subsystems defined by differential (or difference) equations as well as a switching rule that supervises the switching among the subsystems. The switching specifying which subsystem is activated every instant can be either arbitrary or restricted (e.g., obeying a certain probability distribution constraint) [40]. In recent years, by combining the theory of switched systems with DNNs, various mathematical models of switched DNNs have been introduced and a number of theoretical achievements have been reported; see, for instance, [22, 23, 33, 42]. Particularly, in the context of passive filtering for switched delayed neural networks (SDNNs), an error passivation method was put forward in [18], where it was ensured that the corresponding filtering error system is passive and asymptotically stable; the exponential passive filtering for SDNNs was addressed in [2], where a sufficient condition for the needed filter was derived in the form of linear matrix inequalities (LMIs).

In traditional communications, one often assumes that data is transmitted through perfect communication network channels. In practice, however, as a result of the limitation of storage and digital communication bandwidth among nodes, the original data needs to be quantized before transmission. Quantization can be regarded as a map from continuous signals to discrete finite sets [32]. The quantized control strategy is able to save channel resources and cut down both the amount of transmitted data and channel blocking [36]. During the past few years, the design of quantized filters has been a hot topic and a variety of outstanding results have been acquired. To name a few, in [8], a quantized H_∞ filter for time-varying switched systems was designed via employing the gridding method. In [9], based on a sector bound method, both H_∞ and $l_2 - l_\infty$ filtering designs for a class of discrete switched system with quantized measurements were investigated. To the best of our knowledge, nevertheless, there is no relevant report on quantized passive filtering for SDNNs in the open literature, which inspires our current research.

From the above discussions this paper addresses the quantized passive filtering for SDNNs with noise interference. Both arbitrary switching and semi-Markov switching are taken into account. By choosing Lyapunov functionals and applying several inequality techniques, sufficient conditions are proposed to ensure the filtering error system to be not only exponentially stable, but also exponentially passive from the noise interference to the output error. The gain matrix for the proposed quantized passive filter is able to be determined through the feasible solution of LMIs, which are computationally tractable with the help of popular convex optimization tools. The remainder of this paper is as follows: in Section 2, we give the SDNN model, the quantized filter, as well as two types of switching rules under consideration. In Section 3, we propose quantized passive filter design methods for SDNNs under arbitrary switching and semi-Markov switching, respectively. In Section 4, we provide two numerical examples to illustrate the usefulness of the quantized passive filter design methods. Section 5 summarizes our conclusions.

Notations. Throughout the present study, we apply $\mathbb{R}^n$ to represent a n-dimensional Euclidean space with norm $\|\cdot\|$, $\mathbb{R}^{n\times m}$ to represent the set of all $n\times m$ real matrices, and $\mathbb{Z}^+$ (respectively, $\mathbb{R}^+$) to stand for the set of positive integer numbers (respectively, non-negative real numbers). For any matrix $X\in\mathbb{R}^{n\times m}$, X^{T} denotes its transpose, $\lambda_{\min}(X)$ denotes its smallest eigenvalue, and $X>0$ means that it is symmetric positive definite. In the case when $n=m$, let us define by $*$ the symmetric blocks in X and by $\mathscr{S}(X)$ the sum of X and X^{T}. Moreover, we denote by $\mathcal{E}\{\cdot\}$ the expectation operator, by $\mathrm{diag}\{\cdots\}$ a diagonal matrix, and by I (respectively, 0) the identity (respectively, zero) matrix with appropriate dimension.

2 Preliminaries

Consider a switched system composed of multiple DNNs given by

$$\begin{aligned}\dot{x}(t)&=\mathcal{A}\big(\gamma(t)\big)x(t)+\mathcal{W}\big(\gamma(t)\big)\psi\big(x(t-\theta)\big)+\mathcal{J}\big(\gamma(t)\big)(t)+\mathcal{G}\big(\gamma(t)\big)\omega(t),\\ y(t)&=\mathcal{C}\big(\gamma(t)\big)x(t)+\mathcal{D}\big(\gamma(t)\big)x(t-\theta)+\mathcal{F}\big(\gamma(t)\big)\omega(t),\end{aligned}\tag{1}$$

where $x(t)\in\mathbb{R}^n$, $y(t)\in\mathbb{R}^m$, and $\omega(t)\in\mathbb{R}^m$ represent the state, output, and noise interference, respectively; $\theta>0$ stands for the time-delay; $\mathcal{A}(\gamma(t))=\mathrm{diag}\{-a_1(\gamma(t)),\dots,-a_n(\gamma(t))\}\in\mathbb{R}^{n\times n}$ ($a_k(\gamma(t))>0$, $k=1,\dots,n$) and $\mathcal{W}(\gamma(t))(t)\in\mathbb{R}^{n\times n}$ are the self-feedback matrix and the delayed connection weight matrix, respectively; $\mathcal{G}(\gamma(t))\in\mathbb{R}^{n\times m}$, $\mathcal{C}(\gamma(t))\in\mathbb{R}^{m\times n}$, $\mathcal{D}(\gamma(t))\in\mathbb{R}^{m\times n}$, and $\mathcal{F}(\gamma(t))\in\mathbb{R}^{m\times m}$ are known constant matrices; $\mathcal{J}(\gamma(t))(t)\in\mathbb{R}^n$ is an external input vector; $\gamma(t)$ is the switching signal which chooses its values in $\varGamma=(1,\dots,N), N\in\mathbb{Z}^+$; $\psi(x(t))$ denotes the activation function, which is assumed to be global Lipschitz continuous with Lipschitz constant $L_\psi>0$ [11, 12], i.e.,

$$\big\|\psi(s_1)-\psi(s_2)\big\|\leqslant L_\psi\big\|s_1-s_2\big\|\quad\forall s_1,s_2\in\mathbb{R}^n.\tag{2}$$

For more general assumptions on the activation function, one may refer to [26, 27, 34].

A quantizer $q(\cdot) : \mathbb{R}^m \to \Phi^m$ is defined as $q(\nu) = [q_1(\nu_1), \dots, q_m(\nu_m)]^{\mathrm{T}}$, where $\Phi = \{\pm\phi_l,\ \phi_l = \chi^l\phi_0,\ l = 0, \pm1, \pm2, \dots\} \cup \{0\}$ with $\phi_0 > 0$ and $0 < \chi < 1$ [5, 19]. For any $\nu_j \in \mathbb{R}$ ($j = 1, \dots, m$), quantizer $q_j(\nu)$ is given by

$$q_j(\nu_j) = \begin{cases} \phi_l, & \phi_l/(1+\delta) < \nu_j \leqslant \phi_l/(1-\delta), \\ 0, & \nu_j = 0, \\ -q_j(-\nu_j), & \nu_j < 0, \end{cases}$$

where $\delta = (1–\chi)/(1+\chi)$. Note that $q(\nu)$ can be expressed by the sector bound method [10]:

$$q(\nu) = (1 + \varDelta)\nu, \quad \varDelta \in [-\delta, \delta]. \tag{3}$$

Remark 1. In networked control practice, owing to the limited transmission capacity of the network, signals need to be quantized before transmission for acquiring better control results. The quantizer can be seen as a coder that transforms the continuous signal into a piecewise continuous one [15].

Considering quantization effect (3), we propose the following filter

$$\begin{aligned} \dot{\breve{x}}(t) &= \mathcal{A}\big(\gamma(t)\big)\breve{x}(t) + \mathcal{W}\big(\gamma(t)\big)\psi\big(\breve{x}(t-\theta)\big) \\ &\quad + \mathcal{J}\big(\gamma(t)\big)(t) + L\big(\gamma(t)\big)q\big(y(t) - \breve{y}(t)\big), \\ \breve{y}(t) &= \mathcal{C}\big(\gamma(t)\big)\breve{x}(t) + \mathcal{D}\big(\gamma(t)\big)\breve{x}(t-\theta), \end{aligned} \tag{4}$$

where $\breve{x}(t) \in \mathbb{R}^n$, $\breve{y}(t) \in \mathbb{R}^m$, and $L(\gamma(t)) \in \mathbb{R}^{n\times m}$ are the filter state vector, the filter output vector, and the filter gain matrix, respectively. If we define by $z(t) = x(t) - \breve{x}(t)$ the filtering error and by $\bar{y}(t) = y(t) - \breve{y}(t)$ the output error, then the filtering error system is able to be represented as follows:

$$\begin{aligned} \dot{z}(t) &= \big(\mathcal{A}(\gamma(t)\big) - L\big(\gamma(t)\big)(1+\varDelta)\mathcal{C}\big(\gamma(t)\big)\big)z(t) \\ &\quad - L\big(\gamma(t)\big)(1+\varDelta)\mathcal{D}\big(\gamma(t)\big)z(t-\theta) + \mathcal{W}\big(\gamma(t)\big)\bar{\psi}\big(z(t-\theta)\big) \\ &\quad + \big(\mathcal{G}\big(\gamma(t)\big) - L\big(\gamma(t)\big)(1+\varDelta)\mathcal{F}\big(\gamma(t)\big)\big)\omega(t), \\ \bar{y}(t) &= \mathcal{C}\big(\gamma(t)\big)z(t) + \mathcal{D}\big(\gamma(t)\big)z(t-\theta) + \mathcal{F}\big(\gamma(t)\big)\omega(t), \end{aligned} \tag{5}$$

where $\bar{\psi}(z(t-\theta)) = \psi(x(t-\theta)) - \psi(\breve{x}(t-\theta))$.

In this paper, the following two types of switching rules are considered:

Case 1. $\gamma(t)$ is a arbitrary switching signal.

Case 2. $\gamma(t)$ is a semi-Markov switching signal; i.e., $(\gamma(t), h \geqslant 0)_{t\geqslant 0} = (\gamma_n, h_n)_{n\in\mathbb{Z}^+}$ represents a continuous-time and discrete-state semi-Markov process, where $(\gamma_n)_{n\in\mathbb{Z}^+}$ is the index of system mode at nth transition selecting values in $\varGamma$, and $(h_n)_{n\in\mathbb{Z}^+}$ is the sojourn time of mode γ_{n-1} between the $(n–1)$th transition and nth transition selecting values in $\mathbb{R}^+$. The entries of transition probability matrix $\varPi(h) = \{\pi_{uv}(h)\}$ is determined by

$$\begin{aligned} &\Pr\big\{\gamma_{n+1} = v,\ h_{n+1} \leqslant h + \alpha \ \big|\ \gamma_n = u,\ h_{n+1} > h\big\} \\ &\quad = \pi_{uv}(h)\alpha + o(\alpha), \quad u \neq v, \\ &\Pr\big\{\gamma_{n+1} = v,\ h_{n+1} > h + \alpha \ \big|\ \gamma_n = u,\ h_{n+1} > h\big\} \\ &\quad = 1 + \pi_{uu}(h)\alpha + o(\alpha), \quad u = v, \end{aligned}$$

where $h > 0$ denotes the sojourn time, $lim_{\alpha \to 0} o(\alpha)/\alpha = 0$, $\pi_{uv}(h) \geqslant 0$ is the transition rate from mode u at time t to mode v at time $t + \alpha$ for $u \neq v$, and $\pi_{uu}(h) = -\sum_{v=1, v \neq u}^{N} \pi_{uv}(h)$.

Remark 2. The semi-Markov switching is a switching process that can be applied to describe sudden structure changes as well as abrupt component errors. Compared with the usual Markov switching [21, 24], the semi-Markov switching is more general since its sojourn-time can follow a nonexponential distribution that results in time-varying transition rates.

3 Main results

In this section, we propose design methods for quantized passive filtering of SDNN (1) under arbitrary switching and semi-Markov switching, respectively.

3.1 Quantized passive filtering under arbitrary switching

The issue of quantized passive filtering under arbitrary switching to be addressed can be formulated explicitly as follows: for the switching rule in Case 1, design a quantized passive filter having the form in (4) such as the filtering error system in (5):

(i) is exponentially stable when $\omega(t) = 0$;
(ii) is exponentially passive for $\omega(t) \neq 0$ [2]; i.e., for a given scalar $\beta > 0$,

$$\int_0^t \omega^{\mathrm{T}}(s)\hat{y}(s)\,\mathrm{d}s + \beta \geqslant \int_0^t H\big(z(s)\big)\,\mathrm{d}s$$

holds, in which $\hat{y}(s) = \mathrm{e}^{\kappa s}\bar{y}(s)$, $\kappa > 0$ is a real scalar, and $H(z(s))$ is a positive semi-definite storage function.

Define the indicator function as $\zeta(t) = [\zeta_1(t), \ldots, \zeta_N(t)]^{\mathrm{T}}$, where

$$\zeta_u(t) = \begin{cases} 1 & \text{when the switch system is described by the uth mode} \\ & (\mathcal{A}_u, \mathcal{W}_u, \mathcal{J}_u, \mathcal{G}_u, \mathcal{C}_u, \mathcal{D}_u, \mathcal{F}_u), \\ 0 & \text{otherwise} \end{cases}$$

with $u \in \Gamma$. Then the filtering error system can be rewritten as

$$\begin{aligned} \dot{z}(t) = {} & \sum_{u=1}^{N} \zeta_u(t)\big\{\big(\mathcal{A}_u - L_u(1+\Delta)\mathcal{C}_u\big)z(t) - L_u(1+\Delta)\mathcal{D}_u z(t-\theta) \\ & + \mathcal{W}_u\bar{\psi}\big(z(t-\theta)\big) + \big(\mathcal{G}_u - L_u(1+\Delta)\mathcal{F}_u\big)\omega(t)\big\}, \\ \bar{y}(t) = {} & \sum_{u=1}^{N} \zeta_u(t)\big[\mathcal{C}_u z(t) + \mathcal{D}_u z(t-\theta) + \mathcal{F}_u\omega(t)\big]. \end{aligned} \tag{6}$$

Note that $\sum_{u=1}^{N} \zeta_u(t) = 1$.

For the arbitrary switching rule, one can obtain the following result.

Theorem 1. *If there exist matrices $P_u > 0$, $R_u > 0$, $S > 0$, M_u, and scalar $\varepsilon > 0$ such that*

$$\begin{bmatrix} \Theta_{1u} & \Theta_{2u} & \Theta_{3u} & P_u\mathcal{W}_u & M_u \\ * & L_\psi^2 I - \mathrm{e}^{-\kappa\theta}R_u + \varepsilon\mathcal{D}_u^{\mathrm{T}}\mathcal{D}_u & -\frac{1}{2}\mathcal{D}_u^{\mathrm{T}} + \varepsilon\mathcal{D}_u^{\mathrm{T}}\mathcal{F}_u & 0 & 0 \\ * & * & -\frac{1}{2}\mathscr{S}(\mathcal{F}_u) + \varepsilon\mathcal{F}_u^{\mathrm{T}}\mathcal{F}_u & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\frac{\varepsilon}{\delta^2}I \end{bmatrix} < 0 \quad (7)$$

holds true for any $u \in \Gamma$, where

$$\Theta_{1u} = \mathscr{S}(P_u\mathcal{A}_u - M_u\mathcal{C}_u) + \kappa P_u + S + R_u + \varepsilon\mathcal{C}_u^{\mathrm{T}}\mathcal{C}_u,$$
$$\Theta_{2u} = -M_u\mathcal{D}_u + \varepsilon\mathcal{C}_u^{\mathrm{T}}\mathcal{D}_u, \qquad \Theta_{3u} = P_u\mathcal{G}_u - M_u\mathcal{F}_u - \frac{1}{2}\mathcal{C}_u^{\mathrm{T}} + \varepsilon\mathcal{C}_u^{\mathrm{T}}\mathcal{F}_u,$$

then the issue of quantized passive filtering under arbitrary switching is solvable, and the needed gain matrix can be chosen as

$$L_u = P_u^{-1}M_u. \quad (8)$$

Proof. Define

$$\bar{\Theta}_u = \mathscr{S}(P_u\mathcal{A}_u - M_u\mathcal{C}_u - M_u\Delta\mathcal{C}_u) + \kappa P_u + P_u\mathcal{W}_u\mathcal{W}_u^{\mathrm{T}}P_u + S + R_u,$$
$$\check{\Theta}_u = \mathscr{S}(P_u\mathcal{A}_u - M_u\mathcal{C}_u) + \kappa P_u + P_u\mathcal{W}_u\mathcal{W}_u^{\mathrm{T}}P_u + S + R_u,$$
$$\bar{\Omega}_u = \begin{bmatrix} \bar{\Theta}_u & -M_u\mathcal{D}_u - M_u\Delta\mathcal{D}_u & P_u\mathcal{G}_u - M_u\mathcal{F}_u - M_u\Delta\mathcal{F}_u - \frac{1}{2}\mathcal{C}_u^{\mathrm{T}} \\ * & L_\psi^2 I - \mathrm{e}^{-\kappa\theta}R_u & -\frac{1}{2}\mathcal{D}_u^{\mathrm{T}} \\ * & * & -\frac{1}{2}\mathscr{S}(\mathcal{F}_u) \end{bmatrix}.$$

Then, in view of the well-known inequality $XY^{\mathrm{T}} + YX^{\mathrm{T}} \leqslant (1/\varepsilon)XX^{\mathrm{T}} + \varepsilon YY^{\mathrm{T}}$ ($\varepsilon > 0$), one can write

$$\bar{\Omega}_u \leqslant \begin{bmatrix} \check{\Theta}_u & -M_u\mathcal{D}_u & P_u\mathcal{G}_u - M_u\mathcal{F}_u - \frac{1}{2}\mathcal{C}_u^{\mathrm{T}} \\ * & L_\psi^2 I - \mathrm{e}^{-\kappa\theta}R_u & -\frac{1}{2}\mathcal{D}_u^{\mathrm{T}} \\ * & * & -\frac{1}{2}\mathscr{S}(\mathcal{F}_u) \end{bmatrix}$$
$$+ \frac{1}{\varepsilon}\begin{bmatrix} -M_u \\ 0 \\ 0 \end{bmatrix}\Delta^2\begin{bmatrix} -M_u^{\mathrm{T}} & 0 & 0 \end{bmatrix} + \varepsilon\begin{bmatrix} \mathcal{C}_u^{\mathrm{T}} \\ \mathcal{D}_u^{\mathrm{T}} \\ \mathcal{F}_u^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} \mathcal{C}_u & \mathcal{D}_u & \mathcal{F}_u \end{bmatrix}.$$

It follows by $\Delta^2 \leqslant \delta^2$ that

$$\bar{\Omega}_u \leqslant \tilde{\Omega}_u, \quad (9)$$

where

$$\tilde{\Omega}_u = \begin{bmatrix} \tilde{\Theta}_{1u} & \Theta_{2u} & \Theta_{3u} \\ * & L_\psi^2 I - \mathrm{e}^{-\kappa\theta}R_u + \varepsilon\mathcal{D}_u^{\mathrm{T}}\mathcal{D}_u & -\frac{1}{2}\mathcal{D}_u^{\mathrm{T}} + \varepsilon\mathcal{D}_u^{\mathrm{T}}\mathcal{F}_u \\ * & * & -\frac{1}{2}\mathscr{S}(\mathcal{F}_u) + \varepsilon\mathcal{F}_u^{\mathrm{T}}\mathcal{F}_u \end{bmatrix},$$
$$\tilde{\Theta}_{1u} = \mathscr{S}(P_u\mathcal{A}_u - M_u\mathcal{C}_u) + \kappa P_u + P_u\mathcal{W}_u\mathcal{W}_u^{\mathrm{T}}P_u + S + R_u$$
$$+ \varepsilon\mathcal{C}_u^{\mathrm{T}}\mathcal{C}_u + \frac{\delta^2}{\varepsilon}M_uM_u^{\mathrm{T}}.$$

By Schur's complement, (7) is equivalent to $\tilde{\Omega}_u < 0$, which together with (9) ensures that

$$\bar{\Omega}_u < 0. \tag{10}$$

Now, construct multiple Lyapunov functionals as follows:

$$\begin{aligned} V_u\big(z(t), t\big) &= \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u z(t) \\ &\quad + \int_{-\theta}^{0} \mathrm{e}^{\kappa(t+s)} z^{\mathrm{T}}(t+s) R_u z(t+s)\,\mathrm{d}s, \quad u = 1, \dots, N. \end{aligned} \tag{11}$$

Then, along the trajectories of system (6), it can be calculated that

$$\begin{aligned} \dot{V}_u\big(z(t), t\big) &= \kappa \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u z(t) + 2\mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u \dot{z}(t) + \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) R_u z(t) \\ &\quad - \mathrm{e}^{\kappa(t-\theta)} z^{\mathrm{T}}(t-\theta) R_u z(t-\theta) \\ &= \sum_{u=1}^{N} \zeta_u(t) \big\{ \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) \big[2P_u(\mathcal{A}_u - L_u(1+\Delta)\mathcal{C}_u) + \kappa P_u \big] z(t) \\ &\qquad - 2\mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u L_u(1+\Delta)\mathcal{D}_u z(t-\theta) \\ &\qquad + 2\mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u \mathcal{W}_u \bar{\psi}\big(z(t-\theta)\big) \\ &\qquad + 2\mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u \big(\mathcal{G}_u - L_u(1+\Delta)\mathcal{F}_u\big)\omega(t) \big\} \\ &\quad + \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) R_u z(t) - \mathrm{e}^{\kappa(t-\theta)} z^{\mathrm{T}}(t-\theta) R_u z(t-\theta). \end{aligned}$$

By adding and subtracting $\mathrm{e}^{\kappa t}\omega^{\mathrm{T}}(t)[\mathcal{C}_u z(t) + \mathcal{D}_u z(t-\theta) + \mathcal{F}_u\omega(t)]$, one gets

$$\begin{aligned} \dot{V}_u\big(z(t), t\big) &= \sum_{u=1}^{N} \zeta_u(t) \Big\{ \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) \big[2P_u(\mathcal{A}_u - L_u(1+\Delta)\mathcal{C}_u) + \kappa P_u \big] z(t) \\ &\qquad - 2\mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u L_u(1+\Delta)\mathcal{D}_u z(t-\theta) \\ &\qquad + 2\mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u \mathcal{W}_u \bar{\psi}\big(z(t-\theta)\big) \\ &\qquad + 2\mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) \Big[P_u\big(\mathcal{G}_u - L_u(1+\Delta)\mathcal{F}_u\big) - \frac{1}{2}\mathcal{C}_u^{\mathrm{T}} \Big] \omega(t) \\ &\qquad - \mathrm{e}^{\kappa t}\omega^{\mathrm{T}}(t)\mathcal{D}_u z(t-\theta) - \mathrm{e}^{\kappa t}\omega^{\mathrm{T}}(t)\mathcal{F}_u\omega(t) \\ &\qquad + \mathrm{e}^{\kappa t}\omega^{\mathrm{T}}(t)\big[\mathcal{C}_u z(t) + \mathcal{D}_u z(t-\theta) + \mathcal{F}_u\omega(t)\big] \Big\} \\ &\quad + \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) R_u z(t) - \mathrm{e}^{\kappa(t-\theta)} z^{\mathrm{T}}(t-\theta) R_u z(t-\theta). \end{aligned}$$

Using (2), one has

$$\begin{aligned} & 2z^{\mathrm{T}}(t) P_u \mathcal{W}_u \bar{\psi}\big(z(t-\theta)\big) \\ &\quad \leqslant \bar{\psi}\big(z(t-\theta)\big)^{\mathrm{T}} \bar{\psi}\big(z(t-\theta)\big) + z^{\mathrm{T}}(t) P_u \mathcal{W}_u \mathcal{W}_u^{\mathrm{T}} P_u z(t) \\ &\quad \leqslant L_\psi^2 z^{\mathrm{T}}(t-\theta) z(t-\theta) + z^{\mathrm{T}}(t) P_u \mathcal{W}_u \mathcal{W}_u^{\mathrm{T}} P_u z(t). \end{aligned} \tag{12}$$

By $M_u = P_u L_u$ and (12), one can write

$$\dot{V}_u\big(z(t),t\big) \leqslant \sum_{u=1}^{N} \zeta_u(t)\mathrm{e}^{\kappa t}\big\{\eta^{\mathrm{T}}(t)\bar{\Omega}_u\eta(t) - z^{\mathrm{T}}(t)Sz(t) + \omega^{\mathrm{T}}(t) \times \big[\mathcal{C}_u z(t) + \mathcal{D}_u z(t-\theta) + \mathcal{F}_u\omega(t)\big]\big\} \leqslant -\mathrm{e}^{\kappa t} z^{\mathrm{T}}(t)Sz(t) + \omega^{\mathrm{T}}(t)\hat{y}(t), \tag{13}$$

where

$$\eta(t) = \big[z^{\mathrm{T}}(t)\ \ z^{\mathrm{T}}(t-\theta)\ \ \omega^{\mathrm{T}}(t)\big]^{\mathrm{T}},$$

and the second inequality follows from (10).

When $\omega(t) = 0$, from (13) one can get $\dot{V}_u(z(t),t) \leqslant -\mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)Sz(t)$. Owing to the fact that $S > 0$, $\dot{V}_u(z(t),t) < 0$ for any $z(t) \neq 0$. Thus, for any $t > 0$, it can be obtained that

$$V_u\big(z(t),t\big) \leqslant V_u\big(z(0),0\big). \tag{14}$$

In addition, (11) gives

$$V_u\big(z(t),t\big) \geqslant \min_{u\in\Gamma}\big\{\lambda_{\min}(P_u)\big\}\mathrm{e}^{\kappa t}\big\|z(t)\big\|^2. \tag{15}$$

From (14) and (15) one has

$$\big\|z(t)\big\| \leqslant \frac{\sqrt{\max_{u\in\Gamma} V_u(z(0),0)}}{\sqrt{\min_{u\in\Gamma}\{\lambda_{\min}(P_u)\}}}\mathrm{e}^{(-\kappa/2)t}.$$

Thus, the exponential stability of the filtering error system in (6) is guaranteed.

Next, one focuses on the passivity of system (6) with $\omega(t) \neq 0$. Integrating both sides of (13) from t to 0 gives

$$V_u\big(z(t),t\big) - V_u\big(z(0),0\big) \leqslant -\int_0^t \mathrm{e}^{\kappa s}z^{\mathrm{T}}(s)Sz(s)\,\mathrm{d}s + \int_0^t \omega^{\mathrm{T}}(s)\hat{y}(s)\,\mathrm{d}s.$$

Let $\beta = \max_{u\in\Gamma} V_u(z(0),0)$. Then one has

$$\int_0^t \omega^{\mathrm{T}}(s)\hat{y}(s)\,\mathrm{d}s + \beta \geqslant \int_0^t \mathrm{e}^{\kappa s}z^{\mathrm{T}}(s)Sz(s)\,\mathrm{d}s + V_u\big(z(t),t\big) \geqslant \int_0^t \mathrm{e}^{\kappa s}z^{\mathrm{T}}(s)Sz(s)\,\mathrm{d}s,$$

which implies that filtering error system (6) is ensured to be exponentially passive from noise interference $\omega(t)$ to output error $\bar{y}(t)$ under the arbitrary switching rule. This completes the proof. $\square$

When there is no quantization, the passive filter to be applied becomes

$$\begin{aligned}\dot{\breve{x}}(t) &= \mathcal{A}\big(\gamma(t)\big)\breve{x}(t) + \mathcal{W}(\gamma(t))\psi(\breve{x}(t-\theta)) + \mathcal{J}\big(\gamma(t)\big)(t) \\ &\quad + L\big(\gamma(t)\big)\big(y(t) - \breve{y}(t)\big), \\ \breve{y}(t) &= \mathcal{C}\big(\gamma(t)\big)\breve{x}(t) + \mathcal{D}\big(\gamma(t)\big)\breve{x}(t-\theta).\end{aligned} \tag{16}$$

In the case, the filtering error system is represented by

$$\begin{aligned}\dot{z}(t) &= \big(\mathcal{A}\big(\gamma(t)\big) - L\big(\gamma(t)\big)\mathcal{C}\big(\gamma(t)\big)\big)z(t) - L\big(\gamma(t)\big)\mathcal{D}\big(\gamma(t)\big)z(t-\theta) \\ &\quad + \mathcal{W}\big(\gamma(t)\big)\bar{\psi}\big(z(t-\theta)\big) + (\mathcal{G}\big(\gamma(t)\big) - L\big(\gamma(t)\big)\mathcal{F}\big(\gamma(t)\big))\omega(t), \\ \bar{y}(t) &= \mathcal{C}\big(\gamma(t)\big)z(t) + \mathcal{D}\big(\gamma(t)\big)z(t-\theta) + \mathcal{F}\big(\gamma(t)\big)\omega(t),\end{aligned}$$

which corresponds to (5) with $\Delta = 0$. Thus, one can write the following result.

Corollary 1. *If there exist matrices $P_u > 0$, $R_u > 0$, $S > 0$, and M_u such that*

$$\Sigma_u = \begin{bmatrix} \Sigma_{1u} & -M_u\mathcal{D}_u & P_u\mathcal{G}_u - M_u\mathcal{F}_u - \frac{1}{2}\mathcal{C}_u^{\mathrm{T}} & P_u\mathcal{W}_u \\ * & L_\psi^2 I - \mathrm{e}^{-\kappa\theta}R_u & -\frac{1}{2}\mathcal{D}_u^{\mathrm{T}} & 0 \\ * & * & -\frac{1}{2}\mathscr{S}(\mathcal{F}_u) & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{17}$$

holds true for any $u \in \Gamma$, where $\Sigma_{1u} = \mathscr{S}(P_u\mathcal{A}_u - M_u\mathcal{C}_u) + \kappa P_u + S + R_u$, then the issue of passive filtering under arbitrary switching is solvable, and the needed gain matrix of the passive filter can be chosen as (8).

Remark 3. Corollary 1 gives a novel existence criterion for the passive filtering of SDNN (1) without quantization. As going to be shown in Example 1, the criterion in Corollary 1, which is based on multiple Lyapunov functionals, is less conservative than the main result of [2].

3.2 Quantized passive filtering under semi-Markov switching

The issue of quantized passive filtering under semi-Markov switching to be addressed can be formulated explicitly as follows: for the switching rule in Case 2, design a quantized passive filter having the form in (4) such as the filtering error system in (5) is both exponentially stable and exponentially passive in the mean square sense.

Set $\mathcal{A}(\gamma(t)) = \mathcal{A}_u$, $\mathcal{W}(\gamma(t)) = \mathcal{W}_u$, $\mathcal{J}(\gamma(t)) = \mathcal{J}_u$, $\mathcal{G}(\gamma(t)) = \mathcal{G}_u$, $\mathcal{C}(\gamma(t)) = \mathcal{C}_u$, $\mathcal{D}(\gamma(t)) = \mathcal{D}_u$, and $\mathcal{F}(\gamma(t)) = \mathcal{F}_u$. Then the filtering error system changes into

$$\begin{aligned}\dot{z}(t) &= \big(\mathcal{A}_u - L_u(1+\Delta)\mathcal{C}_u\big)z(t) - L_u(1+\Delta)\mathcal{D}_u z(t-\theta) \\ &\quad + \mathcal{W}_u\bar{\psi}\big(z(t-\theta)\big) + \big(\mathcal{G}_u - L_u(1+\Delta)\mathcal{F}_u\big)\omega(t), \\ \bar{y}(t) &= \mathcal{C}_u z(t) + \mathcal{D}_u z(t-\theta) + \mathcal{F}_u\omega(t).\end{aligned}$$

For the quantized passive filtering under semi-Markov switching, one can give the following result.

Theorem 2. *Suppose that there are matrices $P_u > 0$, $R > 0$, $S > 0$, M_u, and scalar $\varepsilon > 0$ such that*

$$\begin{bmatrix} \varTheta_{1u} & \varTheta_{2u} & \varTheta_{3u} & P_u\mathcal{W}_u & M_u \\ * & L_\psi^2 I - \mathrm{e}^{-\kappa\theta}R + \varepsilon\mathcal{D}_u^{\mathrm{T}}\mathcal{D}_u & -\frac{1}{2}\mathcal{D}_u^{\mathrm{T}} + \varepsilon\mathcal{D}_u^{\mathrm{T}}\mathcal{F}_u & 0 & 0 \\ * & * & -\frac{1}{2}\mathscr{S}(\mathcal{F}_u) + \varepsilon\mathcal{F}_u^{\mathrm{T}}\mathcal{F}_u & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\frac{\varepsilon}{\delta^2}I \end{bmatrix} < 0 \tag{18}$$

holds for any $u \in \varGamma$, where

$$\begin{aligned} \varTheta_{1u} &= \mathscr{S}(P_u\mathcal{A}_u - M_u\mathcal{C}_u) + \kappa P_u + \sum_{v=1}^{N} \bar{\pi}_{uv}P_v + S + R + \varepsilon\mathcal{C}_u^{\mathrm{T}}\mathcal{C}_u, \\ \varTheta_{2u} &= -M_u\mathcal{D}_u + \varepsilon\mathcal{C}_u^{\mathrm{T}}\mathcal{D}_u, \qquad \varTheta_{3u} = P_u\mathcal{G}_u - M_u\mathcal{F}_u - \frac{1}{2}\mathcal{C}_u^{\mathrm{T}} + \varepsilon\mathcal{C}_u^{\mathrm{T}}\mathcal{F}_u, \\ \bar{\pi}_{uv} &= \int\limits_0^\infty \pi_{uv}(h)g_u(h)\,\mathrm{d}h \end{aligned}$$

with $g_u(h)$ being the probability density function of sojourn time h at mode u. Then the issue of quantized passive filtering under semi-Markov switching is solvable, and the needed gain matrix can be chosen as (8).

Proof. Define

$$\begin{aligned} \bar{\varTheta}_u &= \mathscr{S}(P_u\mathcal{A}_u - M_u\mathcal{C}_u - M_u\Delta\mathcal{C}_u) + \kappa P_u + \sum_{v=1}^{N} \bar{\pi}_{uv}P_v \\ &\quad + P_u\mathcal{W}_u\mathcal{W}_u^{\mathrm{T}}P_u + S + R, \\ \bar{\varOmega}_u &= \begin{bmatrix} \bar{\varTheta}_u & -M_u\mathcal{D}_u - M\Delta\mathcal{D}_u & P_u\mathcal{G}_u - M_u\mathcal{F}_u - M_u\Delta\mathcal{F}_u - \frac{1}{2}\mathcal{C}_u^{\mathrm{T}} \\ * & L_\psi^2 I - \mathrm{e}^{-\kappa\theta}R & -\frac{1}{2}\mathcal{D}_u^{\mathrm{T}} \\ * & * & -\frac{1}{2}\mathscr{S}(\mathcal{F}_u) \end{bmatrix}. \end{aligned}$$

Along the same line as the proof in Theorem 1, we can write

$$\bar{\varOmega}_u \leqslant \tilde{\varOmega}_u, \tag{19}$$

where

$$\tilde{\varOmega}_u = \begin{bmatrix} \tilde{\varTheta}_{1u} & \varTheta_{2u} & \varTheta_{3u} \\ * & L_\psi^2 I - \mathrm{e}^{-\kappa\theta}R + \varepsilon\mathcal{D}_u^{\mathrm{T}}\mathcal{D}_u & -\frac{1}{2}\mathcal{D}_u^{\mathrm{T}} + \varepsilon\mathcal{D}_u^{\mathrm{T}}\mathcal{F}_u \\ * & * & -\frac{1}{2}\mathscr{S}(\mathcal{F}_u) + \varepsilon\mathcal{F}_u^{\mathrm{T}}\mathcal{F}_u \end{bmatrix},$$

$$\begin{aligned} \tilde{\varTheta}_{1u} &= \mathscr{S}(P_u\mathcal{A}_u - M_u\mathcal{C}_u) + \kappa P_u + \sum_{v=1}^{N} \bar{\pi}_{uv}P_v + P_u\mathcal{W}_u\mathcal{W}_u^{\mathrm{T}}P_u + S + R \\ &\quad + \varepsilon\mathcal{C}_u^{\mathrm{T}}\mathcal{C}_u + \frac{\delta^2}{\varepsilon}M_u^{\mathrm{T}}M_u. \end{aligned}$$

By Schur's complement, the LMI in (18) ensures $\tilde{\Omega}_u < 0$. Then from (19) we have

$$\bar{\Omega}_u < 0. \tag{20}$$

Now, choose a mode-dependent Lyapunov functional as

$$V\bigl(z(t), \gamma(t), t\bigr) = V_1\bigl(z(t), \gamma(t), t\bigr) + V_2\bigl(z(t), t\bigr), \quad \gamma(t) \in \Gamma, \tag{21}$$

where

$$V_1\bigl(z(t), \gamma(t), t\bigr) = \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P\bigl(\gamma(t)\bigr) z(t),$$

$$V_2\bigl(z(t), t\bigr) = \int\limits_{-\theta}^{0} \mathrm{e}^{\kappa(t+s)} z^{\mathrm{T}}(t+s) R z(t+s)\,\mathrm{d}s.$$

Define by $\mathfrak{L}$ the infinitesimal generator [29], i.e.,

$$\begin{aligned}
&\mathfrak{L}V\bigl(z(t), \gamma(t), t\bigr) \\
&\quad = \lim_{\alpha \to 0+} \frac{1}{\alpha} \bigl[\mathcal{E}\bigl\{V\bigl(z(t+\alpha), \gamma(t+\alpha), t+\alpha\bigr) \bigm| z(t), \gamma(t), t\bigr\} - V\bigl(z(t), \gamma(t), t\bigr)\bigr].
\end{aligned}$$

Then, for $\gamma(t) = u$, we can write

$$\begin{aligned}
&\mathfrak{L}V_1\bigl(z(t), \gamma(t), t\bigr) \\
&\quad = \lim_{\alpha \to 0+} \frac{1}{\alpha} \Biggl[\sum_{v=1,\, v \neq u}^{N} \Pr\bigl\{\gamma_{n+1} = v,\ h_{n+1} \leqslant h + \alpha \bigm| \gamma_n = u,\ h_{n+1} > h\bigr\} \\
&\qquad \times \mathrm{e}^{\kappa(t+\alpha)} z^{\mathrm{T}}(t+\alpha) P_v z(t+\alpha) \\
&\qquad + \Pr\bigl\{\gamma_{n+1} = u,\ h_{n+1} > h + \alpha \bigm| \gamma_n = u,\ h_{n+1} > h\bigr\} \\
&\qquad \times \mathrm{e}^{\kappa(t+\alpha)} z^{\mathrm{T}}(t+\alpha) P_u z(t+\alpha) - \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u z(t)\Biggr] \\
&\quad = \lim_{\alpha \to 0+} \frac{1}{\alpha} \Biggl[\sum_{v=1,\, v \neq u}^{N} \bigl(\pi_{uv}(h)\alpha + o(\alpha)\bigr) \mathrm{e}^{\kappa(t+\alpha)} z^{\mathrm{T}}(t+\alpha) P_v z(t+\alpha) \\
&\qquad + \bigl(1 + \pi_{uu}(h)\alpha + o(\alpha)\bigr) \mathrm{e}^{\kappa(t+\alpha)} z^{\mathrm{T}}(t+\alpha) P_u z(t+\alpha) \\
&\qquad - \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u z(t)\Biggr] \\
&\quad = \sum_{v=1}^{N} \pi_{uv}(h) \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_v z(t) \\
&\qquad + \lim_{\alpha \to 0+} \frac{1}{\alpha} \bigl[\mathrm{e}^{\kappa(t+\alpha)} z^{\mathrm{T}}(t+\alpha) P_u z(t+\alpha) - \mathrm{e}^{\kappa t} z^{\mathrm{T}}(t) P_u z(t)\bigr].
\end{aligned}$$

It follows that

$$\mathcal{E}\{\mathfrak{L}V_1(z(t),\gamma(t),t)\} = \sum_{v=1}^{N}\bar{\pi}_{uv}\mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)P_v z(t) + \kappa\mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)P_u z(t)$$
$$+ 2\mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)\big[P_u\big(\mathcal{A}_u - L_u(1+\Delta)\mathcal{C}_u\big)\big]z(t)$$
$$- 2\mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)P_u L_u(1+\Delta)\mathcal{D}_u z(t-\theta)$$
$$+ 2\mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)P_u\mathcal{W}_u\bar{\psi}\big(z(t-\theta)\big)$$
$$+ 2\mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)P_u\big(\mathcal{G}_u - L_u(1+\Delta)\mathcal{F}_u\big)\omega(t). \tag{22}$$

Similarly, it can be obtained that

$$\mathcal{E}\{\mathfrak{L}V_2(z(t),t)\} = \mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)Rz(t) - \mathrm{e}^{\kappa(t-\theta)}z^{\mathrm{T}}(t-\theta)Rz(t-\theta). \tag{23}$$

Thus, for $\gamma(t) = u$, by (12), (22), (23), and $M_u = P_u L_u$, we have

$$\mathcal{E}\{\mathfrak{L}V(z(t),\gamma(t),t)\} \leqslant \mathrm{e}^{\kappa t}\{\eta^{\mathrm{T}}(t)\bar{\Omega}_u\eta(t) - z^{\mathrm{T}}(t)Sz(t)$$
$$+ \omega^{\mathrm{T}}(t)\big[\mathcal{C}_u z(t) + \mathcal{D}_u z(t-\theta) + \mathcal{F}_u\omega(t)\big]\}$$
$$\leqslant -\mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)Sz(t) + \omega^{\mathrm{T}}(t)\hat{y}(t), \tag{24}$$

where the inequality follows by (20).

When $\omega(t) = 0$, noting $S > 0$, from (24) we get $\mathcal{E}\{\mathfrak{L}V(z(t),\gamma(t),t)\} < 0$ for all $z(t) \neq 0$, which, together with Dynkin's formula, yields

$$\mathcal{E}\{(z(t),\gamma(t),t)\} = \mathcal{E}\{V(z(0),\gamma(0),0)\} + \int_0^t \mathcal{E}\{\mathfrak{L}V(z(s),\gamma(s),s)\,\mathrm{d}s\}$$
$$\leqslant \mathcal{E}\{V(z(0),\gamma(0),0)\}. \tag{25}$$

On the other hand, (21) implies

$$\min_{u\in\Gamma}\{\lambda_{\min}(P_u)\}\mathrm{e}^{\kappa t}\{\|z(t)\|^2\} \leqslant V(z(t),\gamma(t),t)$$
$$\leqslant \max_{u\in\Gamma}\{\lambda_{\max}(P_u)\}\{W(z(t),t)\}, \tag{26}$$

where

$$W(z(t),t) = \mathrm{e}^{\kappa t}z^{\mathrm{T}}(t)z(t) + \int_{-\theta}^{0}\mathrm{e}^{\kappa(t+s)}z^{\mathrm{T}}(t+s)Rz(t+s)\,\mathrm{d}s.$$

According to (25) and (26), we have

$$\mathcal{E}\{\|z(t)\|\} \leqslant \frac{\sqrt{\max_{u\in\Gamma}\{\lambda_{\max}(P_u)\}\mathcal{E}\{W(z(0),0)\}}}{\sqrt{\min_{u\in\Gamma}\{\lambda_{\min}(P_u)\}}}\mathrm{e}^{-(\kappa/2)t},$$

which means that the filtering error system is exponentially stable in the mean square sense.

When $\omega(t) \neq 0$, by (24) and Dynkin's formula we can get that

$$\mathcal{E}\{V(z(t),\gamma(t),t)\} \leqslant \mathcal{E}\{V(z(0),\gamma(0),0)\} + \mathcal{E}\left\{-\int_0^t \mathrm{e}^{\kappa s} z^{\mathrm{T}}(s) S z(s)\,\mathrm{d}s + \int_0^t \omega^{\mathrm{T}}(s)\hat{y}(s)\,\mathrm{d}s\right\}.$$

Let $\beta = \max_{u\in\Gamma}\{\lambda_{\max}(P_u)\}\mathcal{E}\{W(z(0),0)\}$. Then we can write

$$\begin{aligned}\mathcal{E}\left\{\int_0^t \omega^{\mathrm{T}}(s)\hat{y}(s)\,\mathrm{d}s\right\} + \beta &\geqslant \mathcal{E}\left\{\int_0^t \mathrm{e}^{\kappa s} z^{\mathrm{T}}(s) S z(s)\,\mathrm{d}s\right\} + \mathcal{E}\{V(z(t),\gamma(t),t)\} \\ &\geqslant \mathcal{E}\left\{\int_0^t \mathrm{e}^{\kappa s} z^{\mathrm{T}}(s) S z(s)\,\mathrm{d}s\right\}.\end{aligned}$$

Thus, the filtering error system is ensured to be exponentially passive in the mean square sense from noise interference $\omega(t)$ to output error $\bar{y}(t)$ under the semi-Markov switching rule. This completes the proof. □

When there is no quantization, we can write the following result:

Corollary 2. *Suppose that there exist matrices $P_u > 0$, $R > 0$, $S > 0$, and M_u such that*

$$\begin{bmatrix} \Theta_u & -M_u\mathcal{D}_u & P_u\mathcal{G}_u - M_u\mathcal{F}_u - \frac{1}{2}\mathcal{C}_u^{\mathrm{T}} & P_u\mathcal{W}_u \\ * & L_\psi^2 I - \mathrm{e}^{-\kappa\theta}R & -\frac{1}{2}\mathcal{D}_u^{\mathrm{T}} & 0 \\ * & * & -\frac{1}{2}\mathscr{S}(\mathcal{F}_u) & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{27}$$

holds for any $u \in \Gamma$, where

$$\Theta_u = \mathscr{S}(P_u\mathcal{A}_u - M_u\mathcal{C}_u) + \kappa P_u + \sum_{v=1}^{N} \bar{\pi}_{uv} P_v + S + R,$$

$$\bar{\pi}_{uv} = \int_0^\infty \pi_{uv}(h) g_u(h)\,\mathrm{d}h$$

with $g_u(h)$ being the probability density function of sojourn time h at mode u. Then the issue of passive filtering under the semi-Markov switching is solvable, and the needed gain matrix of the passive filter can be chosen as (8)*.*

Remark 4. With the aid of multiple Lyapunov functionals and several inequality techniques, design methods for the quantized passive filtering under arbitrary switching and semi-Markov switching are proposed in Theorems 1 and 2, respectively. It is shown that the needed gain matrix is able to be obtained through the feasible solution of LMIs,which are known to be computationally tractable using some popular convex optimization tools. In addition, from the proofs of Theorems 1 and 2 it can be seen that $-\kappa/2$ corresponds to the decay rate. Thus, the larger the scalar κ, the faster the filtering error system converges.

Remark 5. Over the past few decades, there has been an increasing interest in time-delay systems and a great number of research results have been achieved; see., e.g., [16, 35, 37, 41]. To our knowledge, most of the results are based on the Lyapunov functional method. It is worth pointing out that the choice of suitable Lyapunov functionals is of considerable significance. By extending the Lyapunov functionals in Theorems 1 and 2 as [3, 4], it is expected to obtain less conservative conditions. However, this may lead to increases in the dimension of LMIs and the number of decision variables, which in turn will result in higher computational costs.

4 Numerical examples

In this section, we give two numerical examples to show the usefulness of the proposed quantized passive filter design methods for SDNNs under arbitrary switching and semi-Markov switching, respectively.

Example 1. Consider SDNN (1) under arbitrary switching with

$$\mathcal{A}_1 = \begin{bmatrix} -2.2 & 0 \\ 0 & -3.5 \end{bmatrix}, \qquad \mathcal{A}_2 = \begin{bmatrix} -a_1(2) & 0 \\ 0 & -2.8 \end{bmatrix}, \qquad \mathcal{W}_1 = \begin{bmatrix} -1 & 0.4 \\ 0 & -0.1 \end{bmatrix},$$

$$\mathcal{W}_2 = \begin{bmatrix} 0.2 & -0.8 \\ 0.4 & 0.5 \end{bmatrix}, \qquad \mathcal{G}_1 = \mathcal{G}_2 = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}, \qquad \mathcal{F}_1 = \mathcal{F}_2 = 1, \qquad \theta = 1,$$

$$\mathcal{C}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad \mathcal{C}_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad \mathcal{D}_1 = \begin{bmatrix} 0.5 & 1 \end{bmatrix}, \qquad \mathcal{D}_2 = \begin{bmatrix} -1 & 0.3 \end{bmatrix},$$

$$\mathcal{J}_1(t) = \begin{bmatrix} \sin(1.8t) \\ \cos(t) \end{bmatrix}, \qquad \mathcal{J}_2(t) = \begin{bmatrix} 3\cos^2(0.1t) \\ \cos(t) \end{bmatrix}, \qquad \psi(x(t)) = \begin{bmatrix} \tan(x_1(t)) \\ \tan(x_2(t)) \end{bmatrix}.$$

Notice that the activation function satisfies (2) with $L_\psi = 1$ [7].

First, let us consider the case that there is no quantization. When $a_1(2) = 3.9$, it can be verified that the LMIs in (17) are feasible for any $\kappa \leqslant 0.75$, while the condition in Theorem 2 of [2] fails for $\kappa \geqslant 0.48$. This means that, for $\kappa \in [0.48, 0.75]$, Corollary 1 of this paper can be applied for designing passive filter (16) while Theorem 2 of [2] is unavailable. When $a_1(2) = 3.6$, it is found that the maximum allowed values of κ are 0.66 by Corollary 1 and 0.38 by Theorem 2 of [2], respectively. A more detailed comparison of the maximum allowed κ obtained by Corollary 1 of this paper and Theorem 2 of [2] for different choices of $a_1(2)$ is given in Table 1, it can be inferred that the present design method is always less conservative.

Next, we consider the passive filtering with quantization. Set $\kappa = 0.74$ and $\chi = 0.6$. Then, by solving the LMIs in (7), the filter gains can be obtained as follows:

$$L_1 = \begin{bmatrix} 1.0984 \\ -0.4635 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.3990 \\ -0.1297 \end{bmatrix}.$$

Table 1. Maximum allowable κ for different choices of $a_1(2)$.

$a_1(2)$	3.9	3.6	3.3	3.0	2.7
Theorem 2 of [2]	0.47	0.38	0.29	0.18	0.06
Corollary 1	0.75	0.66	0.58	0.47	0.36

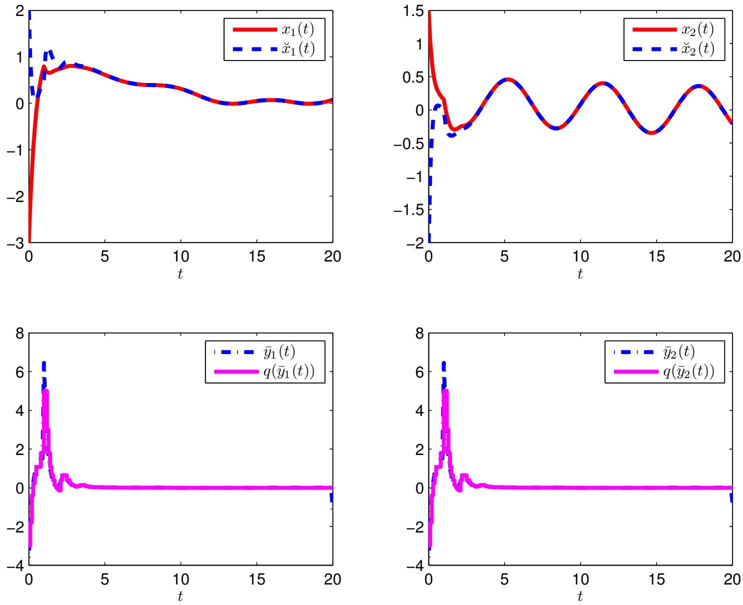

Figure 1. Trajectories of $x(t)$, $\breve{x}(t)$, $\bar{y}(t)$, and $q(\bar{y}(t))$ under arbitrary switching.

Let we set $\gamma(t) = 1$ when $t \in [1, 2]$ and $\gamma(t) = 2$ otherwise, the initial condition to be $x(s) = [-3\ \ 1.5]^{\mathrm{T}}$, $\breve{x}(s) = [2\ \ -2]^{\mathrm{T}}$ ($s \in [-\theta, 0]$), and $\omega(t)$ to be a Gaussian noise subject to mean 0 and variance 1. Then the trajectories of state $x(t)$ and its estimate $\breve{x}(t)$, output error $\bar{y}(t)$ and quantized measurement $q(\bar{y}(t))$ are displayed in Fig. 1. The simulation results show that the quantized passive filter reduces the impact of noise interference $\omega(t)$ on the filtering error system.

Example 2. Consider SDNN (1) under semi-Markov switching with

$$\mathcal{A}_1 = \begin{bmatrix} -1.5 & 0 \\ 0 & -2.0 \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} -3.0 & 0 \\ 0 & -3.0 \end{bmatrix}, \quad \mathcal{W}_1 = \begin{bmatrix} 0.5 & 0.6 \\ 0.1 & 0.2 \end{bmatrix},$$

$$\mathcal{W}_2 = \begin{bmatrix} 0.5 & 0 \\ 0.2 & 0.2 \end{bmatrix}, \qquad \mathcal{G}_1 = \mathcal{G}_2 = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}, \qquad \mathcal{F}_1 = \mathcal{F}_2 = 1, \qquad \theta = 1,$$

$$\mathcal{C}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad \mathcal{C}_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad \mathcal{D}_1 = \begin{bmatrix} 0.5 & 1 \end{bmatrix}, \qquad \mathcal{D}_2 = \begin{bmatrix} -1 & 0.3 \end{bmatrix},$$

$$\mathcal{J}_1(t) = \begin{bmatrix} \sin(t)\cos(t) \\ \sin(t) \end{bmatrix}, \qquad \mathcal{J}_2(t) = \begin{bmatrix} 3\sin^2(0.1t) \\ -\cos(t) \end{bmatrix}, \qquad \psi(x(t)) = \begin{bmatrix} \tan(x_1(t)) \\ \tan(x_2(t)) \end{bmatrix}.$$

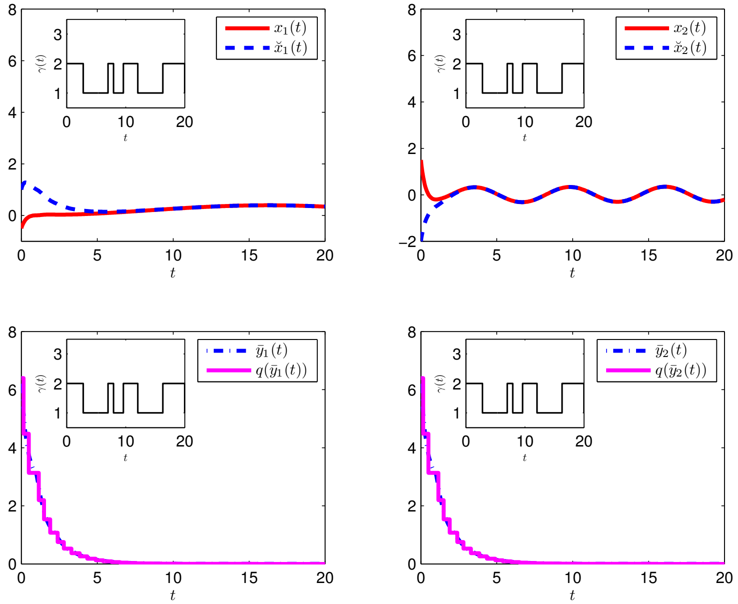

Figure 2. Trajectories of $x(t)$, $\breve{x}(t)$, $\bar{y}(t)$, and $q(\bar{y}(t))$ under semi-Markov switching.

Suppose that sojourn time h obeys the Weibull distribution [39]. Specifically, assume that $h \sim \text{Weibull}(1,2)$ (i.e., $g_1(h) = 2h\exp(-h^2)$) for $u = 1$ and $h \sim \text{Weibull}(1,3)$ (i.e., $g_2(h) = 3h^2\exp(-h^3)$) for $u = 2$. Then the transition probability matrix is given by

$$\varPi(h) = \begin{bmatrix} -2h & 2h \\ 3h^2 & -3h^2 \end{bmatrix}.$$

Consequently, the mathematical expectation of $\varPi(h)$ is able to be acquired as

$$\mathcal{E}\{\varPi(h)\} = \begin{bmatrix} -1.7725 & 1.7725 \\ 2.7082 & -2.7082 \end{bmatrix}.$$

Choose $\kappa = 0.63$ and $\chi = 0.7$. Then, by solving the LMI in (18), the corresponding gains can be obtained as follows:

$$L_1 = \begin{bmatrix} 1.0336 \\ -0.4278 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.6369 \\ -0.1304 \end{bmatrix}.$$

Let the initial condition be $x(s) = [-0.5\ \ 1.5]^{\mathrm{T}}$, $\breve{x}(s) = [1\ \ -2]^{\mathrm{T}}$ ($s \in [-\theta, 0]$), and $\omega(t)$ be a Gaussian noise subject to mean 0 and variance 1. Then the trajectories of state $x(t)$ and its estimate $\breve{x}(t)$, output error $\bar{y}(t)$ and quantized measurement $q(\bar{y}(t))$ are shown in Fig. 2. The simulation results show the usefulness of the proposed quantized passive filter method in Theorem 2.

5 Conclusion

The issue of quantized passive filtering for SDNNs with noise interference has been addressed in this paper. Both arbitrary and semi-Markov switching rules have been discussed. By choosing Lyapunov functionals and applying several inequality techniques, sufficient conditions have been established to ensure the filtering error systems to be not only exponentially stable, but also exponentially passive from the noise interference to the output error. It has been shown that the needed gain matrix for the proposed quantized passive filter can be constructed through the feasible solution of LMIs, which are computationally tractable using some popular convex optimization tools. Finally, two numerical examples have been given to illustrate the usefulness of the present quantized passive filter design methods. It is worth mentioning that the quantizer under consideration is mode-independent. Over the past decade, robust filtering under mode-dependent quantization has received increasing attention. The robust passive filtering for SDNNs with mode-dependent quantization will be considered in our future work.

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