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Fast fixed-time synchronization of T–S fuzzy complex networks*

Shuai Liu[a], **Lingli Zhao**[a], **Wanli Zhang**[b], **Xinsong Yang**[c,1], **Fuad E. Alsaadi**[d]

[a]School of Engineering, Honghe University,
Mengzi 661100, China
liushuai_csu@126.com; zhaolingli_csu@126.com

[b]College of Computer and Information Science,
Chongqing Normal University,
Chongqing 401331, China
mathwlzhang@163.com

[c]College of Electronics and Information Engineering,
Sichuan University,
Chengdu 610065, China
xinsongyang@163.com

[d]Department of Electrical and Computer Engineering,
Faculty of Engineering, King Abdulaziz University,
Jeddah 21589, Saudi Arabia
fuad_alsaadi@yahoo.com

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Abstract. In this paper, fast fixed-time (FDT) synchronization of T–S fuzzy (TSF) complex networks (CNs) is considered. The given control schemes can make the CNs synchronize with the given isolated system more fleetly than the most of existing results. By constructing comparison system and applying new analytical techniques, sufficient conditions are established to derive fast FDT synchronization speedily. In order to give some comparisons, FDT synchronization of the considered CNs is also presented by designing FDT fuzzy controller. Numerical examples are given to illustrate our new results.

Keywords: fixed-time synchronization, complex networks, T–S fuzzy systems, T–S fuzzy control.

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[1]Corresponding author.

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1 Introduction

Recently, there is a rapidly growing interests of fuzzy systems, and many papers have investigated fuzzy systems, for example, [19, 20] and so on. Especially, some researchers have paid their attention to TSF system [6, 10, 24], which is proposed by Takagi and Sugeno [18]. TSF system is always depicted by using some fuzzy IF-THEN rules, simply. However, it can be used to approximate a complex nonlinear system, conveniently. Therefore, it is necessary to investigate TSF system for its important applications.

It is well known that CNs can describe many natural and artificial systems. So, CNs are used widely in internet networks, social networks, and so on [3, 15, 27, 29, 34]. CNs exhibit complicated dynamical behaviors due to complex nodes and their connections. These nodes and connections can present many different objects, which make CNs different from a single node. Note that the TSF CNs can combine the advantage of TSF system and CNs, thus much attention has been paid to TSF CNs [7, 17, 21, 23, 35]. Especially, synchronization of TSF CNs with delays and stochastic perturbations was considered in [35]; by using pinning control, the authors of [17] investigate cluster synchronization of TSF CNs.

As we all know, synchronization is a very important dynamical behavior [33]. At the same time, it brings much attention thanks to its extensive applications in some fields such as biological systems, secure communication, and so on [11, 25, 26, 32, 36]. One can classify various definitions of synchronization into two kinds: (i) synchronization is achieved when time approach to infinity, for example, asymptotic synchronization, exponential synchronization; (ii) synchronization is realized within a finite time, for example, finite-time (FET) synchronization. Considering the convergence rate of synchronization, FET synchronization is optimal [8]. Moreover, the FET control techniques have some other advantages including good robustness properties [4]. Therefore, more and more researchers follow FET synchronization with interest [1, 12, 22, 30, 40, 41].

Note that the settling time of FET synchronization is not stationary if the initial states of systems are not same. In other words, the settling time is heavily rely on initial values. For some case, we always expect the synchronization is achieved within a given time. Nevertheless, if the given time is bound up with initial states, one can not choose easily since the initial values of systems is very hard to obtain generally. Thus, the variable settling time will prohibit the practical application of FET techniques. Not long ago, FDT control was constructed in [16], which is an improved FET control. The convergence time of FDT synchronization can be estimated without the initial conditions. That is to say, settling time is derived only based on control parameters and system's parameters. Then one can make FDT synchronization realize in a prescribed time, which means FDT synchronization more preferable than FET synchronization. As a result, many researchers are devoted to developing FDT control techniques [5, 13, 31, 37, 39]. Particularly, in order to improve convergence rate, the authors presented fast FDT control techniques in [14, 28, 38]. Motivated by these fast control ideas, this paper designs new control schemes, which have advantages in convergence rate over many existing results of FDT control.

From the above analyses this manuscript aims to studying fast FDT synchronization of TSF CNs in this paper. The contributions include: (i) new controllers are designed, which can realize fast FDT synchronization more rapidly; (ii) the TSF CNs and TSF control are considered; (iii) a suitable comparison system is constructed; (iv) fast FDT synchronization criteria are derived. Moreover, some comparisons are presented to show the differences between the fast FDT synchronization criteria established in this paper and the FDT synchronization criteria of this paper and previous papers.

2 Preliminaries

2.1 Notations

$\mathbb{R}^+$ denotes the set of nonnegative real numbers, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ are the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. $B=(b_{ij})_{n\times m}$ stands for a $n\times m$-dimension matrix. $B>0$ ($B<0$) denotes that B is a symmetric and positive (negative) definite matrix, $B^s=(B+B^{\mathrm{T}})/2$, and $\lambda_{\max}(B)$ represents its maximum eigenvalue. $\|\cdot\|$ is the standard Euclidean norm.

2.2 Model description

Considering the singleton fuzzier, product fuzzy inference, and a weighted average defuzzifier, which can be found in [18], a TSF system with controller is presented as

Rule $\boldsymbol{\tau}$: IF $z_1(t)$ is $M_{\tau 1}$, $z_2(t)$ is $M_{\tau 2}$, ..., $z_\mu(t)$ is $M_{\tau\mu}$, THEN

$$\dot{x}_i(t)=\sum_{\tau=1}^{r}h_\tau\bigl(z(t)\bigr)\Biggl[A_\tau x_i(t)+B_\tau f\bigl(x_i(t)\bigr)+\sum_{j=1}^{N}\gamma_{ij}\Phi x_j(t)+U_i^\tau(t)\Biggr],\quad i\in\mathcal{N},\tag{1}$$

$$\dot{y}(t)=\sum_{\tau=1}^{r}h_\tau\bigl(z(t)\bigr)\bigl(A_\tau y(t)+B_\tau f\bigl(y(t)\bigr)\bigr),\tag{2}$$

where $\mathcal{N}=\{1,2,\dots,N\}$, N is the number of nodes, $x_i(t)=(x_{i1}(t),x_{i2}(t),\dots,x_{in}(t))^{\mathrm{T}}\in\mathbb{R}^n$ and $y(t)=(y_1(t),y_2(t),\dots,y_n(t))^{\mathrm{T}}\in\mathbb{R}^n$ denote the state vectors, $f(\cdot):\mathbb{R}^n\to\mathbb{R}^n$ is a continuous function, $A_\tau,B_\tau\in\mathbb{R}^{n\times n}$ are constant matrices. $\Gamma=(\gamma_{ij})_{N\times N}$ satisfies $\gamma_{ij}\geqslant 0$ for $i\neq j$, $\gamma_{ii}=-\sum_{j=1,\,j\neq i}^{N}\gamma_{ij}$, and $\Phi=(\phi_{ij})_{n\times n}$ is inner-coupling matrix. $z(t)=(z_1(t),z_2(t),\dots,z_\mu(t))^{\mathrm{T}}$, z_j and $M_{\tau j}$ ($\tau=1,2,\dots,r$, $j=1,2,\dots,\mu$) are the premise variables and the fuzzy sets. Moreover,

$$h_\tau\bigl(z(t)\bigr)=\frac{w_\tau(z(t))}{\sum_{\tau=1}^{r}w_\tau(z(t))},\qquad w_\tau\bigl(z(t)\bigr)=\prod_{j=1}^{\mu}M_{\tau j}\bigl(z_j(t)\bigr),$$

$w_\tau(z(t))\geqslant 0$ and $\sum_{\tau=1}^{r}w_\tau(z(t))>0$. $M_{\tau j}(z_j(t))$ is the grade of membership function of $z_j(t)$ in $M_{\tau j}$. It is clear that

$$\sum_{\tau=1}^{r}h_\tau\bigl(z(t)\bigr)=1,\quad h_\tau\bigl(z(t)\bigr)\geqslant 0,\ \tau=1,2,\dots,r,\ \text{for any } t\in\mathbb{R}^+,$$

$U_i^\tau(t)$ is the controller. $x_i(0)$ and $y(0)$ are initial values of (1) and (2), respectively.

Based on systems (1) and (2), one can derive

$$\dot{e}_i(t) = \sum_{\tau=1}^{r} h_\tau\big(z(t)\big)\left[A_\tau e_i(t) + B_\tau g\big(e_i(t)\big) + \sum_{j=1}^{N} \gamma_{ij} \varPhi e_j(t) + U_i^\tau(t)\right], \quad i \in \mathcal{N},$$

where $e_i(t) = x_i(t) - y(t)$, $g(e_i(t)) = f(x_i(t)) - f(y(t))$.

This manuscript utilizes the following TSF controller:

$$U_i^\tau(t) = -\xi_i^\tau e_i(t) - \alpha\,\mathrm{sign}\big(e_i(t)\big)\big|e_i(t)\big|^\kappa - \beta e_i^p(t), \quad i \in \mathcal{N}, \tag{3}$$

where $\tau = 1, 2, \ldots, r$, $\kappa = q$ if $\sum_{i=1}^{N} \|e_i(t)\|^2 \geqslant 1$; otherwise, $\kappa = 1$. $q > 1$ and $0 < p < 1$, ξ_i^τ is constant to be determined. $\alpha > 0$ and $\beta > 0$ are tunable constants. $\mathrm{sign}(e_i(t)) = \mathrm{diag}(\mathrm{sign}(e_{i1}(t)), \mathrm{sign}(e_{i2}(t)), \ldots, \mathrm{sign}(e_{in}(t)))$, $|e_i(t)|^\kappa = (|e_{i1}(t)|^\kappa, |e_{i2}(t)|^\kappa, \ldots, |e_{in}(t)|^\kappa)^\mathrm{T}$, and $e_i^p(t) = (e_{i1}^p(t), e_{i2}^p(t), \ldots, e_{in}^p(t))^\mathrm{T}$.

Before considering the FDT synchronization of systems (1) and (2), the needed Definition 1 and Assumption 1 should be stated.

Definition 1. (See [31].) The CN (1) fixed-timely synchronizes onto (2) implies that there exists a constat $\mathcal{T} > 0$ (regardless of initial values $x(0) = (x_1^\mathrm{T}(0), x_2^\mathrm{T}(0), \ldots, x_N^\mathrm{T}(0))^\mathrm{T}$ and $y(0)$) satisfying $\lim_{t\to\mathcal{T}} \|e_i(t)\| = 0$ and $\|e_i(t)\| \equiv 0$ for $t > \mathcal{T}$, $i \in \mathcal{N}$. Here $\mathcal{T}$ denotes settling time.

The following assumption and lemmas will be used.

Assumption 1. There exists a constant $L > 0$ such that

$$\big\|f\big(x(t)\big) - f\big(y(t)\big)\big\| \leqslant L\big\|x(t) - y(t)\big\|, \quad x(t), y(t) \in \mathbb{R}^n.$$

Lemma 1. (*See* [13].) *Let a nonnegative function* $\mathcal{V}(t)$ *satisfy*

$$\dot{\mathcal{V}}(t) \leqslant -\eta \mathcal{V}^p(t) - \xi \mathcal{V}^q(t),$$

Here $\xi > 0$, $\eta > 0$, $1 > p > 0$, $q > 1$. *Then* $\mathcal{V}(t) \equiv 0$ *if*

$$\mathcal{T} \geqslant \frac{1}{\eta(1-p)} + \frac{1}{\xi(q-1)}.$$

Lemma 2. (*See* [9].) *Let* $\eta_1, \eta_2, \ldots, \eta_N \geqslant 0$, $0 < p \leqslant 1$, $q > 1$. *Then*

$$\sum_{i=1}^{N} \eta_i^p \geqslant \left(\sum_{i=1}^{N} \eta_i\right)^p, \qquad \sum_{i=1}^{N} \eta_i^q \geqslant N^{1-q}\left(\sum_{i=1}^{N} \eta_i\right)^q.$$

3 FDT synchronization

3.1 Fast FDT synchronization

In this section, via the designed fuzzy controllers, fast FDT synchronization are derived. Moreover, we also give some comparisons.

Theorem 1. *Let Assumption* 1 *hold. Suppose that control parameter* ξ_i^τ *in the set of controller* (3) *satisfies the following condition*:

$$\Xi_\tau \geqslant \|A_\tau\| I_N + L\|B_\tau\| I_N + \varpi \widehat{\Gamma}^s. \tag{4}$$

Then the CN (1) *can be synchronized onto* (2) *in a fixed time*

$$\mathcal{T} = \frac{1}{\bar{\alpha}(q-1)} + \frac{1}{\bar{\alpha}(1-p)} \ln\left(1 + \frac{\bar{\alpha}}{\beta}\right), \tag{5}$$

where $\widehat{\Gamma} = (\hat{\gamma}_{ij})_{N\times N}$*,* $\hat{\gamma}_{ij} = \gamma_{ij}$*,* $i \neq j$*,* $\hat{\gamma}_{ii} = \rho_{\min}\gamma_{ii}/\varpi$*,* $\varpi = \|\Phi\|$*,* $\rho_{\min}$ *is the minimum eigenvalue of* Φ^s*,* $\Xi_\tau = \operatorname{diag}(\xi_1^\tau, \xi_2^\tau, \dots, \xi_N^\tau)$*, and* $\bar{\alpha} = \alpha(nN)^{(1-q)/2}$*,* $\tau = 1, 2, \dots, r$*,* $i, j = 1, 2, \dots, N$*.*

Proof. Consider Lyapunov function

$$\mathcal{V}(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t). \tag{6}$$

It follows that

$$\dot{\mathcal{V}}(t) = 2\sum_{i=1}^{N}\sum_{\tau=1}^{r} h_\tau\big(z(t)\big) e_i^{\mathrm{T}}(t)\left[A_\tau e_i(t) + B_\tau g\big(e_i(t)\big) + \sum_{j=1}^{N} \gamma_{ij}\Phi e_j(t) + U_i^\tau(t)\right]. \tag{7}$$

By Assumption 1, one derives

$$e_i^{\mathrm{T}}(t) B_\tau g\big(e_i(t)\big) \leqslant L\|B_\tau\| \big\|e_i(t)\big\|^2.$$

Then from (7) we derive

$$\begin{aligned}
\dot{\mathcal{V}}(t) &\leqslant 2\sum_{i=1}^{N}\sum_{\tau=1}^{r} h_\tau\big(z(t)\big)\Bigg[\|A_\tau\|\big\|e_i(t)\big\|^2 + L\|B_\tau\|\big\|e_i(t)\big\|^2 + \rho_{\min}\gamma_{ii}\big\|e_i(t)\big\|^2 \\
&\quad + \sum_{j=1, j\neq i}^{N} \varpi\gamma_{ij}\big\|e_i(t)\big\|\big\|e_j(t)\big\| - \xi_i^\tau\big\|e_i(t)\big\|^2\Bigg] - 2\Theta(t) \\
&\leqslant 2\sum_{\tau=1}^{r} h_\tau\big(z(t)\big)\hat{e}^{\mathrm{T}}(t)\big(\|A_\tau\| I_N + L\|B_\tau\| I_N + \varpi\widehat{\Gamma}^s - \Xi_\tau\big)\hat{e}(t) - 2\Theta(t),
\end{aligned}$$

where $\hat{e}(t) = (\|e_1(t)\|, \|e_2(t)\|, \dots, \|e_N(t)\|)^{\mathrm{T}}$, $\Theta(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)[\alpha \operatorname{sign}(e_i(t)) \times |e_i(t)|^\kappa + \beta e_i^p(t)]$.

Noticing condition (4), one obtains

$$\dot{\mathcal{V}}(t) \leqslant -2\Theta(t). \tag{8}$$

Next, inequality (8) will be separately discussed for two cases.

Case 1. When $\mathcal{V}(t) \geqslant 1$,

$$\varTheta(t) = \alpha \sum_{i=1}^{N} \left[\sum_{j=1}^{N} \left| e_{ij}(t) \right|^{1+q} + \beta \sum_{j=1}^{N} \left| e_{ij}(t) \right|^{1+p} \right].$$

From Lemma 2 it generates

$$\begin{aligned} \varTheta(t) &\geqslant \bar{\alpha} \left(\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \right)^{(1+q)/2} + \beta \left(\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \right)^{(1+p)/2} \\ &= \bar{\alpha} \mathcal{V}^{(1+q)/2}(t) + \beta \mathcal{V}^{(1+p)/2}(t). \end{aligned} \tag{9}$$

Case 2. When $\mathcal{V}(t) < 1$,

$$\varTheta(t) = \alpha \sum_{i=1}^{N} \left[\sum_{i=1}^{N} e_{ij}^2(t) + \beta \sum_{j=1}^{N} \left| e_{ij}(t) \right|^{(1+p)/2} \right].$$

By Lemma 2, it yields

$$\begin{aligned} \varTheta(t) &\geqslant \bar{\alpha} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) + \beta \left(\sum_{i=1}^{N} e_i^{\mathrm{T}}(t) e_i(t) \right)^{(1+p)/2} \\ &= \bar{\alpha} \mathcal{V}(t) + \beta \mathcal{V}^{(1+p)/2}(t). \end{aligned} \tag{10}$$

Form (8)–(10) one derives

$$\dot{\mathcal{V}}(t) \leqslant \begin{cases} -\hat{\alpha} \mathcal{V}^{(1+q)/2}(t) - \hat{\beta} \mathcal{V}^{(1+p)/2}(t), & \mathcal{V}(t) \geqslant 1, \\ -\hat{\alpha} \mathcal{V}(t) - \hat{\beta} \mathcal{V}^{(1+p)/2}(t), & \mathcal{V}(t) < 1, \end{cases} \tag{11}$$

where $\hat{\alpha} = 2\bar{\alpha}$, $\hat{\beta} = 2\beta$.

In order to compare, we give the following system:

$$\begin{aligned} \dot{W}(t) &= \begin{cases} -\hat{\alpha} W^{(1+q)/2}(t) - \hat{\beta} W^{(1+p)/2}(t), & W(t) \geqslant 1, \\ -\hat{\alpha} W(t) - \hat{\beta} W^{(1+p)/2}(t), & 0 < W(t) < 1, \\ 0, & W(t) = 0, \end{cases} \\ W(0) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(0) e_i(0). \end{aligned} \tag{12}$$

By (11) and (12), it is not hard to see that if we find a $\mathcal{T} > 0$ satisfying $W(t) = 0$ for any $t \geqslant \mathcal{T}$, then it also hold for any $t \geqslant \mathcal{T}$, $\mathcal{V}(t) = 0$. By the analysis of Lemma 1 in [14] and Theorem 1 in [38], let $\varrho(t) = W^{(1-p)/2}(t)$, $\varepsilon = (q-1)/(1-p)$, then

$$\dot{\varrho}(t) + \frac{1-p}{2} \hat{\alpha} \varrho^{(q-p)/(1-p)}(t) + \frac{1-p}{2} \hat{\beta} = 0, \quad \varrho(t) \geqslant 1,$$

and

$$\dot{\varrho}(t) + \frac{1-p}{2}\hat{\alpha}\varrho(t) + \frac{1-p}{2}\hat{\beta} = 0, \quad \varrho(t) < 1.$$

By use the similar calculation with [14] or [38], we will obtain the following estimation of $\mathcal{T}$:

$$\mathcal{T} = \frac{1}{\bar{\alpha}(q-1)} + \frac{1}{\bar{\alpha}(1-p)} \ln\left(1 + \frac{\bar{\alpha}}{\beta}\right),$$

and $\mathcal{V}(t) \equiv 0$ for $t \geqslant \mathcal{T}$. Furthermore, $e(t)$ approach to 0 within $\mathcal{T}$. Consequently, the synchronization goal is realized within $\mathcal{T}$ described by (5). The proof is completed. □

Remark 1. The settling time of Theorem 1 does not rely on $x(0)$, y_0, and the fuzzy weighting functions $h_\tau(z(t))$. Moreover, its estimation is more accurate than most existing FDT results. It should be noted that the similar estimation is called the fast FDT results in [14, 28].

Remark 2. In the investigation of FDT synchronization, comparison system is widely used in some papers such as [31, 37, 39] and so on. With the help of those comparison systems, the considered FDT stability or synchronization is transformed to the FDT stability of the corresponding system at 0. Besides, we give the estimation of settling time with the help of comparison system.

3.2 FDT synchronization

In previous investigations, FDT control techniques have been utilized generally. In order to present some comparisons clearly, this paper also establishes FDT synchronization results in Theorem 2 by designing the following FDT control schemes:

$$U_i^\tau(t) = -\xi_i^\tau e_i(t) - \alpha e_i^q(t) - \beta e_i^p(t), \quad i \in \mathcal{N}, \ \tau = 1, 2, \ldots, r, \tag{13}$$

where the definitions of corresponding parameters are similar with (3).

Theorem 2. *Let Assumption* 1 *hold. Suppose that control parameter* ξ_i^τ *in the set of controller* (13) *satisfies condition* (4). *Then the CN* (1) *can be synchronized onto* (2) *in a fixed time. In addition, the settling time are presented by*

$$\mathcal{T} = \frac{1}{\bar{\alpha}(q-1)} + \frac{1}{\beta(1-p)}, \tag{14}$$

where the definitions of corresponding parameters are similar with Theorem 1.

Proof. Consider the same Lyapunov function (6). One derives

$$\begin{aligned}\dot{\mathcal{V}}(t) \leqslant{}& 2\sum_{\tau=1}^{r} h_\tau\big(z(t)\big)\hat{e}^{\mathrm{T}}(t)\big(\|A_\tau\| I_N + L\|B_\tau\| I_N + \varpi\widehat{\Gamma}^s - \Xi_\tau\big)\hat{e}(t)\\ &- 2\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\big[\alpha e_i^q(t) + \beta e_i^p(t)\big],\end{aligned}$$

where $\hat{e}(t)$ is defined in Theorem 1.

From (4) we have

$$\dot{\mathcal{V}}(t) \leqslant -\hat{\alpha}\mathcal{V}^{(1+q)/2}(t) - \hat{\beta}\mathcal{V}^{(1+p)/2}(t), \tag{15}$$

where $\hat{\alpha} = 2\bar{\alpha}$, $\hat{\beta} = 2\beta$.

Based on Lemma 1, it generates $\mathcal{V}(t) \equiv 0$ for $t \geqslant \mathcal{T}$. Furthermore, the synchronization goal can be realized within $\mathcal{T}$, which is given by (14). We complete the proof. □

Remark 3. One can easily see the expression of $\mathcal{T}$ in (5) is more accurate than its expression in (14). Hence, the results of FDT synchronization in many existing papers including [5, 13, 31, 37, 39] are improved. These can be seen from inequalities (11) and (15). From (15) one can derive

$$\dot{\mathcal{V}}(t) \leqslant \begin{cases} -\hat{\alpha}\mathcal{V}^{(1+q)/2}(t) - \hat{\beta}\mathcal{V}^{(1+p)/2}(t), & \mathcal{V}(t) \geqslant 1, \\ -\hat{\alpha}\mathcal{V}^{(1+q)/2}(t) - \hat{\beta}\mathcal{V}^{(1+p)/2}(t), & \mathcal{V}(t) < 1, \end{cases}$$

and when $\mathcal{V}(t) < 1$, then $-\mathcal{V}^{(1+q)/2}(t) > -\mathcal{V}(t)$. Therefore, the conservativeness of estimation by means of (15) is lager.

Remark 4. When $\mathcal{V}(t) \geqslant 1$, $-\hat{\alpha}\mathcal{V}^{(1+q)/2}(t)$ plays an important role, while when $\mathcal{V}(t) < 1$, $-\hat{\beta}\mathcal{V}^{(1+p)/2}(t)$ is a key role. We can give the estimation of setting time with the help of the similar comparison:

$$\dot{W}(t) = \begin{cases} -\hat{\alpha}W^{(1+q)/2}(t), & W(t) \geqslant 1, \\ -\hat{\beta}W^{(1+p)/2}(t), & 0 < W(t) < 1, \\ 0, & W(t) = 0, \end{cases}$$

$$W(0) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(0)e_i(0).$$

Then we can clearly see that $-\hat{\beta}W^{(1+p)/2}(t)$ is omitted when $W(t) \geqslant 1$, while $-\hat{\alpha}W^{(1+q)/2}(t)$ is removed when $0 < W(t) < 1$. Therefore, some conservativeness are caused.

4 Numerical example

Now, we give numerical simulations to verify our synchronization criteria. Here we consider the following TSF systems:

$$\dot{x}_i(t) = \sum_{\tau=1}^{2} h_\tau\big(z(t)\big)\left[A_\tau x_i(t) + B_\tau f\big(x_i(t)\big) + \sum_{j=1}^{30}\gamma_{ij}\Phi x_j(t)\right], \quad i = 1, 2, \dots, 30, \tag{16}$$

$$\dot{y}(t) = \sum_{\tau=1}^{2} h_\tau\big(z(t)\big)\big[A_2 y(t) + B_2 f\big(y(t)\big)\big], \tag{17}$$

where

$$h_1(z(t)) = \begin{cases} \frac{1}{2}(1 - (\sin(z(t)))^2) & \text{if } z(t) \neq 0, \\ 1 & \text{if } z(t) = 0, \end{cases}$$

$$h_2(z(t)) = \begin{cases} \frac{1}{2}(1 + (\sin(z(t)))^2) & \text{if } z(t) \neq 0, \\ 0 & \text{if } z(t) = 0, \end{cases}$$

$$A_1 = \begin{bmatrix} 1 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -15 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -5 & 9 & 0 \\ 1 & -1 & 1 \\ 0 & -13.5 & 0 \end{bmatrix}, \qquad B_1 = B_2 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

and $f(y(t)) = (|y_1(t) + 1| - |y_1(t) - 1|, 0, 0)^{\mathrm{T}}$. Moreover, we take $z(t) = y_1(t)$, $\Phi = \mathrm{diag}(1, 1, 1)$, $\tau = 1, 2$, $\Gamma = (\gamma_{ij})_{30\times 30} = -\mathcal{L}$, where $\mathcal{L}$ is the Laplacian matrix of a BA scale-free network. According to [2], we construct a BA scale-free network. The parameters are given as: an initial graph is complete with $m_0 = 10$ nodes, $m = 3$ edges, and finally, we take $N = 30$. The BA scale-free network is showed in Fig. 1.

Obviously, the function $f(y(t))$ satisfies Assumption 1 with $L = 2$. Accordingly, the chaotic trajectory of the fuzzy system (17) with $y(0) = (0.55, 0.4, 0.6)^{\mathrm{T}}$ is shown in Fig. 2.

The error systems between (16) and (17) with fuzzy controller (3) can be expressed by

$$\dot{e}_i(t) = \sum_{\tau=1}^{2} h_\tau(z(t)) \Biggl[A_\tau e_i(t) + B_\tau g(e_i(t)) + \sum_{j=1}^{N} \gamma_{ij} \Phi e_j(t) - \xi_i^\tau e_i(t) - \alpha e_i^\kappa(t) - \beta e_i^p(t) \Biggr], \quad i = 1, 2, \ldots, 30. \tag{18}$$

Let $\xi^\tau = \min\{\xi_1^\tau, \xi_2^\tau, \ldots, \xi_{30}^\tau\}$, $\tau = 1, 2$. If $\xi^\tau \geqslant \|A_\tau\| + L\|B_\tau\| + \varpi\sigma_{\max}(\widehat{\Gamma}^s)$, then (4) can be satisfied. By simply computation, one can obtain $\xi^1 \geqslant 24.0625$, $\xi^2 \geqslant 22.5175$. Take the control gains $\xi^1 = 25$ and $\xi^2 = 23$. $x(0)$ is chosen from $(-5, 5)$. From Theorem 1 system (16) synchronizes onto (17) under (3), and the time is estimated as $\mathcal{T} = 8.0761$, where $\alpha = 1$, $\beta = 1$, $q = 5/3$, $p = 1/3$. We have presented the trajectories of system (18) in Fig. 3 in which FDT synchronization is achieved before $\mathcal{T} = 8.0761$.

Similarly, under controller (13), system (16) synchronizes onto (17) within $\mathcal{T} = 8.2221$ in view of Theorem 2, which is illustrated by Fig. 4. Here we take $\xi^1 = 25$ and $\xi^2 = 23$. $x(0)$ is taken from $(-5, 5)$, and $\alpha = 1$, $\beta = 1$, $q = 5/3$, $p = 1/3$.

Remark 5. From Theorems 1 and 2 we can see that the estimation of $\mathcal{T}$ in (5) is more accurate than this in (14). However, from Figs. 3 and 4 there are only very few difference. In real life, one can choose the suitable fuzzy control (3) or (13) according to the related conditions.

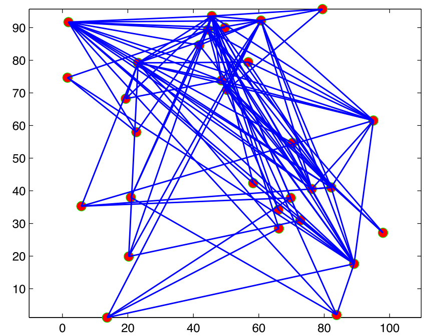

Figure 1. BA scale-free network.

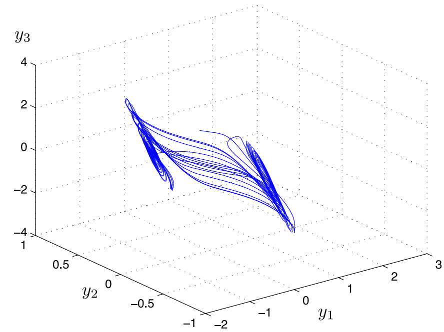

Figure 2. Chaotic trajectory of fuzzy system (17) with $y(0) = (0.55, 0.4, 0.6)^{\mathrm{T}}$.

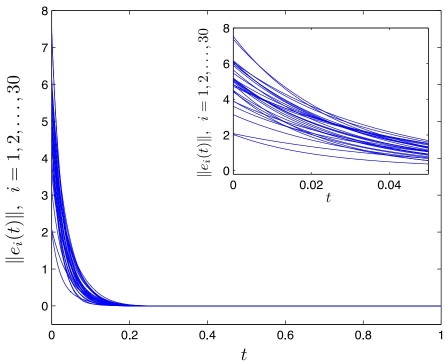

Figure 3. Trajectories $\|e_i(t)\|$ $(i = 1, 2, \dots, 30)$ via fuzzy control (3).

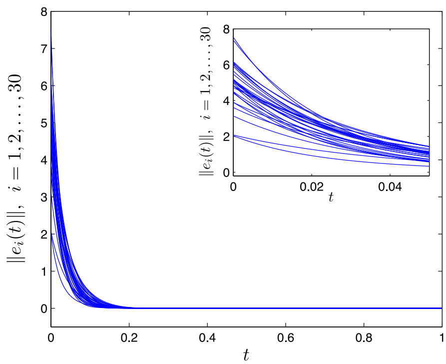

Figure 4. Trajectories $\|e_i(t)\|$ $(i = 1, 2, \dots, 30)$ via fuzzy control (13).

5 Conclusions

This manuscript studies fast FDT synchronization of TSF CNs. New controllers are designed, which can make the CNs synchronize with the given isolated system more fleetly than the most of existing results. By constructing comparison system, sufficient conditions are derived to realize fast FDT. In order to give some comparisons, FDT synchronization of the considered CNs is also presented by designing fuzzy control scheme. Numerical simulations are given to verify our results.

Moreover, uncertain perturbations will bring some difficulties to achieve synchronization of chaotic systems, for example, stochastic perturbations are always considered when the synchronization of CNs is investigated. Considering the FDT synchronization of CNs with stochastic perturbations is interesting, which will be our next research topic.

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