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Projective synchronization analysis for BAM neural networks with time-varying delay via novel control*

Malika Sader[a,b], **Fuyong Wang**[a,b], **Zhongxin Liu**[a,b,1], **Zengqiang Chen**[a,b]

[a]College of Artificial Intelligence, Nankai University, Tianjin, 300350, China

[b]Tianjin Key Laboratory of Intelligent Robotics, Nankai University, Tianjin, 300350, China
lzhx@nankai.edu.cn

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Abstract. In this paper, the projective synchronization of BAM neural networks with time-varying delays is studied. Firstly, a type of novel adaptive controller is introduced for the considered neural networks, which can achieve projective synchronization. Then, based on the adaptive controller, some novel and useful conditions are obtained to ensure the projective synchronization of considered neural networks. To our knowledge, different from other forms of synchronization, projective synchronization is more suitable to clearly represent the nonlinear systems' fragile nature. Besides, we solve the projective synchronization problem between two different chaotic BAM neural networks, while most of the existing works only concerned with the projective synchronization chaotic systems with the same topologies. Compared with the controllers in previous papers, the designed controllers in this paper do not require any activation functions during the application process. Finally, an example is provided to show the effectiveness of the theoretical results.

Keywords: BAM neural networks, projective synchronization, time-varying delay, adaptive controller.

1 Introduction

Over the past several tens of years, more and more scholars devoted many efforts to study the artificial neural networks (NNs) dynamical behaviors since they have application value in many different fields, such as image processing, associative memories, and classification of patterns [9, 16, 18, 22, 23, 27, 42]. NNs fall into several categories

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[1]Corresponding author.

including Hopfield NNs, cellular NNs, BAM NNs (BAMNNs), and Cohen–Grossberg NNs. Among them, Kosko firstly introduce the BAMNNs in 1988 [18]. BAMNNs are constructed of neurons ordered in double layers. Generally speaking, the neurons in one layer are completely incorporated to neurons in another layer. BAMNNs have been applied successfully to pattern recognition due to its generalization of the single-layer auto associative Hebbian correlator to a two-layer pattern-matched hetero associative circuit.

BAMNNs are deemed as one of the most important NNs. Compared with the general NNs, BAMNNs are consisted of neurons distributed in two layers. The neurons distributed in one layer are fully interconnected with the neurons distributed in the other layer, while there is no interconnection between the neurons distributed in the same layer. In implementation of NNs, time delays appeared because of the finite switching speed of amplifiers and neurons, which leads to network instability or oscillatory behavior. For these reasons, more and more researches pay more attention to the stability for NNs with time delays [3–6, 15, 38, 41].

Synchronization is originally introduced by Pecora and Carrol [25] in which two identical systems can be synchronized with different initial values. Recently, many researches studied the synchronization of NNs because of their applications in the rain activity, engineering areas, nonlinear system optimization, and secure communications [34]. In the meantime, scholars proposed many types of synchronization of NNs controllers including the sliding mode controller, feedback controller, impulsive controller, adaptive controller, and active controller [26, 28, 35, 43]. Compared with other controller schemes, adaptive controller has more effectiveness, fast response, and good transient performance. Because of these advantages, many scholars considered the stabilization and synchronization of NNs [7, 12, 19, 24, 33].

Synchronization can be divided into few types including projective synchronization (PS), lag synchronization, phase synchronization, impulsive synchronization, and generalized synchronization [1, 8, 10, 13, 17, 20, 21, 30, 36, 37]. Lag synchronization indicates that the two systems exist a coincidence of shifted-in-time states like $u(t) \to v(t-\sigma)$, $t \to \infty$ (where $\sigma > 0$ is propagation delay). Generalized synchronization means that the drive and response systems have some functional relation like $u(t) \to \phi(v(t))$ (where $t \to \infty$). Complete synchronization means the state variables were equal, while evolving in time like $u(t) \to v(t)$ (where $t \to \infty$). Impulsive synchronization means that the system behaviors abrupt changes at certain moments. PS means that the two systems can be synchronized by a scaling factor P_i like $u(t) \to P_i v(t)$ (where $t \to \infty$). Different from other forms of synchronization, PS is more suitable to clearly represent the nonlinear systems fragile nature, and there is a typical advantage in PS due to unpredictability of proportional constant will additionally enhance the communication security [1]. Therefore, PS of chaotic nonlinear systems received hot research attention [1, 8, 10, 36, 37].

In [8], the writers introduced the analysis for PS of fractional-order memristor-based NNs based on the fractional-order differential inequality and Caputo's fractional derivation. In [10], based on the matrix measure and Halanay inequality, the authors achieved the weak PS of coupled NNs. In [36], the authors studied the PS of fractional-order NNs via adaptive control. In [37], by introducing the sliding mode control, the authors studied the PS os NNs. In [1], authors derived sufficient conditions achieving the GDPS of BAMNNs by applying Lyapunov functional approach and differential inclusion theory. However, there are potential space to discuss the PS of BAMNNs with time-varying delay via adaptive controller.

Based on the above discussion, we study the PS of BAMNNs with time-delays. This paper has the following contributions. First, the definition of PS for BAMNNs is introduced. What is more, based on the adaptive controller and Lyapunov theory [11, 39, 40], some novel and useful conditions are given to ensure the PS of considered NNs. In particular, the synchronization between drive-response systems develops into the complete synchronization as $P_i = 1$. The synchronization between drive-response systems develops into the anti-synchronization as $P_i = -1$. The synchronization problem develops into the chaos problem as $P_i = 0$. Finally, an example is given to prove the adaptability of the theoretical results. Complete synchronization and anti-synchronization are the special case of PS.

The structure of this paper is as follows. Definition, assumptions, lemma, and the system description are given in Section 2. By designing a novel type of adaptive controller, we derive some sufficient criteria of PS in Section 3. In Section 4, an example is given to prove the adaptability of results. Lastly, the conclusion about this paper is given in Section 5.

2 Model description and preliminaries

In this paper, we consider the following BAMNNs system:

$$\begin{aligned}
\dot{x}_i(t) &= -c_i x_i(t) + \sum_{j=1}^{n} a_{ji} f_j\big(y_j(t)\big) + \sum_{j=1}^{n} b_{ji} f_j\big(y_j\big(t-\tau(t)\big)\big) + I_i, \\
\dot{y}_j(t) &= -d_j y_j(t) + \sum_{i=1}^{m} p_{ij} g_i\big(x_i(t)\big) + \sum_{i=1}^{m} h_{ij} g_i\big(x_i\big(t-\sigma(t)\big)\big) + J_j,
\end{aligned} \tag{1}$$

where $x_i(t)$ and $y_j(t)$ indicate the state variable of the neurons at time t, respectively; $f_j(\cdot)$ and $g_i(\cdot)$ are activation functions; $\tau(\cdot)$ and $\sigma(\cdot)$ denote the time-varying delays, which satisfy $0 \leqslant \tau(\cdot) \leqslant \tau$ and $0 \leqslant \sigma(\cdot) \leqslant \sigma$; c_i, d_j denote the self connection of the ith, the jth neurons; a_{ji}, b_{ji}, p_{ij}, and h_{ij} are connection weights; $i \in \mathcal{I} = \{1, 2, \dots, m\}$ and $j \in \mathcal{J} = \{1, 2, \dots, n\}$; $m \geqslant 2$ and $n \geqslant 2$ are the number of units in NNs; I_i and J_j are the input of the ith and jth neurons.

Assumption 1. $\tau(t)$ and $\sigma(t)$ are differential functions with $0 \leqslant \dot{\tau}(t) \leqslant \varepsilon < 1$, $0 \leqslant \dot{\sigma}(t) \leqslant \mu < 1$.

Remark 1. From Assumption 1 we obtained the following inequality:

$$1 \leqslant \frac{1 - \dot{\tau}(t)}{1 - \varepsilon}, \qquad 1 \leqslant \frac{1 - \dot{\sigma}(t)}{1 - \mu}.$$

Assumption 2. Solutions $x_i(t, x_i^0)$, $y_j(t, y_j^0)$, $i \in \mathcal{I}$, $j \in \mathcal{J}$, $t \geqslant 0$, of system (1) are bounded with $x_i^0, y_j^0 \in \mathbb{R}$ being the initial values. That is, there exist positive constants M_i^f and M_j^g such that

$$\left|x_i\left(t, x_i^0\right)\right| \leqslant M_i^f, \qquad \left|y_i\left(t, y_j^0\right)\right| \leqslant M_j^g.$$

Assumption 3. For all $j \in \mathcal{J}$, $u \in \mathbb{R}$, there exist positive constants L_j^f and L_j^g such that

$$\left|\dot{f}_j(u)\right| \leqslant L_j^f, \qquad \left|\dot{g}_j(u)\right| \leqslant L_j^g.$$

Remark 2. If the activation functions satisfy Assumption 3 [2], then from Lagrange mean value theorem, it is not difficult to check that $f_j(u)$ and $g_j(u)$ satisfy the globally Lipschitz condition. That is,

$$\begin{aligned}
\left|f_j(u_1) - f_j(u_2)\right| &\leqslant N_j^f |u_1 - u_2|, \\
\left|g_j(u_1) - g_j(u_2)\right| &\leqslant N_j^g |u_1 - u_2|,
\end{aligned}$$

where $u_1, u_2 \in \mathbb{R}$, $L_j^f = N_j^f > 0$, $L_j^g = N_j^g > 0$.

Consider system (1) as the drive system, then the response system is written by

$$\begin{aligned}
\dot{u}_i(t) &= -c_i u_i(t) + \sum_{j=1}^{n} a_{ji} f_j\left(v_j(t)\right) + \sum_{j=1}^{n} b_{ji} f_j\left(v_j\left(t - \tau(t)\right)\right) \\
&\quad + I_i + q_i(t), \\
\dot{v}_j(t) &= -d_j v_j(t) + \sum_{i=1}^{m} p_{ij} g_i\left(u_i(t)\right) + \sum_{i=1}^{m} h_{ij} g_i\left(u_i\left(t - \sigma(t)\right)\right) \\
&\quad + J_j + r_j(t),
\end{aligned} \tag{2}$$

where $q_i(t)$ and $r_j(t)$ are the controllers to be designed later.

Let $e_i(t) = u_i(t) - P_i x_i(t)$ and $z_j(t) = v_j(t) - \tilde{P}_j y_j(t)$, then the error system can be represented as

$$\begin{aligned}
\dot{e}_i(t) &= -c_i e_i(t) + \sum_{j=1}^{n} a_{ji} \varphi_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ji} \varphi_j\left(z_j\left(t - \tau(t)\right)\right) \\
&\quad + (1 - P_i) I_i + q_i(t), \\
\dot{z}_j(t) &= -d_j z_j(t) + \sum_{i=1}^{m} p_{ij} \psi_i\left(e_i(t)\right) + \sum_{i=1}^{m} h_{ij} \psi_i\left(e_i\left(t - \sigma(t)\right)\right) \\
&\quad + (1 - \tilde{P}_j) J_j + r_j(t),
\end{aligned}$$

where $\varphi_j(z_j(\cdot)) = f_j(v_j(\cdot)) - \tilde{P}_j f_j(y_j(\cdot))$, $\psi_i(e_i(\cdot)) = g_i(u_i(\cdot)) - P_i g_i(x_i(\cdot))$, P_i, and $\tilde{P}_j$ are scaling constants.

Definition 1. (See [31].) The drive-response system (1) and (2) are PS if there exist bounded continuously differentiable scaling constants P_i and $\tilde{P}_j$ satisfying

$$\lim_{t\to\infty}\|e_i(t)\| = \lim_{t\to\infty}|u_i(t) - P_i x_i(t)| = 0,$$
$$\lim_{t\to\infty}\|z_j(t)\| = \lim_{t\to\infty}|v_j(t) - \tilde{P}_j y_j(t)| = 0,$$

where $\|\cdot\|$ stands for the Euclidean vector norm.

Lemma 1. (*See* [29].) *If function* $F(u) : [0,\infty) \to \mathbb{R}$ *is uniformly continuous and* $\lim_{u\to\infty}\int_0^u F(v)\,\mathrm{d}v$ *exists and is bounded, then* $F(u) \to 0$ *as* $u \to +\infty$.

3 Main results

In this section, we will get some effective conditions to achieve the PS between the drive-response systems. Now introduce the controllers $q_i(t)$, $r_j(t)$ as follows:

$$\begin{aligned} q_i(t) &= -\alpha_i(t)\operatorname{sign}(e_i(t)) - \beta_i(t)e_i(t),\\ r_j(t) &= -\gamma_j(t)\operatorname{sign}(z_j(t)) - \xi_j(t)z_j(t), \end{aligned} \tag{3}$$

where

$$\dot{\alpha}_i(t) = k_i|e_i(t)|, \qquad \dot{\gamma}_j(t) = \tilde{k}_j|z_j(t)|,$$
$$\dot{\beta}_i(t) = d_i e_i^2(t), \qquad \dot{\xi}_j(t) = \tilde{d}_j z_j^2(t),$$

k_i, $\tilde{k}_j$, d_i, and $\tilde{d}_j$ are arbitrary positive constants, which to be given later.

According to controller (3), we can derive the following theorem.

Theorem 1. *Suppose that Assumptions* 1, 2, *and* 3 *hold. The drive-response systems* (1) *and* (2) *are PS if the response system* (2) *is controlled under the adaptive controller* (3).

Proof. First, giving the definitions of $\psi_i(e_i(t))$ and $\varphi_j(z_j(t))$, one has

$$\begin{aligned} \varphi_j(z_j(t)) &= f_j(v_j(t)) - f_j(\tilde{P}_j y_j(t)) + f_j(\tilde{P}_j y_j(t)) - \tilde{P}_j f_j(y_j(t)),\\ \varphi_j(z_j(t-\tau(t))) &= f_j(v_j(t-\tau(t))) - f_j(\tilde{P}_j y_j(t-\tau(t)))\\ &\quad + f_j(\tilde{P}_j y_j(t-\tau(t))) - \tilde{P}_j f_j(y_j(t-\tau(t))),\\ \psi_i(e_i(t)) &= g_i(u_i(t)) - g_i(P_i x_i(t)) + g_i(P_i x_i(t)) - P_i g_i(x_i(t)),\\ \psi_i(e_i(t-\sigma(t))) &= g_i(u_i(t-\sigma(t))) - g_i(P_i x_i(t-\sigma(t)))\\ &\quad + g_i(P_i x_i(t-\sigma(t))) - P_i g_i(x_i(t-\sigma(t))). \end{aligned}$$

Based on the Lagrange mean value theorem, one has

$$\begin{aligned} f_j(\tilde{P}_j y_j(\cdot)) - \tilde{P}_j f_j(y_j(\cdot)) &= \dot{f}_j(\eta_j^1)\tilde{P}_j y_j(\cdot) - \tilde{P}_j \dot{f}_j(\eta_j^2) y_j(\cdot) + (1-\tilde{P}_i) f_j(0),\\ g_i(P_i x_i(\cdot)) - P_i g_i(x_i(\cdot)) &= \dot{g}_i(\xi_i^1) P_i x_i(\cdot) - P_i \dot{g}_i(\xi_i^2) x_i(\cdot) + (1-P_i) g_i(0), \end{aligned}$$

where

$$\eta_j^1 \in \bigl(\min\{0, \tilde{P}_j y_j(\cdot)\},\ \max\{0, \tilde{P}_j y_j(\cdot)\}\bigr),$$
$$\eta_j^2 \in \bigl(\min\{0, y_j(\cdot)\},\ \max\{0, y_j(\cdot)\}\bigr),$$
$$\xi_i^1 \in \bigl(\min\{0, P_i x_i(\cdot)\},\ \max\{0, P_i x_i(\cdot)\}\bigr),$$
$$\xi_i^2 \in \bigl(\min\{0, x_i(\cdot)\},\ \max\{0, x_i(\cdot)\}\bigr).$$

Using Assumption 2, we get

$$\dot{f}_j\bigl(\eta_j^1\bigr)\tilde{P}_j y_j(\cdot) \leqslant L_j^f M_j^f |\tilde{P}_j|, \qquad \tilde{P}_i \dot{f}_j\bigl(\eta_j^2\bigr) y_j(\cdot) \leqslant L_j^f M_j^f |\tilde{P}_i|,$$
$$\dot{g}_i\bigl(\xi_i^1\bigr) P_i x_i(\cdot) \leqslant L_i^g M_i^g |P_i|, \qquad P_i \dot{g}_i\bigl(\xi_i^2\bigr) x_i(\cdot) \leqslant L_i^g M_i^g |P_i|,$$

which leads to

$$\bigl|\varphi_{ij}\bigl(z_j(t)\bigr)\bigr| \leqslant L_j^f \bigl|z_j(t)\bigr| + r_{ij}^f,$$
$$\bigl|\psi_{ij}\bigl(e_i(t)\bigr)\bigr| \leqslant L_i^g \bigl|e_i(t)\bigr| + r_{ij}^g,$$
$$\bigl|\varphi_{ij}\bigl(z_j\bigl(t-\tau(t)\bigr)\bigr)\bigr| \leqslant L_j^f \bigl|z_j\bigl(t-\tau(t)\bigr)\bigr| + r_{ij}^f,$$
$$\bigl|\psi_{ij}\bigl(e_i\bigl(t-\tau(t)\bigr)\bigr)\bigr| \leqslant L_i^g \bigl|e_i\bigl(t-\tau(t)\bigr)\bigr| + r_{ij}^g,$$

where

$$r_{ij}^f = L_j^f M_j^f \bigl(|\tilde{P}_j| + |\tilde{P}_i|\bigr) + \bigl|(1-\tilde{P}_i)\bigr|\bigl|f_j(0)\bigr|,$$
$$q_{ij}^g = L_i^g M_i^f \bigl(|P_j| + |P_i|\bigr) + \bigl|(1-P_i)\bigr|\bigl|g_i(0)\bigr|.$$

Now, introduce the following novel Lyapunov functional:

$$V(t) = V_1(t) + V_2(t).$$

Here

$$V_1(t) = \frac{1}{2}\sum_{i=1}^m e_i^2(t) + \frac{1}{2(1-\mu)}\sum_{i=1}^m\sum_{j=1}^n |b_{ij}| L_j^f \int\limits_{t-\sigma(t)}^t e_i^2(s)\,\mathrm{d}s + \frac{1}{2}\sum_{i=1}^m \frac{1}{d_i}\bigl(\beta_i(t)-\beta_i\bigr)^2 + \frac{1}{2}\sum_{i=1}^m \frac{1}{k_i}\bigl(\alpha_i(t)-\alpha_i\bigr)^2,$$

$$V_2(t) = \frac{1}{2}\sum_{j=1}^n z_j^2(t) + \frac{1}{2(1-\varepsilon)}\sum_{i=1}^m\sum_{j=1}^n |h_{ij}| L_j^g \int\limits_{t-\tau(t)}^t z_j^2(s)\,\mathrm{d}s + \frac{1}{2}\sum_{j=1}^n \frac{1}{\tilde{d}_j}\bigl(\xi_j(t)-\xi_j\bigr)^2 + \frac{1}{2}\sum_{j=1}^n \frac{1}{\tilde{k}_j}\bigl(\gamma_j(t)-\gamma_j\bigr)^2,$$

where α_i, β_i, ξ_j, and γ_j are positive constants.

Calculating the $V(t)$ derivation, one has

$$
\begin{aligned}
\dot V_1(t) ={}& \sum_{i=1}^{n} e_i(t)\dot e_i(t) + \frac{1}{k_i}\sum_{i=1}^{n}\bigl(\alpha_i(t)-\alpha_i\bigr)\dot\alpha_i(t) + \sum_{i=1}^{n}\frac{1}{d_i}\bigl(\beta_i(t)-\beta_i\bigr)\dot\beta_i(t)\\
&+\frac{1}{2(1-\mu)}\sum_{i=1}^{n}\sum_{j=1}^{n}|b_{ij}|L_j^f e_i^2(t) - \frac{1-\dot\sigma(t)}{2(1-\mu)}\sum_{i=1}^{n}\sum_{j=1}^{n}|b_{ij}|L_j^f e_i^2\bigl(t-\sigma(t)\bigr),\\
\leqslant{}& \sum_{i=1}^{m}\Biggl\{-c_i e_i^2(t) + \sum_{j=1}^{n}|a_{ji}|\bigl|e_i(t)\bigr|\bigl(L_j^f\bigl|z_j(t)\bigr| + r_{ij}^f\bigr)\\
&+\sum_{j=1}^{n}|b_{ji}|\bigl|e_i(t)\bigr|\bigl(L_j^f\bigl|z_j\bigl(t-\tau(t)\bigr)\bigr| + r_{ij}^f\bigr) + \bigl|e_i(t)\bigr|\bigl|(1-P_i)I_i\bigr| - \alpha_i\bigl|e_i(t)\bigr|\\
&-\beta_i e_i^2(t) + \frac{1}{2(1-\mu)}\sum_{j=1}^{n}|b_{ji}|L_j^f e_i^2(t) - \frac{1}{2}\sum_{j=1}^{n}|b_{ji}|L_j^f e_i^2\bigl(t-\tau(t)\bigr)\Biggr\},\\
\leqslant{}& \sum_{i=1}^{m}\Biggl[-\beta_i - c_i + \frac{1}{2}\sum_{j=1}^{n}|b_{ji}|L_j^f + \frac{1}{2}\sum_{j=1}^{n}|a_{ji}|L_j^f + \frac{1}{2(1-\mu)}\sum_{j=1}^{n}|b_{ji}|L_j^f\Biggr]e_i^2(t)\\
&+\sum_{i=1}^{n}\Biggl[-\alpha_i + \bigl|(1-P_i)I_i\bigr| + \sum_{j=1}^{n}|a_{ji}|r_{ij}^f + \sum_{j=1}^{n}|b_{ji}|r_{ij}^f\Biggr]\bigl|e_i(t)\bigr|\\
&+\frac{1}{2}\sum_{j=1}^{n}|a_{ij}|L_i^f z_j^2(t) + \frac{1}{2}\sum_{j=1}^{n}|b_{ji}|\bigl(L_j^f\bigl|z_j\bigl(t-\tau(t)\bigr)\bigr|^2,\\
\dot V_2(t) ={}& \sum_{j=1}^{n} z_j(t)\dot z_j(t) + \frac{1}{\tilde k_j}\sum_{j=1}^{n}\bigl(\gamma_j(t)-\gamma_j\bigr)\dot\gamma_j(t) + \sum_{j=1}^{n}\frac{1}{\tilde d_j}\bigl(\xi_j(t)-\xi_j\bigr)\dot\xi_j(t)\\
&+\frac{1}{2(1-\varepsilon)}\sum_{i=1}^{m}\sum_{j=1}^{n}|h_{ij}|L_j^g z_j^2(t) - \frac{1-\dot\tau(t)}{2(1-\varepsilon)}\sum_{i=1}^{m}\sum_{j=1}^{n}|h_{ij}|L_j^g z_j^2\bigl(t-\tau(t)\bigr),\\
\leqslant{}& \sum_{j=1}^{n}\Biggl\{-d_j z_j^2(t) + \sum_{i=1}^{m}|p_{ji}|\bigl|z_i(t)\bigr|\bigl(L_i^g\bigl|e_i(t)\bigr| + q_{ij}^g\bigr)\\
&+\sum_{i=1}^{m}|h_{ij}|\bigl|z_i(t)\bigr|\bigl(L_i^g\bigl|e_i\bigl(t-\sigma(t)\bigr)\bigr| + q_{ij}^g\bigr) + \bigl|z_j(t)\bigr|\bigl|(1-\tilde P_j)J_j\bigr| - \gamma_j\bigl|z_j(t)\bigr|\\
&-\xi_j z_j^2(t) + \frac{1}{2(1-\varepsilon)}\sum_{j=1}^{n}|h_{ij}|L_j^g z_j^2(t) - \frac{1}{2}\sum_{j=1}^{n}|h_{ij}|L_j^g z_j^2\bigl(t-\tau(t)\bigr)\Biggr\},\\
\leqslant{}& \sum_{j=1}^{n}\Biggl[-\xi_j - d_j + \frac{1}{2}\sum_{i=1}^{m}|h_{ji}|L_j^g + \frac{1}{2}\sum_{i=1}^{m}|p_{ij}|L_j^g\\
&+\frac{1}{2(1-\varepsilon)}\sum_{i=1}^{m}|h_{ij}|L_j^g\Biggr]z_j^2(t)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\left[-\gamma_j+\left|(1-\tilde{P}_j)J_j\right|+\sum_{i=1}^{m}|p_{ij}|q_{ij}^g+\sum_{i=1}^{m}|h_{ij}|q_{ij}^g\right]\left|z_j(t)\right|\\
&+\frac{1}{2}\sum_{i=1}^{m}|p_{ij}|L_j^g e_i^2(t)+\frac{1}{2}\sum_{i=1}^{m}|h_{ij}|L_i^g\left|e_i\bigl(t-\sigma(t)\bigr)\right|^2,
\end{aligned}
$$

where the inequality $2xy \leqslant x^2+y^2$ for all $x, y \in \mathbb{R}$ is used. Take the α_i, β_i, ξ_j, and γ_j large enough such that

$$
\begin{aligned}
\beta_i &\geqslant -c_i+\frac{1}{2}\sum_{j=1}^{n}|b_{ji}|L_j^f+\frac{1}{2}\sum_{j=1}^{n}|a_{ji}|L_j^f+\frac{1}{2(1-\mu)}\sum_{j=1}^{n}|b_{ji}|L_j^f\\
&\quad+\frac{1}{2}\sum_{i=1}^{m}|p_{ij}|L_j^g+\epsilon_i,\\
\alpha_i &\geqslant \left|(1-p_i)I_i\right|+\sum_{j=1}^{n}|a_{ji}|r_{ij}^f+\sum_{j=1}^{n}|b_{ji}|r_{ij}^f,\\
\xi_j &\geqslant -d_j+\frac{1}{2}\sum_{i=1}^{m}|h_{ji}|L_j^g+\frac{1}{2}\sum_{i=1}^{m}|p_{ij}|L_j^g+\frac{1}{2(1-\varepsilon)}\sum_{i=1}^{m}|h_{ij}|L_j^g\\
&\quad+\frac{1}{2}\sum_{j=1}^{n}|a_{ij}|L_i^f+\nu_j,\\
\gamma_j &\geqslant \left|(1-\tilde{P}_j)J_j\right|+\sum_{i=1}^{m}|p_{ij}|q_{ij}^g+\sum_{i=1}^{m}|h_{ij}|q_{ij}^g,
\end{aligned}
$$

where $\epsilon_i > 0$, $\nu_j > 0$ are arbitrarily chosen constants.

Let $\epsilon = \min_{i\in\mathcal{I}}\{\epsilon_i\} > 0$, $\nu = \min_{j\in\mathcal{J}}\{\nu_j\} > 0$, then we get

$$
\frac{\mathrm{d}V(t)}{\mathrm{d}t} \leqslant -\sum_{i=1}^{m}\epsilon_i e_i^2(t)-\sum_{j=1}^{n}\nu_j z_j^2(t) \leqslant -\epsilon e^{\mathrm{T}}(t)e(t)-\nu z^{\mathrm{T}}(t)z(t).
$$

Therefore,

$$
\begin{aligned}
&e^{\mathrm{T}}(t)e(t)+z^{\mathrm{T}}(t)z(t)\\
&\quad\leqslant 2V(t)=2V(0)+2\int_0^t \dot{V}(s)\,\mathrm{d}s\\
&\quad\leqslant 2V(0)-2\epsilon\int_0^t e^{\mathrm{T}}(s)e(s)\,\mathrm{d}s-2\nu\int_0^t z^{\mathrm{T}}(s)z(s)\,\mathrm{d}s. \qquad (4)
\end{aligned}
$$

Hence, $V(t) \leqslant V(0)$ for all $t \in [0,+\infty)$, which drives that $e(t)$, $z(t)$, $\dot{e}(t)$, and $\dot{z}(t)$ are bounded for all $t \in [0,+\infty)$. Consequently, the derivative of $e(t)^{\mathrm{T}}e(t)$ and $z(t)^{\mathrm{T}}z(t)$

are bounded. By integrating of (4) we get

$$\int_0^t e^{\mathrm{T}}(s)e(s)\,\mathrm{d}s + \int_0^t z^{\mathrm{T}}(s)z(s)\,\mathrm{d}s \leqslant \frac{V(0)}{\epsilon} + \frac{V(0)}{\nu},$$

therefore,

$$\lim_{t\to\infty} \int_0^t e^{\mathrm{T}}(s)e(s)\,\mathrm{d}s + \int_0^t z^{\mathrm{T}}(s)z(s)\,\mathrm{d}s \leqslant \frac{V(0)}{\epsilon} + \frac{V(0)}{\nu} < +\infty.$$

Based on the Lemma 1, we get

$$\lim_{t\to\infty} e(t)^{\mathrm{T}}e(t) = 0, \qquad \lim_{t\to\infty} z(t)^{\mathrm{T}}z(t) = 0.$$

Based on the Definition 1, the drive-response systems can achieve PS under the adaptive control law (3). The proof is thus completed. □

Remark 3. In [8, 10, 36, 37], the authors considered the PS of NNs based on some types of stability techniques, for example, Halanay inequality, Lyapunov–Krasovskii, and linear inequality. Very recently, the GDPS (general decay projective synchronization) of NNs was investigated in [1]. However, the Lagrange mean value theorem is introduced in this work. From this point, the results in this paper are quite distinct from the previous works.

Corollary 1. *Let $\tau = \tau(t)$, $\sigma = \sigma(t)$, then system* (1) *and* (2) *become as*

$$\begin{aligned}
\dot{x}_i(t) &= -c_i x_i(t) + \sum_{j=1}^{n} a_{ji} f_j\big(y_j(t)\big) + \sum_{j=1}^{n} b_{ji} f_j\big(y_j(t-\tau)\big) + I_i, \\
\dot{y}_j(t) &= -d_j y_j(t) + \sum_{i=1}^{m} p_{ij} g_i\big(x_i(t)\big) + \sum_{i=1}^{m} h_{ij} g_i\big(x_i(t-\sigma)\big) + J_j
\end{aligned}$$

and

$$\begin{aligned}
\dot{u}_i(t) &= -c_i u_i(t) + \sum_{j=1}^{n} a_{ji} f_j\big(v_j(t)\big) + \sum_{j=1}^{n} b_{ji} f_j\big(v_j(t-\tau)\big) + I_i + q_i(t), \\
\dot{v}_j(t) &= -d_j v_j(t) + \sum_{i=1}^{m} p_{ij} g_i\big(u_i(t)\big) + \sum_{i=1}^{m} h_{ij} g_i\big(u_i(t-\sigma)\big) + J_j + r_j(t).
\end{aligned}$$

If Assumptions 1, 2, *and* 3 *hold, the above drive-response NNs are PS under the above adaptive controller.*

Assumption 4. If $g_j(v)$ and $h_j(v)$ are bounded, then there exist positive constants G_j and W_j such that

$$\big|g_j(v)\big| \leqslant G_j, \quad \big|h_j(v)\big| \leqslant W_j \quad \forall v \in \mathbb{R},\ j \in \mathcal{J}.$$

Corollary 2. *If Assumption* 4 *holds, then the NNs* (1) *and* (2) *are PS according to the adaptive controller* (3).

Remark 4. It is worth to point out that in most of the existing works [14, 31, 32], the authors achieved the PS by introducing very specific but complex controllers, which are sometimes too difficult to implement physically. For instance, in order to offset the unmatched terms caused by scaling factor P_i when computing the derivative of error system, most of the above-mentioned works constructed very complex controllers, which were consisted by linear terms $e_i(t)$, $z_j(t)$, $e_i(t-\sigma(t))$, and $z_j(t-\tau(t))$ relevant to the activation functions $f_j(e_j(t))$, $g_i(z_i(t))$, $f_j(e_j(t-\sigma(t)))$, and $g_i(z_i(t-\tau(t)))$. However, in some special cases, for example, when the solutions of drive a system are bounded, we can optimize the controller by removing the terms relevant to the terms $f_j(e_j(t))$, $g_i(z_i(t))$, $f_j(e_j(t-\sigma(t)))$, and $g_i(z_i(t-\tau(t)))$. Thus, it is interesting to develop a simple and easy implementing controller for realizing PS between derive-response chaotic systems.

4 Numerical simulations

In this section, a numerical example is shown to describe the adaptability of the derived results in the paper.

Example. When $n = 2$, consider the following BAMNNs system:

$$\begin{aligned}\dot{x}_i(t) &= -c_i x_i(t) + \sum_{j=1}^{2} a_{ji} f_j\bigl(y_j(t)\bigr) + \sum_{j=1}^{2} b_{ji} f_j\bigl(y_j\bigl(t-\tau(t)\bigr)\bigr) + I_i,\\ \dot{y}_j(t) &= -d_j y_j(t) + \sum_{i=1}^{2} p_{ij} g_i\bigl(x_i(t)\bigr) + \sum_{i=1}^{2} h_{ij} g_i\bigl(x_i\bigl(t-\sigma(t)\bigr)\bigr) + J_j,\end{aligned} \tag{5}$$

where $f_1(s) = f_2(s) = g_1(s) = g_2(s) = \tanh(s)$, $c_1 = c_2 = 1$, $d_1 = d_2 = 1.05$, $a_{11} = 2$, $a_{12} = -0.1$, $a_{21} = -5$, $a_{22} = 3$, $b_{11} = -1.5$, $b_{12} = -0.1$, $b_{21} = -0.2$, $b_{22} = -2.5$, $p_{11} = 2.08$, $p_{12} = -0.104$, $p_{21} = -5.2$, $p_{22} = 3.12$, $h_{11} = -1.56$, $h_{12} = -0.104$, $h_{21} = -0.208$, $h_{22} = -2.6$, $\sigma(t) = \mathrm{e}^t/(5 + \mathrm{e}^t)$, $\tau(t) = \mathrm{e}^t/(0.01 + \mathrm{e}^t)$, $I_i = 0 = J_j = 0$ $(i, j = 1, 2)$.

The chaotic attractor of the drive system (5) with the initial values $x_1(s) = 0.5$, $x_2(s) = -0.1$, $y_1(s) = 0.5$, and $y_2(s) = -0.1$ ($s \in [-1, 0]$) are shown in Fig. 1.

The response system is given as

$$\begin{aligned}\dot{u}_i(t) &= -c_i u_i(t) + \sum_{j=1}^{2} a_{ji} f_j\bigl(v_j(t)\bigr) + \sum_{j=1}^{2} b_{ji} f_j\bigl(v_j\bigl(t-\tau(t)\bigr)\bigr) + I_i + q_i(t),\\ \dot{v}_j(t) &= -d_j v_j(t) + \sum_{i=1}^{2} p_{ij} g_i\bigl(u_i(t)\bigr) + \sum_{i=1}^{2} h_{ij} g_i\bigl(u_i\bigl(t-\sigma(t)\bigr)\bigr) + J_j + r_j(t),\end{aligned} \tag{6}$$

where c_i, d_i, a_{ji}, b_{ji}, p_{ij}, h_{ij}, f_j, g_j, $\sigma(t)$, $\tau(t)$, I_i, and J_j have the same value as in those system (5). The controllers $q_i(t)$ and $r_j(t)$ are written as follows:

$$q_i(t) = -\alpha_i(t)\,\mathrm{sign}\big(e_i(t)\big) - \beta_i(t)e_i(t),$$
$$r_j(t) = -\gamma_j(t)\,\mathrm{sign}\big(z_j(t)\big) - \xi_j(t)z_j(t),$$

where $e_i(t) = u_i(t) - P_i x_i(t)$, $z_j(t) = v_j(t) - \tilde{P}_j y_j(t)$.

It can be easily checked that Assumptions 1 and 3 hold with $L_1^f = L_2^f = 1$, $L_1^g = L_2^g = 1$, $\varepsilon = \mu = 1$. Moreover, from system (5) in Fig. 1 we can easy find that the solutions of system (5) are bounded and Assumption 3 holds. Consequently, based on Theorem 1, systems (5) and (6) are PS. When $P_i = \tilde{P}_j = 1$, the estimation of synchronization errors are given in Fig. 2, and the state trajectories of drive-response systems are given in Fig. 3. In Fig. 4, the time evolution of the controllers gains β_i, α_i, γ_i, ξ_i is given. Figures 5 and 6 show the time estimations of synchronization curves and errors for $\tilde{p}_j = -1$. The time evolution of the controllers gains as $P_i = \tilde{P}_j = -1$ are shown in Fig. 7.

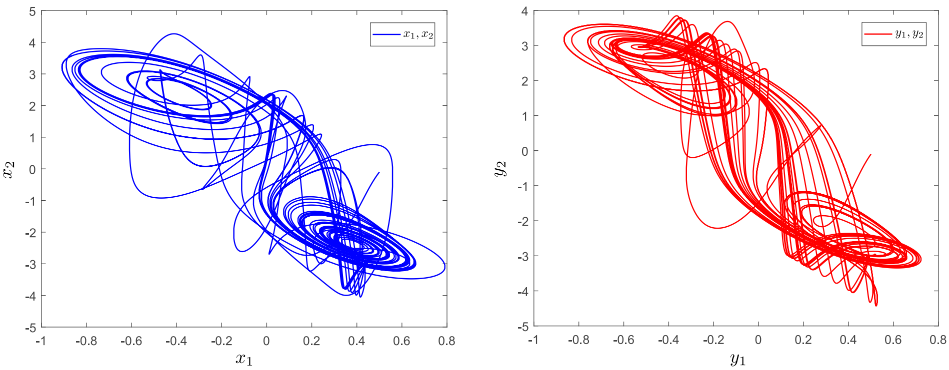

Figure 1. The phase behavior of system (5).

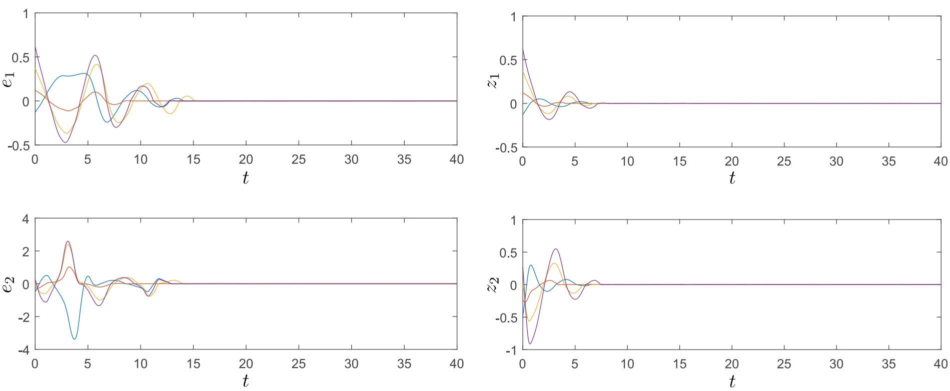

Figure 2. Synchronization errors as $P_i = 1$, $\tilde{P}_j = 1$.

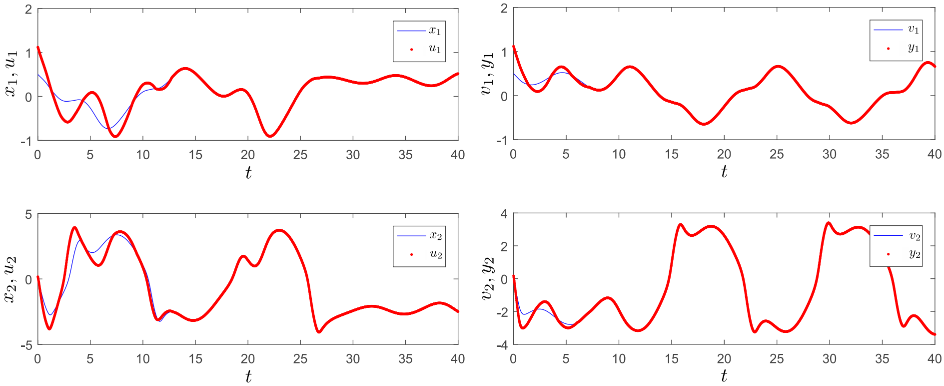

Figure 3. Synchronization curves as $P_i = 1$, $\tilde{P}_j = 1$.

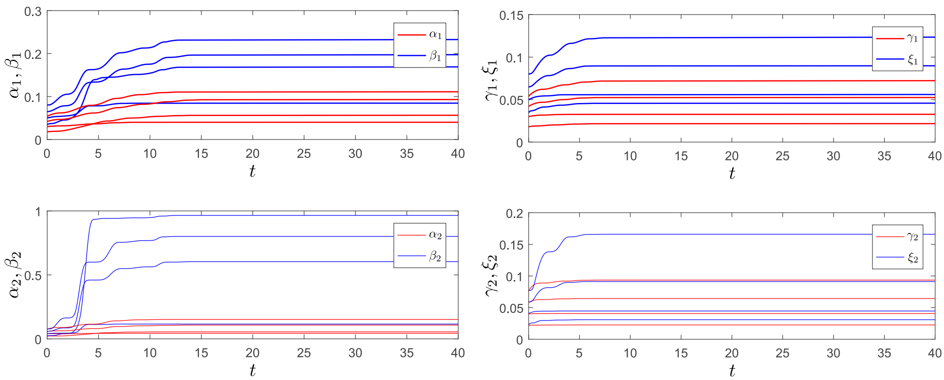

Figure 4. Controller gains β_i, α_i, γ_j, and ξ_j as $P_i = 1$, $\tilde{P}_j = 1$.

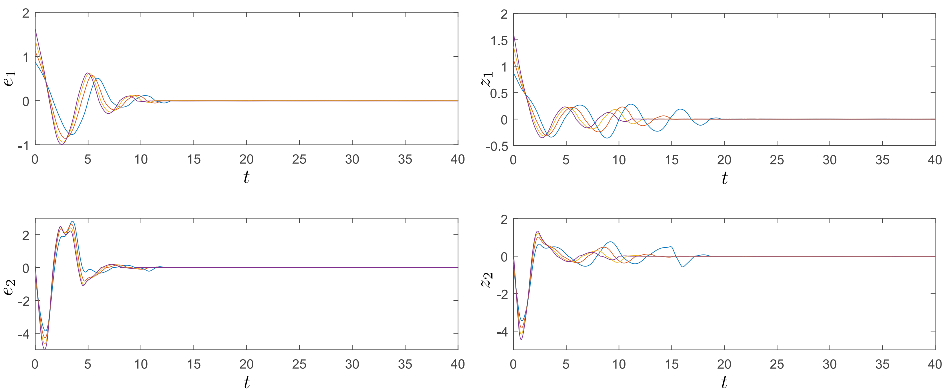

Figure 5. Synchronization errors as $P_i = -1$, $\tilde{P}_j = -1$.

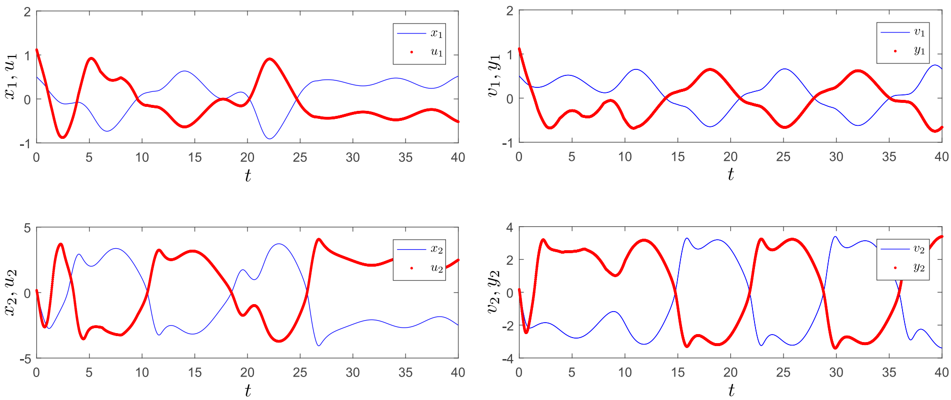

Figure 6. Synchronization errors as $P_i = -1$, $\tilde{P}_j = -1$.

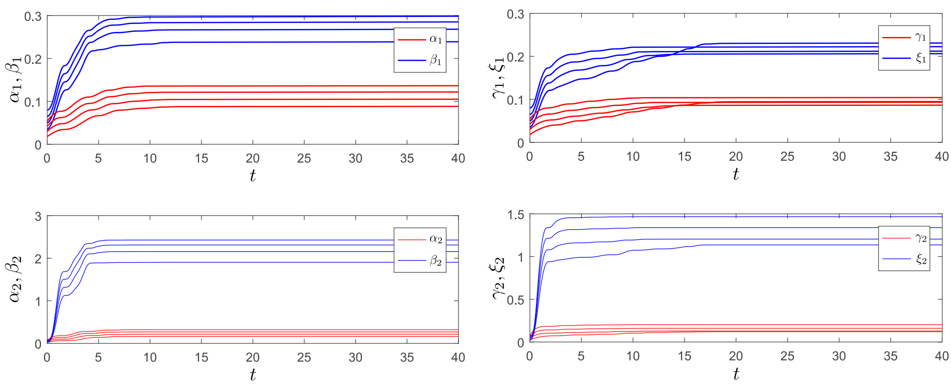

Figure 7. Controller gains β_i, α_i, γ_j, and ξ_j for $P_i = -1$, $\tilde{P}_j = -1$.

5 Conclusion

In this paper, the PS problem of BAMNNs with time-varying delay is studied by adapting a novel adaptive controller. Some sufficient conditions are given by using inequality technique and Lyapunov theory. Finally, an example is given to prove the effectiveness of the obtained results. The results given in this paper can be seen as the extension and improvement of some existing works on the PS of BAMNNs with or without time-varying delays.

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