Nonlinear Analysis: Modelling and Control, Vol. 28, No. 4, 760–779
https://doi.org/10.15388/namc.2023.28.32277

Event-triggered leader-following formation control of general linear multi-agent systems with distributed infinite input time delays*

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Received: November 30, 2022 / **Revised:** May 4, 2023 / **Published online:** May 23, 2023

Abstract. By employing event-triggered control technique, this paper investigates the leader-following formation control problem of general linear multi-agent systems with distributed infinite input time delays. To decrease computing costs, a novel event-triggered formation protocol taking into consideration of the distributed infinite time delays between agents is put forward. Under the designed triggering function and triggering condition, a sufficient condition on leader-following formation is obtained, and then the Zeno-behavior of triggering time sequences is excluded for the concerned closed-loop system. The continuous update of controller for each agent is avoided. Finally, the correctness and the effectiveness of these theoretical results are demonstrated by two numerical examples.

Keywords: event-triggered, formation, distributed infinite delays, Zeno-behavior.

1 Introduction

Distributed cooperative control of multi-agent systems (MASs) [4,11,21] has been widely concerned owing to its many advantages including cost reduction, higher reliability, efficiency and flexibility, and so on. As one of the most important problems in cooperative control of MASs, formation control has been intensively studied due to its broad

*This research was jointly supported by National Natural Science Foundation of China under grant No. 61673080, Science and Technology Research Program of Chongqing Municipal Education Commission under grant No. KJZD-K202000601, Chongqing Talent Plan under grant No. cstc2021ycjh-bgzxm0044.

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application in various areas, such as formation navigation of autonomous vehicles, attitude tracking of spacecrafts, collaborative load transport of robots, cooperative surveillance of aircrafts, just to name a few. The objective of formation control is to design distributed control protocol for each agent so that all agents can achieve and retain some given geometric pattern [5, 23, 31].

To achieve distributed formation, communication among agents plays a key role. For MASs, exchanging information among agents, time delays occur [14, 20]. Time delays may result in performance degradation and even instability of the closed-loop system. Therefore, formation problem of MASs featuring time delays has long attracted interest; see [10, 13, 16] and the references therein.

All of the aforementioned formation control of MASs with time delays focused on bounded delays. However, in formation control problems, the current state of the system may be related to all the previous history. Moreover, MASs may have different delay intervals so that a unified model (such as the pantograph equation in physics) or a complete influence of the whole past of the state is required. Consequently, it is necessary to consider MASs with distributed infinite delay in formation problems, and modeling of such distributed infinite delays is needed. The distributed infinite delays have been utilized to model many practical applications, i.e., remote control of mechatronic systems [24], HIV-1 infection passes [2], population dynamic systems [9], traffic flow dynamics [18], neural networks [25], and so on. More recently, the consensus of MASs with distributed infinite time delays starts to receive increasing attention [27, 28], which is the basis of formation problem of MASs.

On the other hand, the key of implement of information exchange among the agents is to choice an appropriate control mechanism. Due to the superiority of reducing the controller update and maintaining a satisfactory control performance of the closed-loop system, the event-triggered control (ETC) mechanism [7, 26, 29] has attracted persistent attention to explore the formation control problem of MASs with time delays; see [3, 6, 12, 17]. For instance, researchers in [3] designed a distributed event-triggered formation scheme based on complex-valued Laplacian to study MASs with time delays. In consideration of time delays, event-triggered control strategies [12,17] were proposed to cope with formation control problem of second-order MASs. It is worth noting that only bounded time delays were considered in these aforementioned event-triggered formation results. Nevertheless, as far as the authors' knowledge, the event-triggered formation control of MASs with distributed infinite time delays still remains open.

Motivated by the aforementioned discussions, our work aims at designing an event-triggered controller for the formation control of general linear MASs with distributed infinite input time delays. The main contributions of this paper are summarized:

(i) The formation protocol considering distributed infinite input time delays is developed, which not only has practical advantages, but also include bounded distributed input delays as special cases.

(ii) The ETC approach is applied to deal with the leader-following formation control of general linear MASs. The result shows that continuous update of controller is avoided, which means the improvement of the usage of a limited bandwidth resource. Moreover, the Zeno-behavior of the event-triggered time sequence is excluded.

(iii) A sufficient condition for achieving formation is presented, and the convergence rate is estimated. To make the theoretical results more convincing, a practical example is introduced, i.e., the leader-following formation of nonholonomic vehicles of unicycle type.

Notations. $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ stand for the set of n-dimensional real column vectors and $n \times m$-dimensional real matrices, respectively. Given a matrix A, $\lambda_i(A)$ denotes the ith eigenvalues of matrix A, and $\operatorname{Re}\lambda_i(\cdot)$ be the real part of the ith eigenvalue. A^{T} represents the transpose of A. $A > 0$ implies that A is a positive definite matrix. $A \otimes B$ means Kronecker product of matrices A and B. $\|\cdot\|$ represents the Euclidean norm for vectors or the induced 1-norm for matrix. $\operatorname{diag}\{a_1, \dots, a_n\}$ denotes a diagonal matrix, where a_i, $i = 1, \dots, n$, are its diagonal elements.

2 Preliminaries

Consider the following general linear multi-agent systems:

$$\begin{aligned} &\text{Leader:} && \dot{x}_0(t) = Ax_0(t), \\ &\text{Followers:} && \dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \dots, N, \end{aligned} \tag{1}$$

where $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$. $x_0(t) \in \mathcal{R}^n$ represents the state of agent 0. $x_i(t) \in \mathcal{R}^n$ and $u_i(t) \in \mathcal{R}^m$ denote the state and control input of agent i, respectively.

The communication topology among the N followers with one leader is represented by a digraph (directed graph) $\widetilde{G} = (\widetilde{V}, E, \tilde{A})$ with a set of nodes $\widetilde{V} = \{0, 1, 2, \dots, N, \}$. $\tilde{A} = [a_{ij}]_{N\times N}$ is the adjacency matrix of followers, where $a_{ij} > 0$ if and only if $(j, i) \in E$, and $a_{ij} = 0$, otherwise. $\tilde{B} = \operatorname{diag}\{b_1, \dots, b_N\}$ is the leader adjacency matrix associated with $\widetilde{G}$, where $b_i > 0$ if node 0 is a in-neighbor of agent i, and $b_i = 0$ otherwise. The degree matrix is given by $D = \operatorname{diag}\{d_1, d_2, \dots, d_N\}$ with $d_i = \sum_{j=1}^{N} a_{ij}$. The Laplacian is defined as $L = D - \tilde{A}$, and then $H = L + \tilde{B}$.

Definition 1. Let $h = [h_0^{\mathrm{T}}, h_1^{\mathrm{T}}, \dots, h_N^{\mathrm{T}}]^{\mathrm{T}}$ be the desired leader-following formation. The MASs (1) is said to achieve the desired leader-following formation h if for $i = 1, \dots, N$, $\lim_{t\to\infty} \|(x_i(t) - h_i) - (x_0(t) - h_0)\| = 0$ holds with any given initial conditions.

The ETC strategy with distributed infinite input time delays is designed as

$$u_i(t) = -F \int\limits_0^{+\infty} k_i(\tau) q_i\bigl(t_k^i - \tau\bigr) + Q(h_i - h_0), \quad t \in \bigl[t_k^i, t_{k+1}^i\bigr) \tag{2}$$

in which $k_i(\tau) : [0, +\infty) \to [0, +\infty)$ is the delay kernel satisfying $\int_0^{+\infty} k_i(\tau)\, \mathrm{d}\tau = 1$, and $\int_0^{+\infty} k_i(\tau) \mathrm{e}^{\mu\tau}\, \mathrm{d}\tau < +\infty$ for $\mu > 0$. $F \in \mathcal{R}^{m\times n}$ is a control gain matrix, $Q \in \mathcal{R}^{m\times n}$

is also a control gain matrix that makes $(A + BQ)(h_i - h_0) = 0$ [6, 30].

$$q_i(t) = \sum_{j=1}^{N} a_{ij}\big(\big(x_i(t) - h_i\big) - \big(x_j(t) - h_j\big)\big) + b_i\big(\big(x_i(t) - h_i\big) - \big(x_0(t) - h_0\big)\big)$$

is the combining measurement information. t_k^i is called to be triggering time instant for each agent i, which is defined as follows:

$$t_{k+1}^i = \inf\big\{t_k^i > 0\colon\ f_i(t) > 0\big\}, \tag{3}$$

where

$$f_i(t) = \big\|e_i(t)\big\| - \beta_1\big\|q_i\big(t_k^i\big)\big\| - \beta_2 \mathrm{e}^{-\lambda(t-t_0)} \tag{4}$$

is the triggering function with $\beta_1 > 0$, $\beta_2 > 0$, $\lambda > 0$, and the measurement error $e_i(t) = q_i(t_k^i) - q_i(t)$. $e_i(t)$ is reset to 0 at $t = t_k^i$.

Let $\xi_i(t) = (x_i(t) - h_i) - (x_0(t) - h_0)$, $\xi(t) = [\xi_1^{\mathrm{T}}, \dots, \xi_N^{\mathrm{T}}]^{\mathrm{T}}$, and $e(t) = [e_1^{\mathrm{T}}(t), \dots, e_N^{\mathrm{T}}(t)]^{\mathrm{T}}$. In terms of (1) and (2), one has

$$\begin{aligned}
\dot{\xi}_i(t) &= \dot{x_i}(t) - \dot{x_0}(t) + Bu_i(t) \\
&= A\big((x_i(t) - h_i) - (x_0(t) - h_0)\big) + (A + BQ)(h_i - h_0) \\
&\quad - BF\int_0^{+\infty} k_i(\tau) q_i(t-\tau)\,\mathrm{d}\tau - BF\int_0^{+\infty} k_i(\tau) e_i(t-\tau)\,\mathrm{d}\tau \\
&= A\xi_i(t) - BF\int_0^{+\infty} k_i(\tau) q_i(t-\tau)\,\mathrm{d}\tau - BF\int_0^{+\infty} k_i(\tau) e_i(t-\tau)\,\mathrm{d}\tau \\
&= A\xi_i(t) - BF\int_0^{+\infty} k_i(\tau)\left(\sum_{j=1}^{N} a_{ij}\big(\xi_i(t) - \xi_j(t)\big) + b_i\xi_i(t)\right)\mathrm{d}\tau \\
&\quad - BF\int_0^{+\infty} k_i(\tau) e_i(t-\tau)\,\mathrm{d}\tau.
\end{aligned}$$

Then, according to Kronecker product of matrix [8], it holds

$$\begin{aligned}
\dot{\xi}(t) &= (I_N \otimes A)\xi(t) - \int_0^{+\infty} \big(\hat{H}(\tau) \otimes BF\big)\xi(t-\tau)\,\mathrm{d}\tau \\
&\quad - \int_0^{+\infty} \big(\hat{K}(\tau) \otimes BF\big)e(t-\tau)\,\mathrm{d}\tau
\end{aligned} \tag{5}$$

in which $\hat{K}(\tau) = \operatorname{diag}\{k_1(\tau), \dots, k_N(\tau)\}$ and $\hat{H}(\tau) = [\hat{h}_{ij}(\tau)]_{N\times N}$ with

$$\hat{h}_{ij}(\tau) = \begin{cases} (\sum_{j=1}^{N} a_{ij} + b_i)k_i(\tau), & i = j, \\ -a_{ij}k_i(\tau), & i \neq j. \end{cases}$$

Definition 2. The event-triggering time sequence $\{t_k^i\}$ is free of Zeno-behavior if for all i, $\inf_k\{t_{k+1}^i - t_k^i\} > 0$.

Assumption 1. The communication graph $\widetilde{G}$ has a spanning tree with the leader as the root.

Remark 1. (See [19].) Based on Assumption 1, $\operatorname{Re}\lambda_i(H) > 0$, $i = 1, \dots, N$. Obviously, a positive constant η can be found to make sure $\eta \operatorname{Re}\lambda_i(H) \geqslant 1$, $i = 1, \dots, N$.

Assumption 2. (A, B) is stabilization.

Remark 2. (See [32].) Based on Assumption 2, for any given positive definite matrix $M = M^{\mathrm{T}}$, a unique positive definite matrix $P = P^{\mathrm{T}}$ can be found to ensure the algebraic Riccati equation below holds:

$$PA + AP - PBB^{\mathrm{T}}P + M = 0. \tag{6}$$

As a result, according to linear optimal control theory, the eigenvalues $\lambda_i(A - BB^{\mathrm{T}}P)$ are positive.

Thus, owing to Assumptions 1 and 2, $\operatorname{Re}\lambda_i(A - \eta\lambda_i(H)BB^{\mathrm{T}}P) < 0$, $i = 1, \dots, N$. Furthermore, $\operatorname{Re}\lambda_i(I_N \otimes A - \eta H \otimes BB^{\mathrm{T}}P) < 0$. Hence, for $t \geqslant t_0$, we can take these positive constants W and ϖ satisfying

$$\left\| \mathrm{e}^{(I_N \otimes A - \eta H \otimes BB^{\mathrm{T}}P)(t-t_0)} \right\| \leqslant W\mathrm{e}^{-\varpi(t-t_0)}. \tag{7}$$

3 Main results

A sufficient condition on leader-following formation control of MASs (1) is obtained in this section, and the Zeno-behavior of $\{t_k^i\}$ is excluded for any agent.

In order to reading conveniently, let

$$v = \frac{\mu + \varpi + W\phi\|BF\| - \sqrt{(\mu + \varpi + W\phi\|BF\|)^2 - 4\mu\varpi}}{2},$$

$$y = \frac{(\mu - \lambda)(\varpi - \lambda) - W\phi\|BF\|\lambda}{(\mu - \lambda)(\varpi - \lambda) - W\phi\|BF\|\lambda + W\|H\|\sqrt{N}\|BF\|\psi(\mu - \lambda)},$$

$$\int_0^{+\infty} \left\|\hat{H}(\tau)\right\| \mathrm{e}^{\mu\tau}\,\mathrm{d}\tau = \phi \quad \text{and} \quad \int_0^{+\infty} \left\|\hat{K}(\tau)\right\| \mathrm{e}^{\mu\tau}\,\mathrm{d}\tau = \psi.$$

We have the following lemma.

Lemma 1. *A positive constant λ satisfying $0 < \lambda < v$ can be found to make sure $(\mu - \lambda)(\varpi - \lambda) - W\phi\|BF\|\lambda > 0$, $\lambda < \min(\mu, \varpi)$, and $y \in (0, 1)$.*

Proof. By the definition of v, for any $\lambda \in (0, v)$, it is clear that $\lambda^2 - (\mu + \varpi + W\phi\|BF\|)\lambda + \mu\varpi > 0$ and $\lambda < \min(\mu, \varpi)$. Thus, we have $(\mu - \lambda)(\varpi - \lambda) - W\phi\|BF\|\lambda > 0$ and then $0 < y < 1$. The proof is completed. □

Now, we are in the position to state the main results in this paper.

Theorem 1. *Consider the leader-following MASs* (1) *with event-triggered formation protocol* (2)*, where* $\{t_k^i\}$ *for each agent* i *is determined by* (3)*. The control gain matrix* $F = \eta B^{\mathrm{T}} P$ *with* P *satisfies* (6) *and* $\eta \geqslant 1/\operatorname{Re}\lambda_i(H)$*, and control gain matrix* Q *satisfies* $(A + BQ)(h_i - h_0) = 0$*. Then MASs* (1) *can achieve leader-following formation for* $0 < \lambda < v$*,* $\beta_1 \in (0, y)$*,* $\beta_2 > 0$*.*

Proof. From (5) it holds

$$\begin{aligned}\dot{\xi}(t) = {}& \left((I_N \otimes A) - \int_0^{+\infty} \big(\hat{H}(\tau) \otimes BF\big)\,\mathrm{d}\tau\right)\xi(t) \\ &+ \int_0^{+\infty} \big(\hat{H}(\tau) \otimes BF\big)\big(\xi(t) - \xi(t-\tau)\big)\,\mathrm{d}\tau - \int_0^{+\infty} \big(\hat{K}(\tau) \otimes BF\big) e(t-\tau)\,\mathrm{d}\tau. \qquad (8)\end{aligned}$$

Since $\int_0^{+\infty} k_i(\tau)\,\mathrm{d}\tau = 1$, we have $\sum_{j=1}^N \int_0^{+\infty} a_{ij} k_i(\tau)\,\mathrm{d}\tau = \sum_{j=1}^N a_{ij} = d_i$ and $\int_0^{+\infty} b_i k_i(\tau)\,\mathrm{d}\tau = b_i \int_0^{+\infty} k_i(\tau)\,\mathrm{d}\tau = b_i$. Therefore, $\int_0^{+\infty} \hat{H}(\tau)\,\mathrm{d}\tau = D - \tilde{A} + \tilde{B} = H$.

Consequently, (8) becomes

$$\begin{aligned}\dot{\xi}(t) = \mathcal{W}\xi(t) &+ \int_0^{+\infty} \big(\hat{H}(\tau) \otimes BF\big)\big(\xi(t) - \xi(t-\tau)\big)\,\mathrm{d}\tau \\ &- \int_0^{+\infty} \big(\hat{K}(\tau) \otimes BF\big) e(t-\tau)\,\mathrm{d}\tau,\end{aligned}$$

where $\mathcal{W} = I_N \otimes A - H \otimes BF$.

Utilizing the variation of parameter formula, we get

$$\begin{aligned}\xi(t) = {}& \mathrm{e}^{\mathcal{W}(t-t_0)}\xi(t_0) + \int_{t_0}^{t}\int_0^{+\infty} \mathrm{e}^{\mathcal{W}(t-s)}\big(\hat{H}(\tau) \otimes BF\big)\big(\xi(s) - \xi(s-\tau)\big)\,\mathrm{d}\tau\,\mathrm{d}s \\ &- \int_{t_0}^{t}\int_0^{+\infty} \mathrm{e}^{\mathcal{W}(t-s)}\big(\hat{K}(\tau) \otimes BF\big) e(s-\tau)\,\mathrm{d}\tau\,\mathrm{d}s.\end{aligned}$$

Recalling (7) yields

$$\begin{aligned}\|\xi(t)\| \leqslant {}& W\mathrm{e}^{-\varpi(t-t_0)}\big\|\xi(t_0)\big\| \\ &+ W\int_{t_0}^{t}\int_0^{+\infty} \mathrm{e}^{-\varpi(t-s)}\big\|\hat{H}(\tau) \otimes BF\big\|\big\|\xi(s) - \xi(s-\tau)\big\|\,\mathrm{d}\tau\,\mathrm{d}s \\ &+ W\int_{t_0}^{t}\int_0^{+\infty} \mathrm{e}^{-\varpi(t-s)}\big\|\hat{K}(\tau) \otimes BF\big\|\big\|e(s-\tau)\big\|\,\mathrm{d}\tau\,\mathrm{d}s. \qquad (9)\end{aligned}$$

The following inequality is enforced by triggering condition (4)

$$\begin{aligned}
\|e_i(t)\| &\leqslant \beta_1 \|q_i(t_k^i)\| + \beta_2 \mathrm{e}^{-\lambda(t-t_0)} = \beta_1 \|q_i(t) + e_i(t)\| + \beta_2 \mathrm{e}^{-\lambda(t-t_0)} \\
&\leqslant \beta_1 \Bigg\| \sum_{j=1}^{N} a_{ij}\big(\big(x_i(t) - h_i\big) - \big(x_j(t) - h_j\big)\big) \\
&\quad + b_i\big(\big(x_i(t) - h_i\big) - \big(x_0(t) - h_0\big)\big) \Bigg\| + \beta_1 \|e_i(t)\| + \beta_2 \mathrm{e}^{-\lambda(t-t_0)} \\
&\leqslant \beta_1 \|H\| \|\xi(t)\| + \beta_1 \|e_i(t)\| + \beta_2 \mathrm{e}^{-\lambda(t-t_0)}.
\end{aligned}$$

Thus,

$$\|e(t)\| \leqslant \frac{\beta_1 \|H\| \sqrt{N}}{1 - \beta_1} \|\xi(t)\| + \frac{\beta_2 \sqrt{N}}{1 - \beta_1} \mathrm{e}^{-\lambda(t-t_0)}. \tag{10}$$

Substituting (10) into (9), we have

$$\begin{aligned}
\|\xi(t)\| &\leqslant W \mathrm{e}^{-\varpi(t-t_0)} \|\xi(t_0)\| \\
&\quad + W \int_{t_0}^{t} \int_{0}^{+\infty} \mathrm{e}^{-\varpi(t-s)} \|\hat{H}(\tau) \otimes BF\| \|\xi(s) - \xi(s-\tau)\| \,\mathrm{d}\tau \,\mathrm{d}s \\
&\quad + \frac{W\beta_1 \|H\| \sqrt{N}}{1 - \beta_1} \int_{t_0}^{t} \int_{0}^{+\infty} \mathrm{e}^{-\varpi(t-s)} \|\hat{K}(\tau) \otimes BF\| \|\xi(s-\tau)\| \,\mathrm{d}\tau \,\mathrm{d}s \\
&\quad + \frac{W\beta_2 \sqrt{N}}{1 - \beta_1} \int_{t_0}^{t} \int_{0}^{+\infty} \mathrm{e}^{-\varpi(t-s)} \|\hat{K}(\tau) \otimes BF\| \mathrm{e}^{-\lambda(s-t_0-\tau)} \,\mathrm{d}\tau \,\mathrm{d}s \\
&= W \mathrm{e}^{-\varpi(t-t_0)} \|\xi(t_0)\| \\
&\quad + W\|BF\| \int_{t_0}^{t} \int_{0}^{+\infty} \mathrm{e}^{-\varpi(t-s)} \|\hat{H}(\tau)\| \|\xi(s) - \xi(s-\tau)\| \,\mathrm{d}\tau \,\mathrm{d}s \\
&\quad + \frac{W\|BF\|\beta_1 \|H\| \sqrt{N}}{1 - \beta_1} \int_{t_0}^{t} \int_{0}^{+\infty} \mathrm{e}^{-\varpi(t-s)} \|\hat{K}(\tau)\| \|\xi(s-\tau)\| \,\mathrm{d}\tau \,\mathrm{d}s \\
&\quad + \frac{W\|BF\|\beta_2 \sqrt{N}}{1 - \beta_1} \int_{t_0}^{t} \int_{0}^{+\infty} \mathrm{e}^{-\varpi(t-s)} \|\hat{K}(\tau)\| \mathrm{e}^{-\lambda(s-t_0-\tau)} \,\mathrm{d}\tau \,\mathrm{d}s.
\end{aligned} \tag{11}$$

Denote

$$r = \frac{W\beta_2 \sqrt{N} \|BF\| \psi(\mu-\lambda)}{(1-\beta_1)(\mu-\lambda)(\varpi-\lambda) - W\big(\beta_1 \|H\| \sqrt{N} \|BF\| \psi(\mu-\lambda) + \lambda\phi \|BF\| (1-\beta_1)\big)}.$$

Since $\beta_1 \in (0, y)$, one has

$$(1-\beta_1)(\mu-\lambda)(\varpi-\lambda) - W\big(\beta_1\|H\|\sqrt{N}\|BF\|\psi(\mu-\lambda)+\lambda\phi\|BF\|(1-\beta_1)\big) > 0.$$

Moreover, due to the fact that $\beta_2 > 0$ and $\lambda < \mu$, we have $r > 0$.

Denote $Z = \max\{r, W\|\xi(t_0)\|\}$. Next, for any $\rho > 1$, we will prove that

$$\big\|\xi(t)\big\| < \rho Z \mathrm{e}^{-\lambda(t-t_0)} \triangleq \zeta(t), \quad t \geqslant t_0. \tag{12}$$

Otherwise, we can find $t^* > t_0$ satisfying $\|\xi(t^*)\| = \zeta(t^*)$ and $\|\xi(t)\| < \zeta(t)$ when $t \in [t_0, t^*)$.

Subsequently, from (11) one has

$$\begin{aligned}
\zeta(t^*) &= \big\|\xi(t^*)\big\| \\
&< \rho W \mathrm{e}^{-\varpi(t^*-t_0)}\big\|\xi(t_0)\big\| \\
&\quad + \rho W Z\|BF\| \int_{t_0}^{t^*}\int_0^{+\infty} \mathrm{e}^{-\varpi(t^*-s)}\big(\mathrm{e}^{-\lambda(s-t_0-\tau)} - \mathrm{e}^{-\lambda(s-t_0)}\big)\big\|\hat{H}(\tau)\big\|\,\mathrm{d}\tau\,\mathrm{d}s \\
&\quad + \frac{Z\beta_1\|H\|+\beta_2}{1-\beta_1}\rho W\sqrt{N}\|BF\| \int_{t_0}^{t^*}\int_0^{+\infty} \mathrm{e}^{-\varpi(t^*-s)}\mathrm{e}^{-\lambda(s-t_0-\tau)}\big\|\hat{K}(\tau)\big\|\,\mathrm{d}\tau\,\mathrm{d}s \\
&= \rho W \mathrm{e}^{-\varpi(t^*-t_0)}\big\|\xi(t_0)\big\| + \rho W Z\|BF\| \int_{t_0}^{t^*} \mathrm{e}^{-\varpi(t^*-s)}\mathrm{e}^{-\lambda(s-t_0)}\,\mathrm{d}s \\
&\quad \times \int_0^{+\infty} \big\|\hat{H}(\tau)\big\|\mathrm{e}^{\mu\tau}\big(\mathrm{e}^{(\lambda-\mu)\tau} - \mathrm{e}^{-\mu\tau}\big)\,\mathrm{d}\tau \\
&\quad + \frac{Z\beta_1\|H\|+\beta_2}{1-\beta_1}\rho W\sqrt{N}\|BF\| \int_{t_0}^{t^*} \mathrm{e}^{-\varpi(t^*-s)}\mathrm{e}^{-\lambda(s-t_0)}\,\mathrm{d}s \\
&\quad \times \int_0^{+\infty} \big\|\hat{K}(\tau)\big\|\mathrm{e}^{\mu\tau}\mathrm{e}^{(\lambda-\mu)\tau}\,\mathrm{d}\tau \\
&\leqslant \rho W \mathrm{e}^{-\varpi(t^*-t_0)}\big\|\xi(t_0)\big\| + \rho W Z\|BF\| \int_{t_0}^{t^*} \mathrm{e}^{-\varpi(t^*-s)}\mathrm{e}^{-\lambda(s-t_0)}\,\mathrm{d}s \\
&\quad \times \int_0^{+\infty} \big\|\hat{H}(\tau)\big\|\mathrm{e}^{\mu\tau}\,\mathrm{d}\tau \cdot \sup_{\tau\geqslant 0}\big(\mathrm{e}^{(\lambda-\mu)\tau} - \mathrm{e}^{-\mu\tau}\big)
\end{aligned}$$

$$+\frac{Z\beta_1\|H\|+\beta_2}{1-\beta_1}\rho W\sqrt{N}\|BF\|\int\limits_{t_0}^{t^*}\mathrm{e}^{-\varpi(t^*-s)}\mathrm{e}^{-\lambda(s-t_0)}\,\mathrm{d}s$$
$$\times\int\limits_0^{+\infty}\big\|\hat{K}(\tau)\big\|\mathrm{e}^{\mu\tau}\,\mathrm{d}\tau\cdot\sup_{\tau\geqslant 0}\mathrm{e}^{(\lambda-\mu)\tau}.$$

Since $\sup_{\tau\geqslant 0}(\mathrm{e}^{(-\mu+\lambda)\tau}-\mathrm{e}^{-\mu\tau})<\lambda/(\mu-\lambda)$ [27] and $\sup_{\tau\geqslant 0}(\mathrm{e}^{(\lambda-\mu)\tau})=1$, one has

$$\begin{aligned}\zeta(t^*)&<\rho W\mathrm{e}^{-\varpi(t^*-t_0)}\big\|\xi(t_0)\big\|\\&\quad+\rho WZ\|BF\|\frac{\lambda\phi}{\mu-\lambda}\frac{1}{\varpi-\lambda}\big(\mathrm{e}^{-\lambda(t^*-t_0)}-\mathrm{e}^{-\varpi(t^*-t_0)}\big)\\&\quad+\frac{Z\beta_1\|H\|+\beta_2}{1-\beta_1}\rho W\sqrt{N}\|BF\|\frac{\psi}{\varpi-\lambda}\big(\mathrm{e}^{-\lambda(t^*-t_0)}-\mathrm{e}^{-\varpi(t^*-t_0)}\big)\\&=\rho\bigg\{W\big\|\xi(t_0)\big\|-\bigg(WZ\bigg(\frac{\beta_1\|H\|\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}+\frac{\lambda\phi}{\mu-\lambda}\frac{\|BF\|}{\varpi-\lambda}\bigg)\\&\quad+\frac{W\beta_2\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}\bigg)\bigg\}\mathrm{e}^{-\varpi(t^*-t_0)}\\&\quad+\rho\bigg(WZ\bigg(\frac{\beta_1\|H\|\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}+\frac{\lambda\phi}{\mu-\lambda}\frac{\|BF\|}{\varpi-\lambda}\bigg)\\&\quad+\frac{W\beta_2\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}\bigg)\mathrm{e}^{-\lambda(t^*-t_0)}.\end{aligned}$$

Case 1. $Z=r$, which indicates that

$$W\big\|\xi(t_0)\big\|\leqslant WZ\bigg(\frac{\beta_1\|H\|\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}+\frac{\lambda\phi}{\mu-\lambda}\frac{\|BF\|}{\varpi-\lambda}\bigg)+\frac{W\beta_2\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}.$$

Then one has

$$\begin{aligned}\zeta(t^*)&<\rho\bigg(WZ\bigg(\frac{\beta_1\|H\|\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}+\frac{\lambda\phi}{\mu-\lambda}\frac{\|BF\|}{\varpi-\lambda}\bigg)+\frac{W\beta_2\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}\bigg)\\&\quad\times\mathrm{e}^{-\lambda(t^*-t_0)}\\&=\zeta(t^*).\end{aligned}\tag{13}$$

Case 2. $Z=W\|\xi(t_0)\|$, which indicates that

$$W\big\|\xi(t_0)\big\|\geqslant WZ\bigg(\frac{\beta_1\|H\|\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}+\frac{\lambda\phi}{\mu-\lambda}\frac{\|BF\|}{\varpi-\lambda}\bigg)+\frac{W\beta_2\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}.$$

Then one has

$$\begin{aligned}
\zeta(t^*) &< \rho\biggl\{W\bigl\|\xi(t_0)\bigr\| - \biggl(WZ\biggl(\frac{\beta_1\|H\|\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)} + \frac{\lambda\phi}{\mu-\lambda}\frac{\|BF\|}{\varpi-\lambda}\biggr) \\
&\quad + \frac{W\beta_2\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}\biggr)\biggr\}\mathrm{e}^{-\lambda(t^*-t_0)} \\
&\quad + \rho\biggl(WZ(\frac{\beta_1\|H\|\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)} + \frac{\lambda\phi}{\mu-\lambda}\frac{\|BF\|}{\varpi-\lambda}\biggr) \\
&\quad + \frac{W\beta_2\sqrt{N}\|BF\|\psi}{(1-\beta_1)(\varpi-\lambda)}\biggr)\mathrm{e}^{-\lambda(t^*-t_0)} \\
&= \rho W\bigl\|\xi(t_0)\bigr\|\mathrm{e}^{-\lambda(t^*-t_0)} = \zeta(t^*).
\end{aligned} \tag{14}$$

The contradictions in (13) and (14) show that inequality (12) is valid for any $\rho > 1$. Letting $\rho \to 1$, the inequality below can be acquired:

$$\bigl\|\xi(t)\bigr\| \leqslant Z\mathrm{e}^{-\lambda(t-t_0)}, \quad t \geqslant t_0, \tag{15}$$

which indicates that the leader-following formation is reached and the convergence rate can be estimated by λ.

The proof is completed. □

Theorem 2. *With the same conditions in Theorem* 1*, there does not exist Zeno-behavior for triggering time sequence* $\{t_k^i\}$.

Proof. Denote

$$\chi_i(t) = u_i(t) - Q(h_i - h_0) = -F\int_0^{+\infty} k_i(\tau)q_i\bigl(t_k^i - \tau\bigr)\,\mathrm{d}\tau, \quad t \in [t_k^i, t_{k+1}^i).$$

Thus,

$$\chi(t) = -\int_0^{+\infty}\bigl(\hat{H}(\tau)\otimes F\bigr)\xi(t-\tau)\,\mathrm{d}\tau - \int_0^{+\infty}\bigl(\hat{K}(\tau)\otimes F\bigr)e(t-\tau)\,\mathrm{d}\tau. \tag{16}$$

For $t \in [t_k^i, t_{k+1}^i)$, computing the upper right-hand Dini derivative of $\|e_i(t)\|$, we get

$$\begin{aligned}
&D^+\bigl\|e_i(t)\bigr\| \\
&\quad \leqslant \bigl\|\dot{e}_i(t)\bigr\| = \bigl\|\dot{q}_i(t)\bigr\| = \Biggl\|\sum_{j=1}^N a_{ij}\bigl(\dot{x}_i(t) - \dot{x}_j(t)\bigr) + b_i\bigl(\dot{x}_i(t) - \dot{x}_0(t)\bigr)\Biggr\| \\
&\quad = \Biggl\|\sum_{j=1}^N a_{ij}\bigl(Ax_i(t) + Bu_i(t) - Ax_j(t) - Bu_j(t)\bigr) + b_i\bigl(Ax_i(t) + Bu_i(t) - Ax_0(t)\bigr)\Biggr\|
\end{aligned}$$

$$
\begin{aligned}
&= \Bigg\| \sum_{j=1}^{N} a_{ij}\big(A((x_i-h_i)-(x_0-h_0)\big)+(A+BQ)(h_i-h_0)+B\chi_i(t) \\
&\quad - A\big((x_j-h_j)-(x_0-h_0)\big)-(A+BQ)(h_j-h_0)-B\chi_j(t)\big) \\
&\quad + b_i\big(A((x_i-h_i)-(x_0-h_0))+(A+BQ)(h_i-h_0)+B\chi_i(t)\big)\Bigg\| \\
&= \Bigg\| \sum_{j=1}^{N} a_{ij}\big(A(\xi_i(t)-\xi_j(t))+B(\chi_i(t)-\chi_j(t))\big)+b_i\big(A\xi_i(t)+B\chi_i(t)\big)\Bigg\| \\
&\leqslant \|H\otimes A\|\big\|\xi(t)\big\|+\|H\otimes B\|\big\|\chi(t)\big\|.
\end{aligned}
\tag{17}
$$

Substituting (16) into (17) yields

$$
\begin{aligned}
D^{+}\big\|e_i(t)\big\| \leqslant{}& \|H\otimes A\|\big\|\xi(t)\big\|+\|H\otimes B\|\int_0^{+\infty}\big\|\hat{H}(\tau)\otimes F\big\|\big\|\xi(t-\tau)\big\|\,\mathrm{d}\tau \\
&+\|H\otimes B\|\int_0^{+\infty}\big\|\hat{K}(\tau)\otimes F\big\|\big\|e(t-\tau)\big\|\,\mathrm{d}\tau.
\end{aligned}
$$

Let $\varrho=\|H\otimes B\|\|F\|\sqrt{N}/(1-\beta_1)$. Using (10) and (15), it holds

$$
\begin{aligned}
D^{+}\big\|e_i(t)\big\| \leqslant{}& \|H\otimes A\|\big\|\xi(t)\big\|+\|H\otimes B\|\|F\|\int_0^{+\infty}\big\|\hat{H}(\tau)\big\|\big\|\xi(t-\tau)\big\|\,\mathrm{d}\tau \\
&+\varrho\beta_1\|H\|\int_0^{+\infty}\big\|\hat{K}(\tau)\big\|\big\|\xi(t-\tau)\big\|\,\mathrm{d}\tau+\varrho\beta_2\int_0^{+\infty}\big\|\hat{K}(\tau)\big\|\mathrm{e}^{-\lambda(t-t_0-\tau)}\,\mathrm{d}\tau \\
\leqslant{}& \|H\otimes A\|Z\mathrm{e}^{-\lambda(t-t_0)} \\
&+\|H\otimes B\|\|F\|Z\mathrm{e}^{-\lambda(t-t_0)}\int_0^{+\infty}\big\|\hat{H}(\tau)\big\|\mathrm{e}^{\mu\tau}\,\mathrm{d}\tau\cdot\sup_{\tau\geqslant 0}\big(\mathrm{e}^{(\lambda-\mu)\tau}\big) \\
&+\varrho\beta_1\|H\|Z\mathrm{e}^{-\lambda(t-t_0)}\int_0^{+\infty}\big\|\hat{K}(\tau)\big\|\mathrm{e}^{\mu\tau}\,\mathrm{d}\tau\cdot\sup_{\tau\geqslant 0}\big(\mathrm{e}^{(\lambda-\mu)\tau}\big) \\
&+\varrho\beta_2\mathrm{e}^{-\lambda(t-t_0)}\int_0^{+\infty}\big\|\hat{K}(\tau)\big\|\mathrm{e}^{\mu\tau}\,\mathrm{d}\tau\cdot\sup_{\tau\geqslant 0}\big(\mathrm{e}^{(\lambda-\mu)\tau}\big) \\
={}& \varXi\mathrm{e}^{-\lambda(t-t_0)},
\end{aligned}
\tag{18}
$$

where $\varXi=\|H\otimes A\|Z+\|H\otimes B\|\|F\|Z\phi+\varrho\beta_1\|H\|Z\psi+\varrho\beta_2\psi$.

By (18) and $e_i(t_k^i) = 0$ one has

$$\|e_i(t)\| \leqslant \frac{\Xi}{\lambda}\left(\mathrm{e}^{-\lambda(t_k^i - t_0)} - \mathrm{e}^{-\lambda(t-t_0)}\right).$$

When the triggering function (4) crosses zero, the next event will be triggered, i.e.,

$$\beta_1\|q_i(t_k^i)\| + \beta_2\mathrm{e}^{-\lambda(t_{k+1}^i - t_0)} = \|e_i(t_{k+1}^i)\| \leqslant \frac{\Xi}{\lambda}\left(\mathrm{e}^{-\lambda(t_k^i - t_0)} - \mathrm{e}^{-\lambda(t_{k+1}^i - t_0)}\right).$$

Hence,

$$\beta_2\mathrm{e}^{-\lambda(t_{k+1}^i - t_0)} \leqslant \frac{\Xi}{\lambda}\left(\mathrm{e}^{-\lambda(t_k^i - t_0)} - \mathrm{e}^{-\lambda(t_{k+1}^i - t_0)}\right).$$

Letting $T_k^i = t_{k+1}^i - t_k^i$, we can obtain that

$$\beta_2\mathrm{e}^{-\lambda T_k^i} \leqslant \frac{\Xi}{\lambda}\left(1 - \mathrm{e}^{-\lambda T_k^i}\right),$$

that is,

$$\mathrm{e}^{-\lambda T_k^i} \leqslant \frac{\frac{\Xi}{\lambda}}{\beta_2 + \frac{\Xi}{\lambda}} < 1,$$

and then

$$T_k^i \geqslant -\frac{1}{\lambda}\ln\frac{\frac{\Xi}{\lambda}}{\beta_2 + \frac{\Xi}{\lambda}} > 0. \tag{19}$$

From (19) one can derive that for any agent i, $\inf_k\{T_k^i\} > 0$, which means that there does not exist Zeno-behavior for any agent.

The proof is completed. □

4 Simulation results

The theoretical results are illustrated through two numerical examples. The simulation is performed by MATLAB software, and the pseudocode of the event-triggered formation control strategy is shown as follows:

```
Input: The initial states x_0(t_0), x_i(t_0), the desired formation h,
       the delay kernels k_i(τ), other parameters, like event-
       triggered parameters β_1, β_2 and the control matrixes F, Q.
Output: The error of states, the event-triggering instants, and
        the evolvement of agents.
 BEGIN
       t_0 = 0;
         for t = 0 : ħ : T − ħ do
           Update the leader's state x_0(t) based on (1);
           Update the followers's state x_i(t) based on (1) under
           control input (2);
```

```
        for i = 1 : N do
          Compute the combined error e_i(t) and the combined
          state q_i(t_k^i);
          If ||e_i(t)|| > β_1||q_i(t_k^i)|| + β_2 e^{-λt} then
            Record the instant as t_{k+1}^i and update the state
            x_i(t) at t_{k+1}^i;
          Else
            The event has not occurred, and update the state
            x_i(t) at instant t ∈ [t_k^i, t_{k+1}^i);
        End
      End
END
```

Example 1. Suppose MASs consist of five follower agents with one leader. Let the connectivity weights $a_{12} = a_{14} = a_{21} = a_{25} = a_{32} = a_{45} = a_{53}$, $b_4 = b_5 = 1$. The other weights are all equal to zero. Obviously, Assumption 1 is satisfied, and

$$H = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}.$$

The delay kernels are given as: $k_1(\tau) = \mathrm{e}^{-\tau}$, $k_2(\tau) = \mathrm{e}^{-\tau}$, $k_3(\tau) = 2\mathrm{e}^{-2\tau}$, $k_4(\tau) = 2\mathrm{e}^{-2\tau}$, $k_5(\tau) = 5\mathrm{e}^{-5\tau}$, which satisfy $\int_0^{+\infty} k_i(\tau)\,\mathrm{d}\tau = 1$ and $\int_0^{+\infty} k_i(\tau)\mathrm{e}^{\mu\tau}\,\mathrm{d}\tau < +\infty$ by taking $\mu < 1$, $i = 1, \dots, 5$. Furthermore, one can compute that $\phi = 3.7521$ and $\psi = 1.1111$ by choosing $\mu = 0.1$.

Assume that

$$A = \begin{bmatrix} -0.5 & -0.1 & 0.2 \\ -0.1 & -0.4 & 0.1 \\ -0.1 & -0.2 & -0.6 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.1 & 0.02 & 0.02 \\ 0.3 & 0.4 & 0.5 \\ 0.6 & -0.5 & -0.4 \end{bmatrix}.$$

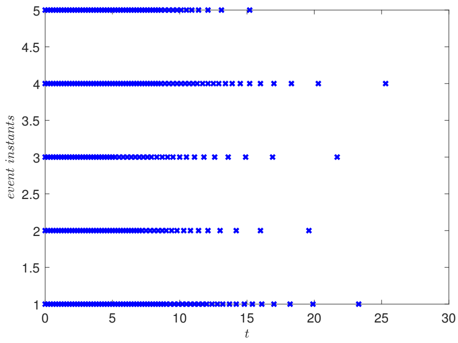
One can verify that (A, B) is stabilization. Given

$$Q = \begin{bmatrix} 5.0000 & 10.3125 & -30.6250 \\ 0.0000 & -0.8958 & 4.4583 \\ 0.0000 & -1.1042 & 2.5417 \end{bmatrix},$$

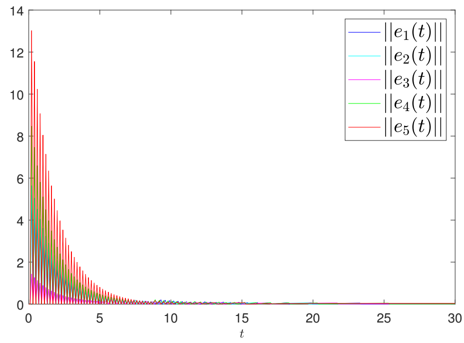
we can get $A + BQ = 0$. Let the desired formation $h = [h_0^{\mathrm{T}}, h_1^{\mathrm{T}}, \dots, h_5^{\mathrm{T}}]^{\mathrm{T}}$, where $h_0 = [50, 20, 80]^{\mathrm{T}}$, $h_1 = [30, 30, 10]^{\mathrm{T}}$, $h_2 = [60, 30, 10]^{\mathrm{T}}$, $h_3 = [70, 20, 10]^{\mathrm{T}}$, $h_4 = [50, 10, 10]^{\mathrm{T}}$, $h_5 = [10, 20, 10]^{\mathrm{T}}$.

Letting $M = 0.001\,\mathrm{diag}\{1, 1, 1\}$, by solving the Riccati equation (6) we have

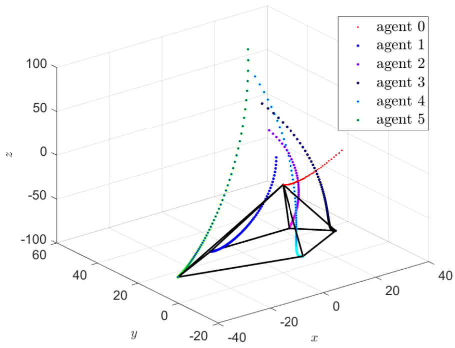
$$P = \begin{bmatrix} 0.0010 & -0.0003 & 0.0001 \\ -0.0003 & 0.0014 & -0.0001 \\ 0.0001 & -0.0001 & 0.0008 \end{bmatrix},$$



(a) Events for followers



(b) Error



(c) Agents converge to a desired formation

Figure 1. Simulation results.

$$F = \eta B^{\mathrm{T}} P = \begin{bmatrix} 0.0003 & 0.0000 & 0.0001 \\ 0.0008 & 0.0014 & 0.0013 \\ 0.0024 & -0.0028 & -0.0007 \end{bmatrix},$$

where

$$\eta = \frac{1}{\min \operatorname{Re}(\lambda_i(H))} = 3.3887.$$

Hence, one has $W = 27.7318$, $\varpi = 0.3337$ according to (7). Then $\upsilon = 0.0312$ and $y = 0.0112$ by choosing $\lambda = 0.03 \in (0, \upsilon)$. Then by Theorem 1 the leader-following formation can be reached for $\beta_1 \in (0, y)$ and $\beta_2 > 0$.

Let $\beta_1 = 0.01$ and $\beta_2 = 0.1$. The initial conditions are set as $x_0(t_0) = [30, 10, 1]^{\mathrm{T}}$, $x_i(t_0) = 5i \cdot [1, 2, 3]^{\mathrm{T}}$, $i = 1, 2, 3, 4, 5$. The simulation results are depicted in Fig. 1.

Event-triggering instants for each follower are shown in Fig. 1(a), which demonstrates that the frequency of controller update is greatly reduced and there is no Zeno-behavior. As described in Fig. 1(b), $\|e_i\|$ for followers all converge to 0. The evolution of all agents is shown in Fig. 1(c), which indicates the accomplishment of the desired formation.

Example 2. Suppose MASs include one leader and six followers of nonholonomic vehicles of unicycle type. For $i = 0, 1, 2, \dots, 6$, each vehicle has the follwing kinematic model:

$$\begin{aligned}\dot{\bar{x}}_i &= \nu_i \cos\vartheta_i,\\ \dot{\bar{y}}_i &= \nu_i \sin\vartheta_i,\\ \dot{\vartheta}_i &= \omega_i,\end{aligned}$$

where $[\bar{x}_i, \bar{y}_i]^{\mathrm{T}}$ denotes the Cartesian coordinates of the center of mass, ϑ_i represents the heading angle in the inertial frame. ν_i and ω_i denote the linear velocity and angular velocity, respectively.

By linearizing the kinematic model for each vehicle [1,22] the leader-following MASs can be described by (1) with the following system parameter matrices [6, 15, 30]:

$$A = \begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}, \qquad B = \begin{bmatrix}0&0\\0&1\\0&0\\0&1\end{bmatrix}.$$

One can verify that (A, B) is stabilization.

Let the connectivity weights $a_{12} = a_{16} = a_{25} = a_{31} = a_{34} = a_{36} = a_{41} = a_{43} = a_{54} = a_{56} = a_{61} = a_{63}$, $b_1 = b_2 = b_6 = 1$. The other weights are all equal to zero. Obviously, Assumption 1 is satisfied, and

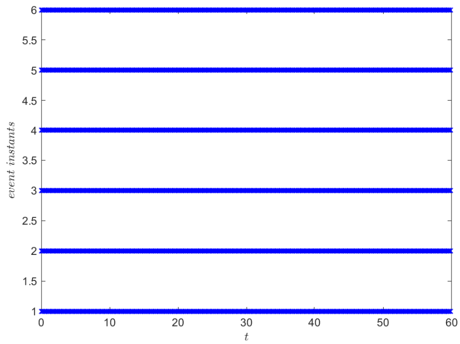
$$H = \begin{bmatrix}3&-1&0&0&0&-1\\0&2&0&0&-1&0\\-1&0&3&-1&0&-1\\-1&0&-1&2&0&0\\0&0&0&-1&2&-1\\-1&0&-1&0&0&3\end{bmatrix}.$$

The delay kernels are given as: $k_1(\tau) = \mathrm{e}^{-\tau}$, $k_2(\tau) = \mathrm{e}^{-\tau}$, $k_3(\tau) = 2\mathrm{e}^{-2\tau}$, $k_4(\tau) = 2\mathrm{e}^{-2\tau}$, $k_5(\tau) = 5\mathrm{e}^{-5\tau}$, $k_6(\tau) = 2\mathrm{e}^{-2\tau}$, which satisfy $\int_0^{+\infty} k_i(\tau)\,\mathrm{d}\tau = 1$ and $\int_0^{+\infty} k_i(\tau)\mathrm{e}^{\mu\tau}\,\mathrm{d}\tau < +\infty$ by taking $\mu < 1$, $i = 1, \dots, 6$. Furthermore, $\phi = 4.8772$ and $\psi = 1.1111$ by choosing $\mu = 0.1$.

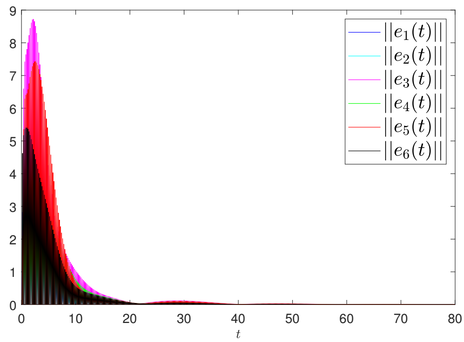
The desired formation is described by $h = [h_0^{\mathrm{T}}, h_1^{\mathrm{T}}, \dots, h_6^{\mathrm{T}}]^{\mathrm{T}}$, where $h_0 = [0, 5, 0, 1]^{\mathrm{T}}$, $h_1 = [-10\sqrt{3}, 5, 30, 1]^{\mathrm{T}}$, $h_2 = [10\sqrt{3}, 5, 30, 1]^{\mathrm{T}}$, $h_3 = [20\sqrt{3}, 5, 0, 1]^{\mathrm{T}}$, $h_4 = [10\sqrt{3}, 5, -30, 1]^{\mathrm{T}}$, $h_5 = [-10\sqrt{3}, 5, -30, 1]^{\mathrm{T}}$, $h_6 = [-20\sqrt{3}, 5, 0, 1]^{\mathrm{T}}$. It is clear that $A(h_i - h_0) = 0$ for any i, thus, Q can be chosen as 0.

Letting $M = 0.01\,\mathrm{diag}\{1, 1, 1, 1\}$, by solving the Riccati equation (6) we have

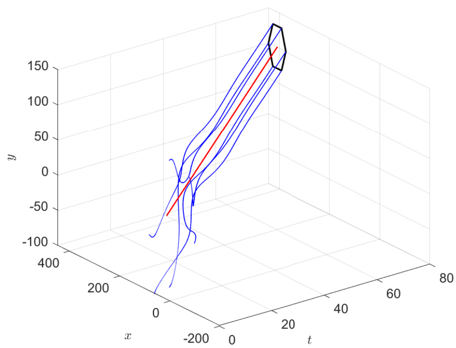
$$P = \begin{bmatrix}0.0458&0.1000&0.0000&0.0000\\0.1000&0.4583&0.0000&0.0000\\0.0000&0.0000&0.0458&0.1000\\0.0000&0.0000&0.1000&0.4583\end{bmatrix},$$



(a) Events for followers



(b) Error



(c) Agents converge to a desired formation

Figure 2. Simulation results.

and then

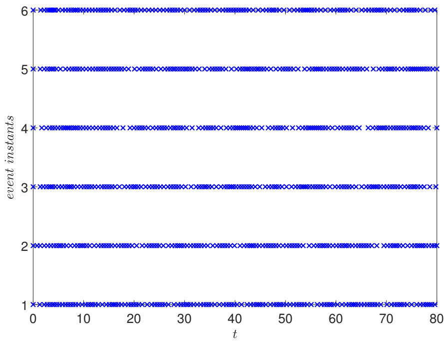
$$F = \eta B^{\mathrm{T}} P = \begin{bmatrix} 0.1779 & 0.8115 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1779 & 0.8115 \end{bmatrix},$$

where

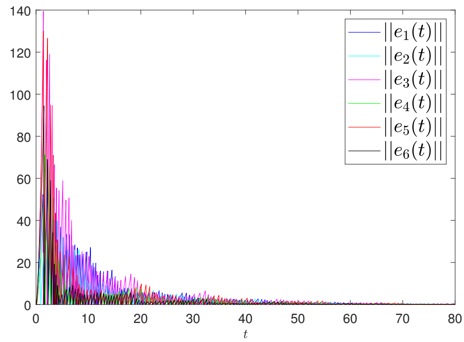
$$\eta = \frac{1}{\min(\operatorname{Re}\lambda_i(H))} = 1.7793.$$

Hence, one has $W = 61.317$, $\varpi = 0.229$ according to (7). Then $v = 9.3818 \cdot 10^{-5}$ and $y = 1.6376 \cdot 10^{-5}$ by choosing $\lambda = 9.0 \cdot 10^{-5} \in (0, v)$. Then by Theorem 1 the leader-following formation can be reached for $\beta_1 \in (0, y)$ and $\beta_2 > 0$.

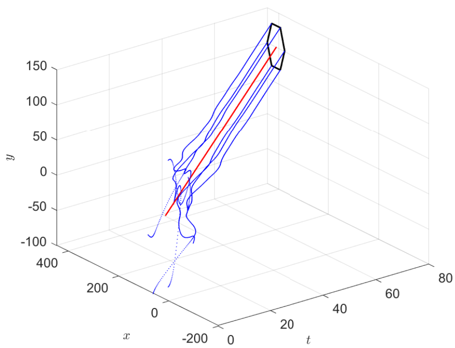
Let $\beta_1 = 1.6 \cdot 10^{-5}$ and $\beta_2 = 1.0 \cdot 10^{-4}$. The initial conditions are set as $x_0(t_0) = [30, 5, 50, 1]^{\mathrm{T}}$, $x_1(t_0) = [0, -3, 100, 3]^{\mathrm{T}}$, $x_2(t_0) = [80, -4, -20, -6]^{\mathrm{T}}$, $x_3(t_0) = [0, -3.5, -80, 6]^{\mathrm{T}}$, $x_4(t_0) = [-100, 2.5, 0, 7]^{\mathrm{T}}$, $x_5(t_0) = [60, 2, -100, 8]^{\mathrm{T}}$, $x_6(t_0) = [-80, 1, 90, -12]^{\mathrm{T}}$. The simulation results are depicted in Fig. 2. Event-triggering instants for each follower are shown in (a), which demonstrates that the frequency of controller update is reduced and the Zeno-behavior is excluded. (b) shows the evolution of $\|e_i\|$ for followers. As described in (c) of the path of all agents, the desired formation is achieved.



(a) Events for followers



(b) Error



(c) Agents converge to a desired formation

Figure 3. Simulation results.

However, we find that the number of triggering time instants for MASs is very large if $\lambda = 9.0 \cdot 10^{-5}$ and $\beta_1 \in (0, 1.6376 \cdot 10^{-5})$ by the above calculation. In fact, the desired formation can still be reached for some large values of β_1 and λ. For example, choosing $\lambda = 0.1$, $\beta_1 = 0.3$, and $\beta_2 = 1$, the simulation results are shown in Fig. 3, from which we can also obtain the desired formation and exclude Zeno-behavior. Thus, it should be noted that $0 < \lambda < v$ and $\beta_1 \in (0, y)$ is only a sufficient condition in Theorem 1. How to release the conservative condition is interested for us in future study.

5 Conclusion

The leader-following formation for general linear MASs with distributed infinite input time delays is explored based on the ETC approach. Considering the distributed infinite time delays between agents, a novel event-triggered formation control protocol is put forward. Utilizing inequality technique, leader-following formation is achieved without Zeno-behavior. It should be noted that this paper only deals with fixed formation for a classic linear MASs model with fixed communication topology, which are too strict in practical application. In practice, MASs may be subject to unmatched nonlinear dynamics, external disturbance and uncertainty, while communication topology may be switching. In addition, the fixed formation is not conducive for agents to adjust relative positions in time while avoiding obstacles and to expand or contract the movement scales. Therefore, the time-varying formation of nonlinear MASs considering external disturbance and uncertainty with switching topology will be investigated in further study.

References

1. A.E. Abouelregal, M. Marin, The response of nanobeams with temperature dependent properties using state-space method via modified couple stress theory, *Symmetry*, **12**(8):1275, 2020, https://doi.org/10.3390/sym12081276.
2. N. Ali, G. Zaman, Asymptotic behavior of HIV-1 epidemic model with infinite distributed intracellular delays, *SpringerPlus*, **5**:324, 2016, https://doi.org/10.1186/s40064-016-1951-9.
3. W. Cao, W. Zhu, Event-triggered formation control of multi-agent systems with time delay via complex-valued Laplacian, in *2019 3rd International Symposium on Autonomous Systems (ISAS), Shanghai, China, May 29–31, 2019*, IEEE, 2019, pp. 451–456, https://doi.org/10.1109/ISASS.2019.8757698.
4. Y. Cao, L. Zhang, C. Li, M.Z.Q. Chen, Observer-based consensus tracking of nonlinear agents in hybrid varying directed topology, *IEEE Trans. Cybern.*, **47**(8):2212–2222, 2017, https://doi.org/10.1109/TCYB.2016.2573138.
5. X. Dong, G. Hu, Time-varying formation tracking for linear multiagent systems with multiple leaders, *IEEE Trans. Autom. Control*, **62**(7):3658–3664, 2017, https://doi.org/10.1109/TAC.2017.2673411.
6. X. Ge, Q.L. Han, Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism, *IEEE Trans. Ind. Electron.*, **64**(10): 8118–8127, 2017, https://doi.org/10.1109/TIE.2017.2701778.
7. X. Ge, Q.L. Han, L. Ding, Dynamic event-triggered distributed coordination control and its applications: A survey of trends and techniques, *IEEE Trans. Syst. Man Cybern., Syst.*, **50**(9): 3112–3125, 2020, https://doi.org/10.1109/TSMC.2020.3010825.
8. R.A. Horn, C.R. Johnson, *Topics in Matrix Analysis*, Cambridge Univ. Press, Cambridge, 1994, https://doi.org/10.1017/CBO9780511840371.
9. Y. Hu, F. Wu, Stochastic kolmogorov-type population dynamics with infinite distributed delays, *Acta Appl. Math.*, **110**(3):1407–1428, 2010, https://doi.org/10.1007/s10440-009-9517-2.
10. W. Jiang, C. Wang, Y. Meng, Fully distributed time-varying formation tracking control of linear multi-agent systems with input delay and disturbances, *Syst. Control Lett.*, **146**:104814, 2022, https://doi.org/10.1016/j.sysconle.2020.104814.
11. S. Knorn, Z. Chen, R.H. Middleton, Overview: Collective control of multiagent systems, *IEEE Trans. Control Netw. Syst.*, **3**(4):334–347, 2016, https://doi.org/10.1109/TCNS.2015.2468991.

12. T. Li, Z. Li, S. Shen, S. Fei, Extended adaptive event-triggered formation tracking control of a class of multi-agent systems with time-varying delay, *Neurocomputing*, **316**:386–398, 2018, `https://doi.org/10.1016/j.neucom.2018.08.019`.
13. J. Liu, P. Li, W. Chen, K. Qin, L. Qi, Distributed formation control of fractional-order multi-agent systems with relative damping and nonuniform time-delays, *ISA Trans.*, **93**:189–198, 2019, `https://doi.org/10.1016/j.isatra.2019.03.012`.
14. K. Liu, A. Selivanov, E. Fridman, Survey on time-delay approach to networked control, *Annu. Rev. Control*, **48**:57–79, 2019, `https://doi.org/10.1016/j.arcontrol.2019.06.005`.
15. T. Liu, Z.P. Jiang, Distributed formation control of nonholonomic mobile robotswithout global position measurements, *Automatica*, **49**:592–600, 2013, `https://doi.org/10.1016/j.automatica.2012.11.031`.
16. X. Liu, F. Deng, W. Wei, F. Wan, Formation tracking control of networked systems with time-varying delays and sampling under fixed and markovian switching topology, *IEEE Trans. Control Netw. Syst.*, **9**(2):601–612, 2022, `https://doi.org/10.1109/TCNS.2022.3158817`.
17. Y. Luo, X. Gao, A. Li, A. Kashkynbayev, Event-triggered finite-time formation control for second-order leader-following multi-agent systems with nonlinear term time delay, *Int. J. Control*, 2022, `https://doi.org/10.1080/00207179.2022.2106898`.
18. W. Michiels, C.I. Morărescu, S.I. Niculescu, Consensus problems with distributed delays, with application to traffic flow models, *SIAM J. Control Optim.*, **48**(1):77–101, 2009, `https://doi.org/10.1137/060671425`.
19. W. Ren, R. W. Beard, *Distributed Consensus in Multi-Vehicle Cooperative Control: Theory and Applications*, Springer, London, 2008, `https://doi.org/10.1007/978-1-84800-015-5`.
20. S.A. Samy, Y. Cao, R. Ramachandran, J. Alzabut, M. Niezabitowski, C.P. Lim, Globally asymptotic stability and synchronization analysis of uncertain multi-agent systems with multiple time-varying delays and impulses, *Int. J. Robust Nonlinear Control*, **32**(2):737–773, 2022, `https://doi.org/10.1002/rnc.5851`.
21. R. Saravanakumar, A. Amini, R. Datta, Y. Cao, Reliable memory sampled-sata consensus of multi-agent systems with nonlinear actuator faults, *IEEE Trans. Circuits Syst. II, Express Briefs*, **69**(4):2201–2205, 2022, `https://doi.org/10.1109/TCSII.2021.3124043`.
22. M.L. Scutaru, S. Vlase1, M. Marin, A. Modrea, New analytical method based on dynamic response of planar mechanical elastic systems, *Bound. Value Probl.*, **2020**:104, 2020, `https://doi.org/10.1186/s13661-020-01401-9`.
23. F. Sun, H. Li, W. Zhu, J. Kurths, Fixed-time formation tracking for multiple nonholonomic wheeled mobile robots based on distributed observer, *Nonlinear Dyn.*, **106**:3331–3349, 2021, `https://doi.org/10.1007/s11071-021-06946-7`.
24. Y. Tipsuwan, M.Y. Chow, Gain scheduler middleware: A methodology to enable existing controllers for networked control and teleoperationpart i: networked control, *IEEE Trans. Ind. Electron.*, **51**(6):1218–1227, 2004, `https://doi.org/10.1109/TIE.2004.837866`.

25. L. Wang, M.F. Ge, J. Hu, G. Zhang, Global stability and stabilization for inertial memristive neural networks with unbounded distributed delays, *Nonlinear Dyn.*, **95**(2):943–955, 2019, https://doi.org/10.1007/s11071-018-4606-2.
26. C. Xia, C. Wang, Fully distributed event-triggered output feedback control for linear multi-agent systems with a derivable leader under directed graphs, *Inf. Sci.*, **619**:562–577, 2023, https://doi.org/10.1016/j.ins.2022.11.044.
27. X. Xu, L. Liu, G. Feng, Consensus of single integrator multi-agent systems with distributed infinite transmission delays, in *2018 Annual American Control Conference (ACC), Milwaukee, USA, June 27–29, 2018*, IEEE, 2018, pp. 1653–1658, https://doi.org/10.23919/ACC.2018.8430862.
28. X. Xu, L. Liu, G. Feng, Consensus of linear multiagent systems with distributed infinite transmission delays: A low gain approach, *IEEE Trans. Autom. Control*, **65**(2):809–816, 2020, https://doi.org/10.1109/TAC.2019.2918119.
29. N. Yu, W. Zhu, Exponential stabilization of fractional-order continuous-time dynamic systems via event-triggered impulsive control, *Nonlinear Anal. Model. Control*, **27**(3):592–608, 2022, https://doi.org/10.15388/namc.2022.27.26638.
30. G. Zhu, K. Liu, H. Gu, J.H. Lu, Event-based formation control for linear multi-agent systems under switching topology, in *2020 15th IEEE Conference on Industrial Electronics and Applications (ICIEA), Kristiansand, Norway, November 9–13, 2020*, IEEE, 2020, pp. 1306–1311, https://doi.org/10.1109/ICIEA48937.2020.9248331.
31. W. Zhu, W. Cao, M. Yan, Q. Li, Event-triggered formation control of multi-agent systems with linear continuous-time dynamic models, *IEEE Trans. Syst. Man Cybern., Syst.*, **52**(10):6235–6245, 2022, https://doi.org/10.1109/TSMC.2022.3144161.
32. W. Zhu, Z.P. Jiang, Event-based leader-following consensus of multi-agent systems with input time delay, *IEEE Trans. Autom. Control*, **60**(5):1362–1367, 2015, https://doi.org/10.1109/TAC.2014.2357131.