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Leader-following identical consensus for Markov jump nonlinear multi-agent systems subjected to attacks with impulse*

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Abstract. The issue of leader-following identical consensus for nonlinear Markov jump multi-agent systems (NMJMASs) under deception attacks (DAs) or denial-of-service (DoS) attacks is investigated in this paper. The Bernoulli random variable is introduced to describe whether the controller is injected with false data, that is, whether the systems are subjected to DAs. A connectivity recovery mechanism is constructed to maintain the connection among multi-agents when the systems are subjected to DoS attack. The impulsive control strategy is adopted to ensure that the systems can normally work under DAs or DoS attacks. Based on graph theory, Lyapunov stability theory, and impulsive theory, using the Lyapunov direct method and stochastic analysis method, the sufficient conditions of identical consensus for Markov jump multi-agent systems (MJMASs) under DAs or DoS are obtained, respectively. Finally, the correctness of the results and the effectiveness of the method are verified by two numerical examples.

Keywords: Markov jump multi-agent systems, DAs, DoS attacks, impulsive control, identical consensus.

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1 Introduction

The multi-agent systems (MASs) refer to network systems formed by a cluster of agent individuals with partial perception, communication, calculation, and execution capabilities, which are established by communicating and coordinating with each other through the communication networks. In recent years, the MASs have attracted much attention in different fields because they are exerted in broad places, for instance, intelligent traffic control [2], unmanned aerial vehicle [20], formation [5], etc. Markov jump systems, as a typical class of hybrid dynamical systems, have important applications in aerospace, industrial processes, biomedical, socio-economic, and other fields. There exist some related reports [14, 29–31]. For example, Luo et al. investigated the stability of MASs with asynchronous Markov jump parameters, the mean-square asymptotically stable sufficient conditions were obtained by constructing suitable Lyapunov function [14]. In [30], Zhou et al. researched the event-based asynchronous filtering issue of the Takagi–Sugeno fuzzy nonhomogeneous Markov jump systems with variable packet dropouts. At the same time, the MASs with Markov jump parameters have also attracted the research interest of many scholars. For instance, Wu et al. studied the problem of identical consensus for MJMASs with random time delays [24]. Liu et al. investigated the cooperative output regulation problem of discrete-time linear MASs with Markov switching topologies based on the stochastic stability scheme [13].

As is known to all, cyber attacks can destroy the stability of systems and even lead to systems paralysis, which makes cyber attacks into one of the biggest dangers threatening systems security. Among the common cyber attacks, there are main replay attacks (RAs) [16, 17, 34], DoS attacks [7, 19, 26], and DAs [8, 23, 28, 32]. The RAs are that when the attacker records a transmitted data sequence and repeatedly transmits this data to overwrite the new data, thus attacking the systems [17]. DoS attacks is one type of cyber attacks by temporarily interrupting or stopping the network service of the systems, hindering the normal transmission of data, and making it impossible for data to reach the destination [19]. In the process of data transmission, DAs tamper with information to make the systems complete the instructions of the attackers or injects false information to attack the systems [23]. Mo et al. analyzed the impact of RAs on the cyber physical system [16]. In [34], Zhu et al. considered the formation control problem for second-order MASs under RAs, put forward a new distributed elastic algorithm that was based on a rolling optimal control method, which was shown to enable vehicles to asymptotically achieve the desired formation under RAs. In [7], the authors proposed an adaptive memory observer-based opposing interference control strategy and an adaptive internal memory event-triggered scheme to address the event-triggered security consensus of the nonlinear MASs under DoS attacks. Yang et al. proposed security consensus control strategy under DoS attacks, which was based on an event-triggered scheme [26]. In [23], the authors investigated the fault-tolerant secure consensus tracking issue of delayed nonlinear MASs, which have DAs, parameter uncertainty and actuator failures, and proposed distributed impulsive control protocol to obtain sufficient conditions of mean-square bounded consensus. He et al. investigated the secure synchronization of MASs under DAs by distributed impulsive control strategy, obtained the mean-square bounded synchronization conditions, and gave the error bound [8]. Mahmou et al. aimed at the distributed DoS attacks and DAs of network physical systems, improved a method based on observation information and then controlled the systems, and carried out simulation experiments to verify the applicability of the method [15]. Compared with other types of attacks, DoS attacks and DAs are more destructive, more hidden, and easier to implement. Therefore, DoS attacks and DAs are commonly used in cyber attacks.

In order to make the systems work normally under cyber attacks, they need to be controlled. Common control methods include event-triggered control [3], intermittent control [35], impulsive control [1, 33], etc. Among them, the impulsive control has been studied broadly. As one of the control approaches in the consistency protocol, impulsive control has the advantages of discontinuity, transient and low energy consumption, which can compensate for the shortcomings of continuous control [11, 25]. In [25], the concept of impulsive control was given for the first time. In [11], the authors researched the exponential stability of nonlinear delay systems by means of event-triggered impulsive control strategy.

The consensus problem [10], as a hot issue of MASs coordinated control, has already caused the concern of many scholars. The consensus is usually classified as identical consensus [12], partial component consensus [27], lag consensus [22], etc. The leader-following identical consensus of MASs is achieved, that is to say, through the mutual communication and coordination among agents, the state values of all followers change with time and, finally, reach the state value of the leader. In [4], Dong et al. constructed the distributed adaptive observer and the common observer to estimate the state of the leader, and after that the authors designed the synchronous controller and the asynchronous controller on the basis of the estimation state and the self-information of the followers to achieve the mean-square leader-following identical consensus. He et al. designed a control protocol of distributed random sampling and gave the leader-following identical consensus conditions of nonlinear MASs [9].

Based on the above discussion, it is clear that the leader-following identical consensus problem of NMJMASs with impulse under DAs or DoS attacks has not been studied. We will try to explore it. The main innovations can be summed up as follows:

(i) The Markov jump parameters are considered in the MASs, which are subjected to DAs and DoS attacks. The identical consensus of the MASs are achieved by impulsive control strategy.

(ii) In [7, 18, 19] and [26], the authors studied the identical consensus of MASs under DoS attacks, however, the DAs and impulse were not considered. In [8] and [23], the authors researched the identical consensus of MASs under DAs, and yet the DoS attacks were not considered. In [21], the authors investigated the consensus of nonlinear MASs with impulse under DoS attacks, however, the DAs and Markov jump parameters were not considered. Moreover, compared with the relevant results in [7, 8, 21, 23, 26], the MASs were not considered with Markov jump parameters.

(iii) In [15], the authors studied the secure control of cyber physical systems under DoS attacks and DAs, the Markov jump parameters and impulse were not considered. Moreover, $N+1$ ($N > 1$) agents are considered in this paper rather than a single agent.

Structure of the paper is shown as below. Section 2 shows the formulation of the questions as well as the corresponding preparatory knowledge. The leader-following bounded identical consensus of NMJMASs with the impulse under DAs is analyzed in Section 3.1. In Section 3.2, the leader-following identical consensus conditions of NMJMASs with the impulse under DoS attacks are obtained. Section 4 shows the validity of the theoretical results obtained by means of two numerical examples. At last, in Section 5, the conclusions and prospects are listed.

Notations. $\mathbb{R}^n$ is the n-dimensional real vector space. I_n stands for the nth order identity matrix. The term $*$ in the matrix denotes a block caused by symmetry. The sign $\otimes$ indicates the Kronecker product. For a matrix $y \in \mathbb{R}^{n\times n}$, $\mathrm{He}(y)$ is to be characterized as $y+y^{\mathrm{T}}$. $\lambda_{\max}(B)$ stands for the maximal eigenvalue of matrix B. $\varphi_{\max}(R)$ is a maximum singular value. $\mathbf{E}(\cdot)$ is the expected value for a stochastic variable. The Euclidean vector norm is represented as $\|\cdot\|$.

2 Problem formulations

Graph theory. The network communication topologies of the agents are presented as $\mathcal{G} = (\varsigma, \chi, \mathcal{C})$, where $\varsigma = (\varsigma_1, \varsigma_2, \dots, \varsigma_N)$ is the nodes set, $\chi = \varsigma \times \varsigma$ is the edges set, and $\mathcal{C} = [a_{ij}] \in \mathbb{R}^{N\times N}$ with $a_{ii} = 0$ stands for the adjacency matrix. It is assumed that a direct edge from agent j to agent i is available, that is, $(j,i) \in \chi$, $a_{ij} > 0$, otherwise, $a_{ij} = 0$. A directed path consists of a series of consecutive directed edges. If $a_{ij} = a_{ji}$, $\mathcal{G}$ is undirected. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ is described by $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$ with $j \neq i$.

Consider the MASs with Markov jump parameters, and the dynamic model of the ith follower is described as

$$\dot{x}_i(\tau) = A_{\sigma(\tau)} x_i(\tau) + B_{\sigma(\tau)} g\big(x_i(\tau)\big) + u_i(\tau), \quad i = 1, 2, \dots, N, \tag{1}$$

where τ stands for time variable, and $\tau > 0$. $x_i(\tau) \in \mathbb{R}^n$ stands for the state vector of the ith follower. $g(\cdot)$ is continuous nonlinear vector function satisfying Assumption 1. $u_i(\tau)$ is the control input vector of the ith follower. $A_{\sigma(\tau)}$ and $B_{\sigma(\tau)}$ are known constant matrices within appropriate dimensions. $\{\sigma(\tau),\, \tau \geqslant 0\}$ denotes a continuous time Markov jump process on the complete probability space $(\Omega, F, \mathbf{P})$ and takes the value in the finite set $\mathbb{M} = \{1, 2, \dots, M\}$, and M is a natural number.

Suppose that all agents are controlled under the same transition rate matrix $\Pi = [\pi_{\sigma n}]$. The transition probabilities are satisfying

$$\mathbf{P}\big\{\sigma(\tau + \mathrm{d}\tau) = n \,\big|\, \sigma(\tau) = \sigma\big\} = \begin{cases} \pi_{\sigma n}\,\mathrm{d}\tau + o(\mathrm{d}\tau), & \sigma \neq n, \\ 1 + \pi_{\sigma\sigma}\,\mathrm{d}\tau + o(\mathrm{d}\tau), & \sigma = n. \end{cases}$$

For the elements in $\varPi$, for all $\sigma \neq n$, $\pi_{\sigma n} \geqslant 0$ and $\pi_{\sigma\sigma} = -\sum_{n=1,\, n\neq\sigma}^{M} \pi_{\sigma n} < 0$. $\mathrm{d}\tau$ is the stopping time, which shows the stopping time for the systems to jump from state σ to state n. $o(\mathrm{d}\tau)$ is defined by $\lim_{\mathrm{d}\tau\to 0} o(\mathrm{d}\tau)/\mathrm{d}\tau = 0$.

At the same time, the dynamics model of the leader is described as

$$\dot{s}(\tau) = A_{\sigma(\tau)} s(\tau) + B_{\sigma(\tau)} g\big(s(\tau)\big), \tag{2}$$

where $s(\tau) \in \mathbb{R}^n$ denotes the state vector of the leader.

To achieve bounded identical consensus of NMJMASs under DAs, the control input of agent i is constructed as

$$\begin{aligned} u_i(\tau) = \sum_{k=1}^{\infty} c \Bigg\{ &\sum_{j=1}^{N} \big[-l_{ij}\big(x_j(\tau) - x_i(\tau)\big) + \vartheta_{ij}(\tau) z_i(\tau)\big] - d_{i0}\big(x_i(\tau) - s(\tau)\big) \Bigg\} \\ &\times f(\tau - \tau_k), \end{aligned} \tag{3}$$

where c shows the coupling strength, $z_i(\tau) \in \mathbb{R}^n$, $i = 1, 2, \dots, N$, denotes an externally injected attack signal of agent i. $Z(\tau) = [z_1^{\mathrm{T}}(\tau), z_2^{\mathrm{T}}(\tau), \dots, z_N^{\mathrm{T}}(\tau)] \in \mathbb{R}^{N\times N}$ satisfies $\|Z(\tau)\|^2 \leqslant z$, and z is a positive constant. d_{i0} denotes the coupling strength both agent i and leader, at the same time, the coupling strength $d_{i0} > 0$ if the leader can transmit information to agent i, otherwise, $d_{i0} = 0$. $f(\cdot)$ is the Dirac function, if $\tau = \tau_k$, then $f(\cdot) = 1$, otherwise, $f(\cdot) = 0$. $\{\tau_k\}_{k=0}^{\infty}$ is the impulse sequence. $\vartheta_{ij}(\tau)$ is the Bernoulli random variable and satisfies

$$\mathbf{P}\big\{\vartheta_{ij}(\tau) = 1\big\} = \rho_{ij}, \qquad \mathbf{P}\big\{\vartheta_{ij}(\tau) = 0\big\} = 1 - \rho_{ij},$$

where it is assumed that $\vartheta_{ii}(\tau) = 0$ and $\rho_{ij} \in [0, 1)$.

Remark 1. DAs occur randomly in the communication channel, and their randomness are described as the independent random variable $\vartheta_{ij}(\tau)$. When the systems are subjected to DAs at time τ, $\vartheta_{ij}(\tau) = 1$, otherwise, $\vartheta_{ij}(\tau) = 0$. $\vartheta_{ij}(\tau)$ and $\sigma(\tau)$ are independent of each other.

Remark 2. In (3), the systems are subjected to DAs, where the attacker injects false signal $z_i(\tau)$ into the controller. Due to the limited energy, the attacker cannot launch attacks arbitrarily, so it is assumed that the external injected signal is bounded, that is, $\|Z(\tau)\|^2 \leqslant z$.

The state vector error can be denoted as $\delta_i(\tau) = x_i(\tau) - s(\tau)$. When $\tau \neq \tau_k$, $f(\tau - \tau_k) = 0$, one has $u_i(\tau) = 0$. Let $g(\delta_i(\tau), s(\tau)) = g(x_i(\tau)) - g(s(\tau))$. Because $\delta_i(\tau) = x_i(\tau) - s(\tau)$, combining (1) and (2), one can obtain

$$\begin{aligned} \dot{\delta}_i(\tau) &= \dot{x}_i(\tau) - \dot{s}(\tau) \\ &= A_{\sigma(\tau)} x_i(\tau) + B_{\sigma(\tau)} g\big(x_i(\tau)\big) - A_{\sigma(\tau)} s(\tau) - B_{\sigma(\tau)} g\big(s(\tau)\big) \\ &= A_{\sigma(\tau)}\big(x_i(\tau) - s(\tau)\big) + B_{\sigma(\tau)}\big(g\big(x_i(\tau)\big) - g\big(s(\tau)\big)\big) \\ &= A_{\sigma(\tau)} \delta_i(\tau) + B_{\sigma(\tau)} g\big(\delta_i(\tau), s(\tau)\big). \end{aligned} \tag{4}$$

When $\tau = \tau_k$, $f(\tau - \tau_k) = 1$, one gets

$$u_i(\tau) = c\left\{ \sum_{j=1}^{N} \left[-l_{ij}\big(x_j(\tau) - x_i(\tau)\big) + \vartheta_{ij}(\tau) z_i(\tau) \right] - d_{i0}\big(x_i(\tau) - s(\tau)\big) \right\}. \quad (5)$$

Because $\Delta\delta_i(\tau_k) = \delta_i(\tau_k^+) - \delta_i(\tau_k^-)$, $\delta_i(\tau_k) = \delta_i(\tau_k^+) = \lim_{h\to 0^+} \delta_i(\tau_k + h)$ and $\delta_i(\tau_k^-) = \lim_{h\to 0^-} \delta_i(\tau_k + h)$. One can derive

$$\begin{aligned}\Delta\delta_i(\tau_k) &= \delta_i(\tau_k) - \delta_i(\tau_k^-) \\ &= \big(x_i(\tau_k) - s(\tau_k)\big) - \big(x_i(\tau_k^-) - s(\tau_k^-)\big) \\ &= x_i(\tau_k) - x_i(\tau_k^-) = \Delta x_i(\tau_k) = u_i(\tau_k^-).\end{aligned}$$

According to (5), one can get

$$\begin{aligned}\Delta\delta_i(\tau_k) &= c\sum_{j=1}^{N} \left[-l_{ij}\big(x_j(\tau_k^-) - x_i(\tau_k^-)\big) + \vartheta_{ij}(\tau_k^-) z_i(\tau_k^-) \right] \\ &\quad - c d_{i0}\big(x_i(\tau_k^-) - s(\tau_k^-)\big) \\ &= c\sum_{j=1}^{N} \left[-l_{ij}\big(\big(x_j(\tau_k^-) - s(\tau_k^-)\big) - \big(x_i(\tau_k^-) - s(\tau_k^-)\big)\big) + \vartheta_{ij}(\tau_k) z_i(\tau_k) \right] \\ &\quad - c d_{i0}\big(x_i(\tau_k^-) - s(\tau_k^-)\big) \\ &= -c\sum_{j=1}^{N} \left[-l_{ij}\big(\delta_j(\tau_k^-) - \delta_i(\tau_k^-)\big) - d_{i0}\delta_i\big(\tau_k^-\big) \right] + c\sum_{j=1}^{N} \vartheta_{ij}(\tau_k) z_i(\tau_k). \quad (6)\end{aligned}$$

According to (4) and (6), the following error system is obtained:

$$\begin{aligned}\dot{\delta}_i(\tau) &= A_{\sigma(\tau)}\delta_i(\tau) + B_{\sigma(\tau)} g\big(\delta_i(\tau), s(\tau)\big), \quad \tau \neq \tau_k, \\ \Delta\delta_i(\tau_k) &= -c\sum_{j=1}^{N} \left[l_{ij}\big(\delta_j(\tau_k^-) - \delta_i(\tau_k^-)\big) + d_{i0}\delta_i(\tau_k^-) \right] \\ &\quad + c\sum_{j=1}^{N} \vartheta_{ij}(\tau_k) z_j(\tau_k).\end{aligned} \quad (7)$$

Let $\delta(\tau) = [\delta_1^{\mathrm{T}}(\tau), \delta_2^{\mathrm{T}}(\tau), \dots, \delta_N^{\mathrm{T}}(\tau)]^{\mathrm{T}}$. Combining with $\delta_i(\tau) = \Delta\delta_i(\tau_k) + \delta_i(\tau_k^-)$, (7) is rewritten as

$$\begin{aligned}\dot{\delta}_i(\tau) &= (I_N \otimes A_{\sigma(\tau)})\delta(\tau) + (I_N \otimes B_{\sigma(\tau)}) G\big(\delta(\tau), s(\tau)\big), \quad \tau \neq \tau_k, \\ \delta(\tau_k) &= \big((I_N - cH) \otimes I_n\big)\delta(\tau_k^-) + \big(c\Upsilon(\tau_k) \otimes I_n\big) Z(\tau_k),\end{aligned} \quad (8)$$

where $G(\delta(\tau), s(\tau)) = [g(\delta_1(\tau), s(\tau))^{\mathrm{T}}, g(\delta_2(\tau), s(\tau))^{\mathrm{T}}, \dots, g(\delta_i(\tau), s(\tau))^{\mathrm{T}}]$, $H = L + D$, $\Upsilon(\tau_k) = [\vartheta_{ij}(\tau_k)]_{N\times N}$. The expectation of the random matrix $\Upsilon(\tau_k)$ can be denoted as $\mathbf{E}\{\Upsilon(\tau_k)\} = W$, where

$$W = \begin{bmatrix} 0 & w_{12} & \cdots & w_{1N} \\ w_{21} & 0 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ w_{N1} & w_{N2} & \cdots & 0 \end{bmatrix}.$$

Assumption 1. (See [19].) For any $x_1, x_2 \in \mathbb{R}^n$, there exists a constant $\rho > 0$ such that

$$\|g(x_1) - g(x_2)\| \leqslant \rho\|x_1 - x_2\|.$$

Assumption 2. (See [23].) The network communication topologies of MASs are connected. There exists at least one path from the leader to other followers.

Lemma 1. (*See* [23].) *Given permutation matrices A, B, and C such that*

$$\begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix} < 0,$$

it is equivalent to $C < 0$ and $A - BC^{-1}B^{\mathrm{T}} < 0$.

Lemma 2. (*See* [23].) *If there exist any real matrices A and B, any matrix $C > 0$, and any constant $\omega > 0$, A, B, and C have the appropriate dimensions, then the following inequality is satisfied*:

$$A^{\mathrm{T}}B + B^{\mathrm{T}}A \leqslant \omega^{-1}A^{\mathrm{T}}CA + \omega B^{\mathrm{T}}C^{-1}B.$$

Definition 1. (See [6, 8].) The NMJMASs under DAs can achieve bounded identical consensus, suppose that there exist a bounded constant $\phi > 0$ and a set $\mathcal{M}$ such that for any given initial state $x_i(0), s(0) \in \mathbb{R}^n$, $\delta_i(\tau) = x_i(\tau) - s(\tau)$ converges to the set

$$\mathcal{M} = \{\delta(\tau) \in \mathbb{R}^{N\times N} \mid \mathbf{E}\{\|\delta(\tau)\|^2\} < \phi\}$$

when $\tau \to \infty$.

Definition 2. (See [21].) For $T > 0$, $\tau > 0$, and $\tau < T$, the average value of impulsive interval $\{\tau_{k+1} - \tau_k\}$ $(k = 1, 2, \dots)$ is denoted as T_a $(T_a > 0)$, and assume that T_a is bounded if there exists an integer $N_0 > 0$ such that

$$\frac{T-\tau}{T_a} - N_0 \leqslant N(\tau, T) \leqslant \frac{T-\tau}{T_a} + N_0,$$

where $N(\tau, T)$ is the impulsive times appear within (τ, T).

Definition 3. (See [19].) The NMJMASs under DoS attacks can achieve identical consensus if the following inequality holds:

$$\mathbf{E}\{\|\delta_i(\tau)\|^2\} \leqslant \gamma\exp(-\varrho\tau)\mathbf{E}\{\|\delta_i(0)\|^2\}, \quad i = 1, 2, \dots, N,$$

where $\gamma > 0$, $\varrho > 0$ are both scalars, and ϱ stands for the decay rate.

3 Main results

3.1 The bounded identical consensus of NMJMASs under DAs

Theorem 1. *According to Assumptions* 1 *and* 2 *listed above, the NMJMASs can achieve bounded identical consensus, that is to say, the trajectory of error system* (7) *converges to set* $\mathcal{M}$*, where*

$$\mathcal{M} = \left\{ \delta(\tau) \mid \mathbf{E}\{\|\delta(\tau)\|^2\} \leqslant \frac{v_\sigma u^{-1} \exp((1-N_0)\beta T_a)}{\varpi(u^{-1}\exp(\beta T_a) - 1)} \right\},$$

if there exist matrices $P_\sigma > 0$ $(\sigma \in \mathbb{M})$ *and scalars* β, $\rho > 0$, $\eta > 0$, $\varpi > 0$ *such that the following conditions hold*:

$$\begin{bmatrix} A_\sigma Q_\sigma + Q_\sigma A_\sigma^{\mathrm{T}} + B_\sigma B_\sigma^{\mathrm{T}} + \pi_{\sigma\sigma} Q_\sigma + Q_\sigma \beta & Q_\sigma & \aleph \\ * & -\rho^{-2} I_N & 0 \\ * & * & \Sigma \end{bmatrix} < 0, \tag{9}$$

$$\frac{\ln u}{T_a} < \beta, \quad 0 < u < 1,$$
$$\varpi I \leqslant P_\sigma \leqslant \eta I, \tag{10}$$

where

$$\begin{aligned}
\aleph &= (\sqrt{\pi_{\sigma 1}} Q_\sigma, \sqrt{\pi_{\sigma 2}} Q_\sigma, \dots, \sqrt{\pi_{\sigma\sigma-1}} Q_\sigma, \sqrt{\pi_{\sigma\sigma+1}} Q_\sigma, \dots, \sqrt{\pi_{\sigma M}} Q_\sigma), \\
\Sigma &= -\operatorname{diag}\{Q_1, \dots, Q_{\sigma-1}, Q_{\sigma+1}, \dots, Q_M\}, \qquad Q_\sigma = P_\sigma^{-1}, \\
u &= \varphi_{\max}^2 (I_N - cH) + \omega^{-1}, \\
v_\sigma &= \omega c^2 z \varphi_{\max}^2 \{W^{\mathrm{T}}(I_N - cH)\}\eta + z c^2 \varphi_{\max}^2 (W^{\mathrm{T}} W)\eta.
\end{aligned}$$

Proof. At first, the Lyapunov function is constructed as follows:

$$V(\delta(\tau), \tau, \sigma(\tau)) = \delta^{\mathrm{T}}(\tau)(I_N \otimes P_{\sigma(\tau)})\delta(\tau). \tag{11}$$

An infinitesimal operator $\mathcal{A}$ is defined by

$$\begin{aligned}
&\mathcal{A}V(\delta(\tau), \tau, \sigma(\tau)) \\
&\quad = \lim_{\mathrm{d}\tau \to 0^+} \frac{1}{\mathrm{d}\tau} \{\mathbf{E}[V(\delta(\tau+\mathrm{d}\tau), \tau + \mathrm{d}\tau, \sigma(\tau+\mathrm{d}\tau)) \mid \delta(\tau), \tau, \sigma(\tau)] \\
&\qquad - V(\delta(\tau), \tau, \sigma(\tau))\}.
\end{aligned}$$

Define $\sigma(\tau) = \sigma$ $(\sigma \in \mathbb{M})$. Applying the total probability formula as well as the conditional expectation formula, one has

$$\begin{aligned}
&\mathbf{E}\{\mathcal{A}V(\delta(\tau), \tau, \sigma)\} \\
&\quad = \mathrm{He}\{\delta^{\mathrm{T}}(\tau)(I_N \otimes P_\sigma)[(I_N \otimes A_\sigma)\delta(\tau) + (I_N \otimes B_\sigma)G(\delta(\tau), s(\tau))]\} \\
&\qquad + \delta^{\mathrm{T}}(\tau) \sum_{n=1}^{z} \pi_{\sigma n}(I_N \otimes P_n)\delta(\tau).
\end{aligned} \tag{12}$$

Based on Assumption 1, it can be obtained the following:

$$\begin{aligned}
&2\delta(\tau)(I_N \otimes P_\sigma B_\sigma)G\bigl(\delta(\tau), s(\tau)\bigr)\\
&\quad = 2\sum_{i=1}^{N}\delta_i(\tau)P_\sigma B_\sigma G\bigl(\delta_i(\tau), s(\tau)\bigr)\\
&\quad \leqslant \sum_{i=1}^{N}\delta_i^{\mathrm{T}}(\tau)P_\sigma B_\sigma B_\sigma^{\mathrm{T}}P_\sigma\delta_i(\tau) + G^{\mathrm{T}}\bigl(\delta_i(\tau), s(\tau)\bigr)G\bigl(\delta_i(\tau), s(\tau)\bigr)\\
&\quad \leqslant \sum_{i=1}^{N}\delta_i^{\mathrm{T}}(\tau)P_\sigma B_\sigma B_\sigma^{\mathrm{T}}P_\sigma\delta_i(\tau) + \rho^2\delta_i^{\mathrm{T}}(\tau)\delta_i(\tau)\\
&\quad = \delta_i^{\mathrm{T}}(\tau)\bigl(I_N \otimes \bigl(P_\sigma B_\sigma B_\sigma^{\mathrm{T}}P_\sigma + \rho^2 I_N\bigr)\bigr)\delta_i(\tau).
\end{aligned} \tag{13}$$

According to Lemma 1, for (9), one has

$$\mathrm{He}(A_\sigma P_\sigma) + \sum_{n=1}^{M}\pi_{\sigma n}P_n + P_\sigma B_\sigma B_\sigma^{\mathrm{T}}P_\sigma + \rho^2 I_N + \beta P_\sigma < 0.$$

Combine (9), (12), and (13), then

$$\begin{aligned}
&\mathbf{E}\bigl\{\mathcal{A}V\bigl(\delta(\tau), \tau, \sigma\bigr)\bigr\}\\
&\quad \leqslant \delta^{\mathrm{T}}(\tau)\left[I_N \otimes \left(\mathrm{He}(A_\sigma P_\sigma) + \sum_{n=1}^{M}\pi_{\sigma n}P_n + P_\sigma B_\sigma B_\sigma^{\mathrm{T}}P_\sigma + \rho^2 I_N\right)\right]\delta(\tau)\\
&\quad \leqslant -\beta\delta^{\mathrm{T}}(\tau)(I_N \otimes P_\sigma)\delta(\tau) = -\beta\mathbf{E}\bigl\{V\bigl(\delta(\tau), \tau, \sigma\bigr)\bigr\}.
\end{aligned} \tag{14}$$

Then, by integrating (14), one gets

$$\begin{aligned}
&\mathbf{E}\bigl\{V\bigl(\delta(\tau), \tau, \sigma\bigr)\bigr\}\\
&\quad \leqslant \exp\bigl(-\beta(\tau_k - \tau_{k-1})\bigr)\mathbf{E}\bigl\{V\bigl(\delta(\tau_{k-1}), \tau_{k-1}, \sigma\bigr)\bigr\}, \quad \tau \in [\tau_{k-1}, \tau_k).
\end{aligned} \tag{15}$$

Based on (8), one has

$$\begin{aligned}
&\mathbf{E}\bigl\{\delta^{\mathrm{T}}(\tau_k)(I_N \otimes P_\sigma)\delta(\tau_k)\bigr\}\\
&\quad = \mathbf{E}\bigl\{\bigl(\bigl((I_N - cH) \otimes I_n\bigr)\delta(\tau_k^-) + (c\Upsilon\tau_k \otimes I_n)Z(\tau_k)\bigr)^{\mathrm{T}}\\
&\qquad \times (I_N \otimes P_\sigma)\bigl(\bigl((I_N - cH) \otimes I_n\bigr)\delta(\tau_k^-) + \bigl(c\Upsilon(\tau_k) \otimes I_n\bigr)Z(\tau_k)\bigr)\bigr\}\\
&\quad = \mathbf{E}\bigl\{\delta^{\mathrm{T}}(\tau_k^-)\Gamma^{\mathrm{T}}(I_N \otimes P_\sigma)\Gamma\delta(\tau_k^-) + \delta^{\mathrm{T}}(\tau_k^-)\Gamma^{\mathrm{T}}(I_N \otimes P_\sigma)\zeta(\tau_k)\\
&\qquad + \zeta^{\mathrm{T}}(\tau_k)(I_N \otimes P_\sigma)\Gamma\delta(\tau_k^-) + \zeta^{\mathrm{T}}(\tau_k)(I_N \otimes P_\sigma)\zeta(\tau_k)\bigr\},
\end{aligned} \tag{16}$$

where $\Gamma = (I_N - cH) \otimes I_n$, $\zeta(\tau_k) = (c\Upsilon(\tau_k) \otimes I_n)Z(\tau_k)$. (16) is analyzed item by item as follows.

Based on the definition of $\varGamma$, one has

$$\begin{aligned}
&\mathbf{E}\bigl\{\delta^{\mathrm{T}}(\tau_k^-)\varGamma^{\mathrm{T}}(I_N\otimes P_\sigma)\varGamma\delta(\tau_k^-)\bigr\}\\
&\quad=\mathbf{E}\bigl\{\delta^{\mathrm{T}}(\tau_k^-)\bigl[(I_N-cH)^{\mathrm{T}}(I_N-cH)\otimes P_\sigma\bigr]\delta(\tau_k^-)\bigr\}\\
&\quad\leqslant\varphi_{\max}^2(I_N-cH)\mathbf{E}\bigl\{V\bigl(\delta(\tau_k^-),\tau_k^-,\sigma\bigr)\bigr\}.
\end{aligned}\tag{17}$$

According to Lemma 2, it yields

$$\begin{aligned}
&\mathbf{E}\bigl\{\delta^{\mathrm{T}}(\tau_k^-)\varGamma^{\mathrm{T}}(I_N\otimes P_\sigma)\zeta(\tau_k)+\zeta^{\mathrm{T}}(\tau_k)(I_N\otimes P_\sigma)\varGamma\delta(\tau_k^-)\bigr\}\\
&\quad\leqslant\omega^{-1}\mathbf{E}\bigl\{\delta^{\mathrm{T}}(\tau_k^-)(I_N\otimes P_\sigma)\delta(\tau_k^-)\bigr\}\\
&\qquad+\omega Z(\tau_k)^{\mathrm{T}}(c\varUpsilon(\tau_k)\otimes I_n)^{\mathrm{T}}\bigl\{\bigl[(I_N-cH\otimes I_n)(I_N-cH\otimes I_n)^{\mathrm{T}}\bigr]\otimes P_\sigma\bigr\}\\
&\qquad\times\bigl(c\varUpsilon(\tau_k)\otimes I_n\bigr)Z(\tau_k)\\
&\quad=\omega^{-1}\mathbf{E}\bigl\{V\bigl(\delta(\tau_k^-),\tau_k^-,\sigma\bigr)\bigr\}+\omega c^2z\varphi_{\max}^2\bigl\{W^{\mathrm{T}}(I_N-cH)\bigr\}\lambda_{\max}(P_\sigma),
\end{aligned}\tag{18}$$

where z is a limited attack energy.

Based on the definition of $\zeta(\tau_k)$, one gets

$$\begin{aligned}
&\mathbf{E}\bigl\{\zeta^{\mathrm{T}}(\tau_k)(I_N\otimes P_\sigma)\zeta(\tau_k)\bigr\}\\
&\quad=\mathbf{E}\bigl\{Z(\tau_k)^{\mathrm{T}}\bigl(c\varUpsilon(\tau_k)\otimes I_n\bigr)^{\mathrm{T}}(I_N\otimes P_\sigma)\bigl(c\varUpsilon(\tau_k)\otimes I_n\bigr)Z(\tau_k)\bigr\}\\
&\quad\leqslant zc^2\varphi_{\max}^2\bigl(W^{\mathrm{T}}W\bigr)\lambda_{\max}(P_\sigma).
\end{aligned}\tag{19}$$

Combining (17), (18), and (19), from (16) one gets

$$\begin{aligned}
\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\}\leqslant{}&\bigl(\varphi_{\max}^2(I_N-cH)+\omega^{-1}\bigr)\mathbf{E}\bigl\{V\bigl(\delta(\tau_k^-),\tau_k^-,\sigma\bigr)\bigr\}\\
&+\omega c^2z\varphi_{\max}^2\bigl\{W^{\mathrm{T}}(I_N-cH)\bigr\}\lambda_{\max}(P_\sigma)\\
&+zc^2\varphi_{\max}^2\bigl(W^{\mathrm{T}}W\bigr)\lambda_{\max}(P_\sigma).
\end{aligned}$$

Let $u=\varphi_{\max}^2(I_N-cH)+\omega^{-1}$, combining with (10), one has

$$v_\sigma=\omega c^2z\varphi_{\max}^2\bigl\{W^{\mathrm{T}}(I_N-cH)\bigr\}\lambda_{\max}\eta+zc^2\varphi_{\max}^2\bigl(W^{\mathrm{T}}W\bigr)\eta,\tag{20}$$

then one can obtain that

$$\mathbf{E}\bigl\{V\bigl(\delta(\tau_k),\tau_k,\sigma\bigr)\bigr\}=u\mathbf{E}\bigl\{V\bigl(\delta(\tau_k^-),\tau_k^-,\sigma\bigr)\bigr\}+v_\sigma.\tag{21}$$

Based on (15) and (21), it yields

$$\begin{aligned}
&\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\}\leqslant\exp\bigl(-\beta(\tau-\tau_{k-1})\bigr)\\
&\qquad\qquad\times\mathbf{E}\bigl\{V\bigl(\delta(\tau_{k-1}),\tau_{k-1},\sigma\bigr)\bigr\},\quad\tau\in[\tau_{k-1},\tau_k),\\
&\mathbf{E}\bigl\{V\bigl(\delta(\tau_k),\tau_k,\sigma\bigr)\bigr\}=u\mathbf{E}\bigl\{V\bigl(\delta(\tau_k^-),\tau_k^-,\sigma\bigr)\bigr\}+v_\sigma.
\end{aligned}\tag{22}$$

When $\tau\in[\tau_0,\tau_1)$, the following inequality holds:

$$\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\}\leqslant\exp\bigl(-\beta(\tau-\tau_0)\bigr)\mathbf{E}\bigl\{V\bigl(\delta(\tau_0),\tau_0,\sigma\bigr)\bigr\}.\tag{23}$$

Thus, in terms of (22) and (23), one has

$$\mathbf{E}\big\{V\big(\delta(\tau_1),\tau_1,\sigma\big)\big\} \leqslant u\exp\big(-\beta(\tau_1-\tau_0)\big)\mathbf{E}\big\{V\big(\delta(\tau_0),\tau_0,\sigma\big)\big\} + v_\sigma.$$

When $\tau \in [\tau_1,\tau_2)$, one can obtain that

$$\begin{aligned}\mathbf{E}\big\{V\big(\delta(\tau),\tau,\sigma\big)\big\} &\leqslant u\exp\big(-\beta(\tau-\tau_0)\big)\mathbf{E}\big\{V\big(\delta(\tau_0),\tau_0,\sigma\big)\big\} \\ &\quad + v_\sigma\exp\big(-\beta(\tau-\tau_1)\big), \\ \mathbf{E}\big\{V\big(\delta(\tau_2^-),\tau_2^-,\sigma\big)\big\} &\leqslant u\exp\big(-\beta(\tau_2-\tau_0)\big)\mathbf{E}\big\{V\big(\delta(\tau_0),\tau_0,\sigma\big)\big\} \\ &\quad + v_\sigma\exp\big(-\beta(\tau_2-\tau_1)\big), \\ \mathbf{E}\big\{V\big(\delta(\tau_2),\tau_2,\sigma\big)\big\} &\leqslant u^2\exp\big(-\beta(\tau_2-\tau_0)\big)\mathbf{E}\big\{V\big(\delta(\tau_0),\tau_0,\sigma\big)\big\} \\ &\quad + uv_\sigma\exp\big(-\beta(\tau_2-\tau_1)\big) + v_\sigma.\end{aligned}$$

When $\tau \in [\tau_2,\tau_3)$, one gets

$$\begin{aligned}\mathbf{E}\big\{V\big(\delta(\tau),\tau,\sigma\big)\big\} &\leqslant u^2\exp\big(-\beta(\tau-\tau_0)\big)\mathbf{E}\big\{V\big(\delta(\tau_0),\tau_0,\sigma(\tau_0)\big)\big\} \\ &\quad + uv_\sigma\exp\big(-\beta(\tau-\tau_1)\big) + v_\sigma\exp\big(-\beta(\tau-\tau_2)\big).\end{aligned}$$

Suppose that when $\tau \in [\tau_{k-1},\tau_k)$, the following formulas hold:

$$\begin{aligned}\mathbf{E}\big\{V\big(\delta(\tau),\tau,\sigma\big)\big\} &\leqslant u^{k-1}\exp\big(-\beta(\tau-\tau_0)\big)\mathbf{E}\big\{V\big(\delta(\tau_0),\tau_0,\sigma\big)\big\} \\ &\quad + u^{k-2}v_\sigma\exp\big(-\beta(\tau-\tau_1)\big) + \cdots \\ &\quad + v_\sigma\exp\big(-\beta(\tau-\tau_{k-1})\big), \\ \mathbf{E}\big\{V\big(\delta(\tau_k^-),\tau_k^-,\sigma\big)\big\} &\leqslant u^{k-1}\exp\big(-\beta(\tau_k-\tau_0)\big)\mathbf{E}\big\{V\big(\delta(\tau_0),\tau_0,\sigma\big)\big\} \\ &\quad + u^{k-2}v_\sigma\exp\big(-\beta(\tau_k-\tau_1)\big) + \cdots \\ &\quad + v_\sigma\exp\big(-\beta(\tau_k-\tau_{k-1})\big), \\ \mathbf{E}\big\{V\big(\delta(\tau_k),\tau_k,\sigma\big)\big\} &\leqslant u^{k}\exp\big(-\beta(\tau_k-\tau_0)\big)\mathbf{E}\big\{V\big(\delta(\tau_0),\tau_0,\sigma\big)\big\} \\ &\quad + u^{k-1}v_\sigma\exp\big(-\beta\big(\tau_k-\tau_1)\big) \\ &\quad + u^{k-2}v_\sigma\exp\big(-\beta(\tau_k-\tau_2)\big) + \cdots \\ &\quad + v_\sigma\exp\big(-\beta(\tau_k-\tau_{k-1})\big).\end{aligned} \tag{24}$$

Thus, when $\tau \in [\tau_k,\tau_{k+1})$, in terms of (22) and (24), the following formula holds:

$$\begin{aligned}&\mathbf{E}\big\{V\big(\delta(\tau),\tau,\sigma\big)\big\} \\ &\quad \leqslant \exp\big(-\beta(\tau-\tau_k)\big)\mathbf{E}\big\{V(\delta(\tau_k),\tau_k,\sigma\big)\big\} \\ &\quad \leqslant \exp\big(-\beta(\tau-\tau_k)\big)\big[u^k\exp\big(-\beta(\tau_k-\tau_0)\big)\mathbf{E}\big\{V\big(\delta(\tau_0),\tau_0,\sigma\big)\big\} \\ &\qquad + u^{k-1}v_\sigma\exp\big(-\beta(\tau_k-\tau_1)\big) + u^{k-2}v_\sigma\exp\big(-\beta(\tau_k-\tau_2)\big) + \cdots \\ &\qquad + v_\sigma\exp\big(-\beta(\tau_k-\tau_{k-1})\big)\big] \\ &\quad = u^k\exp\big(-\beta(\tau-\tau_0)\big)\mathbf{E}\big\{V(\delta(\tau_0),\tau_0,\sigma\big)\big\} \\ &\qquad + u^{k-1}v_\sigma\exp\big(-\beta(\tau-\tau_1)\big) + u^{k-2}v_\sigma\exp\big(-\beta(\tau-\tau_2)\big) + \cdots \\ &\qquad + v_\sigma\exp\big(-\beta(\tau-\tau_k)\big).\end{aligned}$$

According to mathematical induction, for any $\tau \in [\tau_k, \tau_{k+1})$ $(k = 1, 2, \dots)$, the following inequality holds:

$$\begin{aligned} \mathbf{E}\{V(\delta(\tau), \tau, \sigma)\} \leqslant{}& u^k \exp(-\beta(\tau - \tau_0))\mathbf{E}\{V(\delta(\tau_0), \tau_0, \sigma)\} \\ &+ u^{k-1} v_\sigma \exp(-\beta(\tau - \tau_1)) \\ &+ u^{k-2} v_\sigma \exp(-\beta(\tau - \tau_2)) + \cdots \\ &+ v_\sigma \exp(-\beta(\tau - \tau_k)). \end{aligned} \tag{25}$$

If $0 < u < 1$, for $\tau \in [\tau_k, \tau_{k+1})$, based on Definition 2 and (25), one gets

$$\begin{aligned} &\mathbf{E}\{V(\delta(\tau), \tau, \sigma)\} \\ &\quad\leqslant u^{(\tau-\tau_0)/T_a - N_0} \exp(-\beta(\tau - \tau_0))\mathbf{E}\{V(\delta(\tau_0), \tau_0, \sigma)\} \\ &\qquad + v_\sigma u^{(\tau-\tau_0)/T_a - N_0 - 1} \exp(-\beta(\tau - \tau_0)) \exp(\beta(\tau_1 - \tau_0)) \\ &\qquad + v_\sigma u^{(\tau-\tau_0)/T_a - N_0 - 2} \exp(-\beta(\tau - \tau_0)) \exp(\beta(\tau_2 - \tau_0)) + \cdots \\ &\qquad + v_\sigma u^{(\tau-\tau_0)/T_a - N_0 - k} \exp(-\beta(\tau - \tau_0)) \exp(\beta(\tau_k - \tau_0)). \end{aligned} \tag{26}$$

Therefore, (26) can be reformulated as

$$\begin{aligned} &\mathbf{E}\{V(\delta(\tau), \tau, \sigma)\} \\ &\quad\leqslant u^{(\tau-\tau_0)/T_a - N_0} \exp(-\beta(\tau - \tau_0))\mathbf{E}\{V(\delta(\tau_0), \tau_0, \sigma)\} \\ &\qquad + v_\sigma u^{(\tau-\tau_0)/T_a - N_0 - 1} \exp(-\beta(\tau - \tau_0)) \exp(\beta T_a) \\ &\qquad + v_\sigma u^{(\tau-\tau_0)/T_a - N_0 - 2} \exp(-\beta(\tau - \tau_0)) \exp(2\beta T_a) + \cdots \\ &\qquad + v_\sigma u^{(\tau-\tau_0)/T_a - N_0 - k} \exp(-\beta(\tau - \tau_0)) \exp(k\beta T_a) \\ &\quad= u^{(\tau-\tau_0)/T_a - N_0} \exp(-\beta(\tau - \tau_0))\mathbf{E}\{V(\delta(\tau_0), \tau_0, \sigma)\} \\ &\qquad + v_\sigma \exp(-\beta(\tau - \tau_0)) \frac{u^{(\tau-\tau_0)/T_a - N_0 - 1} \exp(\beta T_a)(1 - u^{-k} \exp(k\beta T_a))}{1 - u^{-1} \exp(\beta T_a)}. \end{aligned} \tag{27}$$

Based on Definition 2 and (27), one gets

$$\begin{aligned} \mathbf{E}\{V(\delta(\tau), \tau, \sigma)\} \leqslant{}& u^{(\tau-\tau_0)/T_a - N_0} \exp(-\beta(\tau - \tau_0))\mathbf{E}\{V(\delta(\tau_0), \tau_0, \sigma)\} \\ &+ \frac{v_\sigma \exp(-\beta(\tau - \tau_0 - T_a)) u^{(\tau-\tau_0)/T_a - N_0 - 1}}{1 - u^{-1} \exp(\beta T_a)} \\ &+ \frac{v_\sigma u^{-1} \exp((1 - N_0)\beta T_a)}{u^{-1} \exp(\beta T_a) - 1}. \end{aligned} \tag{28}$$

When $0 < u < 1$, the inequality $\ln u/T_a - \beta < 0$ holds, the right side of (28) will converge to $v_\sigma u^{-1} \exp(1 - N_0)\beta T_a/(u^{-1} \exp(\beta T_a) - 1)$ as $\tau \to \infty$. Furthermore, according to (11), one gets

$$\mathbf{E}\{\|\delta(\tau)\|^2\} \leqslant \frac{1}{\varpi}\mathbf{E}\{V(\delta(\tau), \tau, \sigma)\}.$$

When $\tau \to \infty$, the error system (8) is derived as

$$\mathcal{M} = \left\{ \delta(\tau) \in \mathbb{R}^{N\times N} \big| \mathbf{E}\{\|\delta(\tau)\|^2\} \leqslant \frac{1}{\varpi} \frac{v_\sigma u^{-1} \exp(1-N_0)\beta T_a}{u^{-1}\exp(\beta T_a) - 1} \right\}. \tag{29}$$

Consequently, depending on Definition 1, the bounded leader-following identical consensus of the NMJMASs under DAs can be achieved. This completes the proof. □

Remark 3. According to (20) and (29), the attack energy z and the average value of impulsive interval T_a influence the error bound. The smaller z will yield the smaller v_σ, and then the smaller $\mathcal{M}$ will be obtained. From parameter v_σ it can be seen that the DAs have an influence on the design of the impulse sequence.

3.2 The identical consensus of NMJMASs under DoS attacks

In this section, the NMJMASs are suffered from DoS attacks, which will cause the agents unable to communicate with each other. In order to achieve identical consensus of NMJMASs under DoS attacks, in what follows, the control input of agent i is constructed as

$$u_i(\tau) = \begin{cases} \sum_{k=1}^{\infty} c[\sum_{j=1}^{N} -l_{ij}x_j(\tau) - d_{i0}(x_i(\tau) - s(\tau))]f(\tau - \tau_k), \\ \qquad \tau \in [h_k,\, h_k + t_k), \\ 0, \quad \tau \in [h_k + t_k, h_{k+1}), \end{cases} \tag{30}$$

where the meanings of c, l_{ij}, d_{i0}, and τ_k are the same as those explained before. The time interval is divided into intervals $[h_k, h_{k+1})$. t_k is the time when the systems can communicate normally in the interval $[h_k, h_{k+1})$.

$$u_i(\tau) = \begin{cases} c\sum_{j=1}^{N} -l_{ij}x_j(\tau) - cd_{i0}(x_i(\tau) - s(\tau)) & \text{if } \tau = \tau_k,\ f(\cdot) = 1, \\ 0 & \text{if } \tau \neq \tau_k,\ f(\cdot) = 0. \end{cases}$$

Remark 4. In (30), a connectivity recovery mechanism is used for the case where the NMJMASs are subjected to DoS attacks. At the moment $h_k + t_k$, the NMJMASs suffered from DoS attacks, which takes some time to repair or recover. At time h_{k+1}, the effects of DoS attacks are eliminated. The interval $[h_k,\, h_k + t_k)$ indicates that the systems can run normally, the interval $[h_k + t_k,\, h_{k+1})$ indicates that the systems are in recovery time, and there is no impulse in $[h_k + t_k,\, h_{k+1})$. The next time interval $[h_{k+1},\, h_{k+1} + t_{k+1})$, the network topology of the multi-agent works properly before the next DoS attacks occurs to ensure that the systems can function normally.

Based on (1), (2), and (30), when $\tau \in [h_k, h_k + t_k)$, the error system is obtained as follows:

$$\begin{aligned} \dot{\delta}_i(\tau) &= A_{\sigma(\tau)}\delta_i(\tau) + B_{\sigma(\tau)} g\big(\delta_i(\tau), s(\tau)\big), \quad \tau \neq \tau_k, \\ \varDelta\delta_i(\tau_k) &= -c\sum_{j=1}^{N} l_{ij}\delta_j(\tau_k^-) - cd_{i0}\delta_i(\tau_k^-). \end{aligned}$$

Theorem 2. *Under Assumptions* 1 *and* 2*, the NMJMASs subjected to DoS attacks achieve identical consensus if there exist matrices* $P_\sigma > 0$ $(\sigma \in \mathbb{M})$ *and scalars* β, $\rho > 0$ *such that the following conditions hold*:

$$\begin{bmatrix} A_\sigma Q_\sigma + Q_\sigma A_\sigma^{\mathrm{T}} + B_\sigma B_\sigma^{\mathrm{T}} + \pi_{\sigma\sigma} Q_\sigma + Q_\sigma \beta & Q_\sigma & \aleph \\ * & -\rho^{-2} I_N & 0 \\ * & * & \Sigma \end{bmatrix} < 0, \tag{31}$$

$$\frac{T_b}{\tau} \leqslant \frac{\varepsilon + (-\beta + \frac{\ln \xi}{T_a})}{\frac{\ln \xi}{T_a}}, \tag{32}$$

$$\frac{\ln \xi}{T_a} < \beta, \tag{33}$$

$$0 < \xi < 1, \tag{34}$$

$$\varepsilon > 0, \tag{35}$$

where

$$\aleph = \big(\sqrt{\pi_{\sigma 1}} Q_\sigma, \sqrt{\pi_{\sigma 2}} Q_\sigma, \dots, \sqrt{\pi_{\sigma\sigma-1}} Q_\sigma, \sqrt{\pi_{\sigma\sigma+1}} Q_\sigma, \dots, \sqrt{\pi_{\sigma M}} Q_\sigma\big),$$
$$\Sigma = -\operatorname{diag}\{Q_1, \dots, Q_{\sigma-1}, Q_{\sigma+1}, \dots, Q_M\}, \qquad Q_\sigma = P_\sigma^{-1},$$
$$\xi = \varphi_{\max}^2 (I_N - cH).$$

Proof. Construct the same Lyapunov function as in Theorem 1, therefore, one can get that the Lyapunov function is constructed as follows:

$$V\big(\delta(\tau), \tau, \sigma(\tau)\big) = \delta^{\mathrm{T}}(\tau)(I_N \otimes P_{\sigma(\tau)})\delta(\tau).$$

In the next step, define $\sigma(\tau) = \sigma$ $(\sigma \in \mathbb{M})$, for $\tau \neq \tau_k$, by the total probability formula as well as the conditional expectation formula, one gets

$$\begin{aligned} \mathbf{E}\big\{\mathcal{A}V\big(\delta(\tau), \tau, \sigma\big)\big\} &= \mathrm{He}\big\{\delta^{\mathrm{T}}(\tau)(I_N \otimes P_\sigma)\big[(I_N \otimes A_\sigma)\delta(\tau) \\ &\quad + (I_N \otimes B_\sigma) G\big(\delta(\tau), s(\tau)\big)\big]\big\} \\ &\quad + \delta^{\mathrm{T}}(\tau) \sum_{n=1}^{z} \pi_{\sigma n}(I_N \otimes P_n)\delta(\tau). \end{aligned} \tag{36}$$

According to Assumption 1, then

$$\begin{aligned} &2\delta(\tau)(I_N \otimes P_\sigma B_\sigma) G\big(\delta(\tau), s(\tau)\big) \\ &\quad \leqslant \delta_i^{\mathrm{T}}(\tau)\big(I_N \otimes (P_\sigma B_\sigma B_\sigma^{\mathrm{T}} P_\sigma + \rho^2 I_N)\big)\delta_i(\tau). \end{aligned} \tag{37}$$

Based on Lemma 1, according to (31), (36), and (37),

$$\begin{aligned} &\mathbf{E}\big\{\mathcal{A}V\big(\delta(\tau), \tau, \sigma\big)\big\} \\ &\quad \leqslant \delta^{\mathrm{T}}(\tau)\left[I_N \otimes \left(\mathrm{He}(A_\sigma P_\sigma) + \sum_{n=1}^{M} \pi_{\sigma n} P_n + P_\sigma B_\sigma B_\sigma^{\mathrm{T}} P_\sigma + \rho^2 I_N\right)\right]\delta(\tau) \\ &\quad \leqslant -\beta\delta^{\mathrm{T}}(\tau)(I_N \otimes P_\sigma)\delta(\tau) = -\beta \mathbf{E}\big\{V\big(\delta(\tau), \tau, \sigma\big)\big\}. \end{aligned} \tag{38}$$

Then, by integrating (38), it is easy to obtain

$$\mathbf{E}\{V(\delta(\tau),\tau,m)\} \leqslant \exp(-\beta(\tau-\tau_{k-1})) \\ \times \mathbf{E}\{V(\delta(\tau_{k-1}),\tau_{k-1},m)\}, \quad \tau \in [\tau_{k-1},\tau_k).$$

In what follows, for $\tau = \tau_k$, by calculation, the following formula can be obtained:

$$\begin{aligned} &\mathbf{E}\{V(\delta(\tau_k),\tau_k,\sigma)\} \\ &\quad = \mathbf{E}\{\delta^{\mathrm{T}}(\tau_k)(I_N\otimes P_\sigma)\delta(\tau_k)\} \\ &\quad = \mathbf{E}\{[((I_N-cH)\otimes I_n)\delta(\tau_k^-)]^{\mathrm{T}}(I_N\otimes P_\sigma)[((I_N-cH)\otimes I_n)\delta(\tau_k^-)]\} \\ &\quad = \mathbf{E}\{\delta^{\mathrm{T}}(\tau_k^-)((I_N-cH)\otimes I_n)^{\mathrm{T}}(I_N\otimes P_\sigma)((I_N-cH)\otimes I_n)\delta(\tau_k^-)\} \\ &\quad \leqslant \varphi_{\max}^2(I_N-cH)\mathbf{E}\{V(\delta(\tau_k^-),\tau_k^-,\sigma)\} = \xi\mathbf{E}\{V(\delta(\tau_k^-),\tau_k^-,\sigma)\}. \end{aligned}$$

If $\tau \in [h_k,\ h_k+t_k)$, based on (33), one has

$$\mathcal{Q}(\tau,s) = \exp(\tau-s)\prod_{s\leqslant\tau_k\leqslant\tau}\xi \leqslant \xi^{-N_0}\exp\left(\left(-\beta+\frac{\ln\xi}{T_a}\right)(\tau-s)\right),$$

then

$$\mathbf{E}\{V(\delta(\tau),\tau,\sigma)\} \leqslant \mathcal{Q}(\tau,h_l)\mathbf{E}\{V(\delta(h_l),h_l,\sigma)\} \\ = \xi^{-N_0}\exp\left(\left(-\beta+\frac{\ln\xi}{T_a}\right)(\tau-h_l)\right)\mathbf{E}\{V(\delta(h_l),h_l,\sigma)\}. \quad (39)$$

When $\tau \in [h_{k-1}+t_{k-1},\ h_k)$, because there exist DoS attacks, the impulsive control is invalid. Then one can obtain that

$$\mathbf{E}\{V(\delta(\tau),\tau,\sigma)\} \leqslant \exp(-\beta(\tau-h_l-t_l))\mathbf{E}\{V(\delta(h_{l-1}),h_{l-1},\sigma)\}. \quad (40)$$

For $\tau \in [h_0,\ h_0+t_0)$, according to (39), it can deduce

$$\mathbf{E}\{V(\delta(\tau),\tau,\sigma)\} \leqslant \xi^{-N_0}\exp\left(\left(-\beta+\frac{\ln\xi}{T_a}\right)(\tau-h_0)\right)\mathbf{E}\{V(\delta(h_0),h_0,\sigma)\}. \quad (41)$$

For $\tau \in [h_0+t_0,\ h_1)$, combining (40) and (41), one has

$$\mathbf{E}\{V(\delta(\tau),\tau,\sigma)\} \leqslant \xi^{-N_0}\exp(-\beta(\tau-h_0-t_0))\exp\left(\left(-\beta+\frac{\ln\xi}{T_a}\right)t_0\right) \\ \times \mathbf{E}\{V(\delta(h_0),h_0,\sigma)\}. \quad (42)$$

For $\tau \in [h_1,\ h_1+t_1)$, based on (39) and (42), one gets

$$\mathbf{E}\{V(\delta(\tau),\tau,\sigma)\} \leqslant \xi^{-N_0}\exp(-\beta(\tau-h_0-t_0)) \\ \times \exp\left(\left(-\beta+\frac{\ln\xi}{T_a}\right)(\tau-h_1+t_0)\right)\mathbf{E}\{V(\delta(h_0),h_0,\sigma)\}. \quad (43)$$

For $\tau \in [h_1 + t_1,\ h_2)$, according to (40) and (43), it yields

$$\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\} \leqslant \xi^{-N_0}\exp\bigl(-\beta(\tau-h_0-t_0)\bigr) \\ \times \exp\biggl(\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)(t_0+t_1)\biggr)\mathbf{E}\bigl\{V\bigl(\delta(h_0),h_0,\sigma\bigr)\bigr\}.$$

Assuming $\tau \in [h_{l-1},\ h_{l-1}+t_{l-1})$, the following inequality

$$\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\} \leqslant \xi^{-N_0}\exp\bigl(-\beta(\tau-h_0-t_0)\bigr) \\ \times \exp\biggl(\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)\bigl(\tau-h_{l-1}+t_0+t_1+\cdots+t_{l-2}\bigr)\biggr) \\ \times \mathbf{E}\bigl\{Vv(\delta(h_0),h_0,\sigma)\bigr\} \tag{44}$$

holds. Then, for $\tau \in [h_{l-1}+t_{l-1},\ h_l)$, from (40) and (44) one can derive the inequality as follows:

$$\begin{aligned}
&\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\} \\
&\quad\leqslant \exp\bigl(-\beta\bigl(\tau-h_{l-1}-t_{l-1}\bigr)\bigr)\mathbf{E}\bigl\{V\bigl(\delta(h_{l-1}),h_{l-1},\sigma\bigr)\bigr\} \\
&\quad\leqslant \exp\bigl(-\beta\bigl(\tau-h_{l-1}-t_{l-1}\bigr)\bigr)\xi^{-N_0}\exp\bigl(-\beta(h_{l-1}-t_{l-1}-h_0-t_0)\bigr) \\
&\qquad\times \exp\biggl(\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)(h_{l-1}+t_{l-1}-h_{l-1}+t_0+t_1+\cdots+t_{l-2})\biggr) \\
&\qquad\times \mathbf{E}\bigl\{V\bigl(\delta(h_0),h_0,\sigma\bigr) \\
&\quad= \xi^{-N_0}\exp\bigl(-\beta(\tau-h_0-t_0)\bigr)\exp\biggl(\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)(t_0+t_1+\cdots+t_{l-1})\biggr) \\
&\qquad\times \mathbf{E}\bigl\{V(\delta(h_0),h_0,\sigma)\bigr\}.
\end{aligned} \tag{45}$$

Therefore, for $\tau \in [h_l, h_l + t_l)$, according to (39) and (45), one can derive the following inequality:

$$\begin{aligned}
&\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\} \\
&\quad\leqslant \xi^{-N_0}\exp\bigl(-\beta(\tau-h_l)\bigr)\mathbf{E}\bigl\{V\bigl(\delta(h_l),h_l,\sigma\bigr)\bigr\} \\
&\quad\leqslant \xi^{-N_0}\exp\bigl(-\beta(\tau-h_l)\bigr)\xi^{-N_0}\exp\bigl(-\beta(\tau-h_0-t_0)\bigr) \\
&\qquad\times \exp\biggl(\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)(t_0+t_1+\cdots+t_{l-1})\biggr)\mathbf{E}\bigl\{V(\delta(h_0),h_0,\sigma)\bigr\} \\
&\quad= \xi^{-N_0}\exp\bigl(-\beta(\tau-h_0-t_0)\bigr) \\
&\qquad\times \exp\biggl(\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)(\tau-h_l+t_0+t_1+\cdots+t_{l-1})\biggr) \\
&\qquad\times \mathbf{E}\bigl\{V\bigl(\delta(h_0),h_0,\sigma\bigr)\bigr\}.
\end{aligned}$$

Based on mathematical induction, for any $\tau \in [h_l,\ h_l+t_l)$, $l = 1, 2, \dots$, the following inequality

$$\begin{aligned}\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\} \leqslant \xi^{-N_0}&\exp\bigl(-\beta(\tau-h_0-t_0)\bigr)\\ &\times\exp\biggl(\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)(\tau-h_l+t_0+t_1+\dots+t_{l-1})\biggr)\\ &\times\mathbf{E}\bigl\{V\bigl(\delta(h_0),h_0,\sigma\bigr)\bigr\}\end{aligned}\tag{46}$$

holds. Let $T_b = \theta_0 + \theta_1 + \dots + \theta_{l-1}$, where θ_i indicates the total attack duration of interval $[h_i + t_i,\ h_{i+1})$ $(i = 0, \dots, l-1)$. Combining with (46), one has

$$\begin{aligned}\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\} &\leqslant \xi^{-N_0}\exp\bigl(-\beta(\theta_0+\theta_1+\dots+\theta_{l-1})\bigr)\\ &\quad\times\exp\biggl(\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)(\tau-h_l+t_0+t_1+\dots+t_{l-1})\biggr)\\ &\quad\times\mathbf{E}\bigl\{V(\delta(h_0),h_0,\sigma)\bigr\}\\ &\leqslant \xi^{-N_0}\exp\bigl(-\beta T_b\bigr)\exp\biggl(\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)(\tau-T_b)\biggr)\\ &\quad\times\mathbf{E}\bigl\{V(\delta(h_0),h_0,\sigma)\bigr\}.\end{aligned}\tag{47}$$

According to (32), (34), and (35), it holds

$$\biggl(-\beta+\frac{\ln\xi}{T_a}\biggr)(\tau-T_b)-\beta T_b \leqslant -\varepsilon\tau,\tag{48}$$

thus, according to (47) and (48), one has

$$\mathbf{E}\bigl\{V\bigl(\delta(\tau),\tau,\sigma\bigr)\bigr\} \leqslant \xi^{-N_0}\exp\bigl(-\varepsilon\tau\bigr)\mathbf{E}\bigl\{V\bigl(\delta(h_0),h_0,\sigma\bigr)\bigr\}.$$

Combining with (35), the state decay estimation of the identical consensus tracking error is obtained as follows:

$$\mathbf{E}\bigl\{\bigl\|\delta_i(\tau)\bigr\|^2\bigr\} \leqslant \frac{\xi^{-N_0}\exp(-\varepsilon\tau)\mathbf{E}\{V(\delta(h_0),h_0,\sigma)\}}{\varpi}.$$

Consequently, according to Definition 3, the leader-following identical consensus of the NMJMASs under DoS attacks can be achieved. So the proof is completed. □

Remark 5. According to (34), an appropriate coupling strength c is selected within a certain range to achieve the leader-following identical consensus of NMJMASs under DoS attacks.

Remark 6. Compared with [8,23], the DoS attacks were not considered. The MASs were considered without Markov jump parameters in [7, 8, 21, 23, 26]. Compared with [15], impulse was not considered.

4 Numerical examples

Example 1. Consider that NMJMASs have one leader (node 0) as well as four followers (nodes 1, 2, 3, and 4). The network communication topology of multi-agent is described in Fig. 1. At the same time, the topology diagram is a directed topology diagram. Suppose $\mathbb{M} = \{1, 2\}$. Let the constant coefficient matrices A_1, A_2, B_1, and B_2 be as follows:

$$A_1 = \begin{bmatrix} -1.1 & 2 & 0 \\ 2 & -1.3 & 0 \\ 4 & 0 & - \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1.14 & -1.4 & -1.5 \\ -1.4 & 1.16 & -1.0 \\ -1.1 & 2.0 & 1.16 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.71 & -1.9 & 0.03 \\ 2.1 & -0.9 & 0.01 \\ 3.5 & 0.04 & -1.5 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -1.65 & -1.8 & 2.8 \\ -1.16 & 1.1 & -0.3 \\ 0.7 & -2.4 & 1.2 \end{bmatrix},$$

$$\varPi = \begin{bmatrix} -0.3 & 0.1 & 0.2 \\ 0.4 & -0.6 & 0.2 \\ 0.3 & 0.2 & -0.5 \end{bmatrix}.$$

Assume that the state dimension of each agent is 3, then $x_i(\tau) = [x_{i1}, x_{i2}, x_{i3}]^{\mathrm{T}} \in \mathbb{R}^3$, $i = 1, 2, 3, 4$. The nonlinear function $f_i(x(\tau)) = (\tanh(x_{i1}(\tau)), \tanh(x_{i2}(\tau)), \tanh(x_{i3}(\tau)))^{\mathrm{T}}$. It is assumed that the initial states are described by

$$s_0(0) = [0.2, -0.1, 0.1]^{\mathrm{T}},$$

$$x_1(0) = [1.2, -0.7, 2.5]^{\mathrm{T}}, \qquad x_2(0) = [1, 1.2, -1.5]^{\mathrm{T}},$$
$$x_3(0) = [0.4, -1, -2]^{\mathrm{T}}, \qquad x_4(0) = [-1, 1.2, 0.1]^{\mathrm{T}}.$$

In terms of the communication topological diagram Fig. 1, it is easy to obtain the adjacency matrix $D = \mathrm{diag}([1, 0, 1, 0])$ of the leader and the corresponding Laplacian matrix

$$L = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & -1 & 0 & 2 \end{bmatrix}.$$

With other parameters $c = 0.4$, $\omega = 15$, $z = 1.2$, $\varpi = 0.058$, $\eta = 1.9$, $\rho = 8.5$, and $\beta = 10.21$, according to Theorem 1, one obtains $T_a < 0.884$. Then the impulsive interval $T_a = 0.884$, and W is given as

$$W = \begin{bmatrix} 0 & 0.5 & 0 & 0.2 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0.4 & 0 & 0.6 & 0 \end{bmatrix}.$$

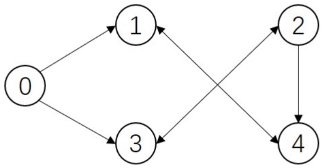


Figure 1. The communication topological structure of NMJMASs under DAs.

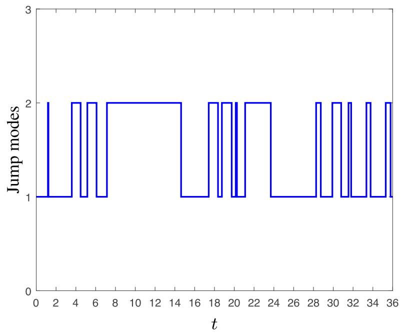


Figure 2. The jumping mode of Markov chain $\sigma(\tau)$.

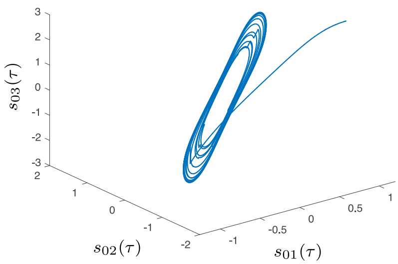


Figure 3. The trajectory diagram of the leader.

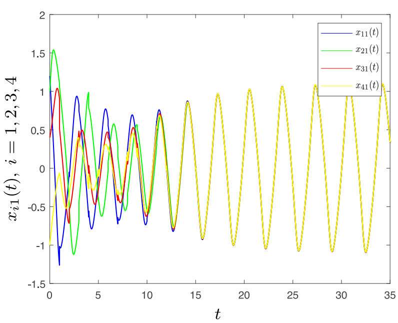
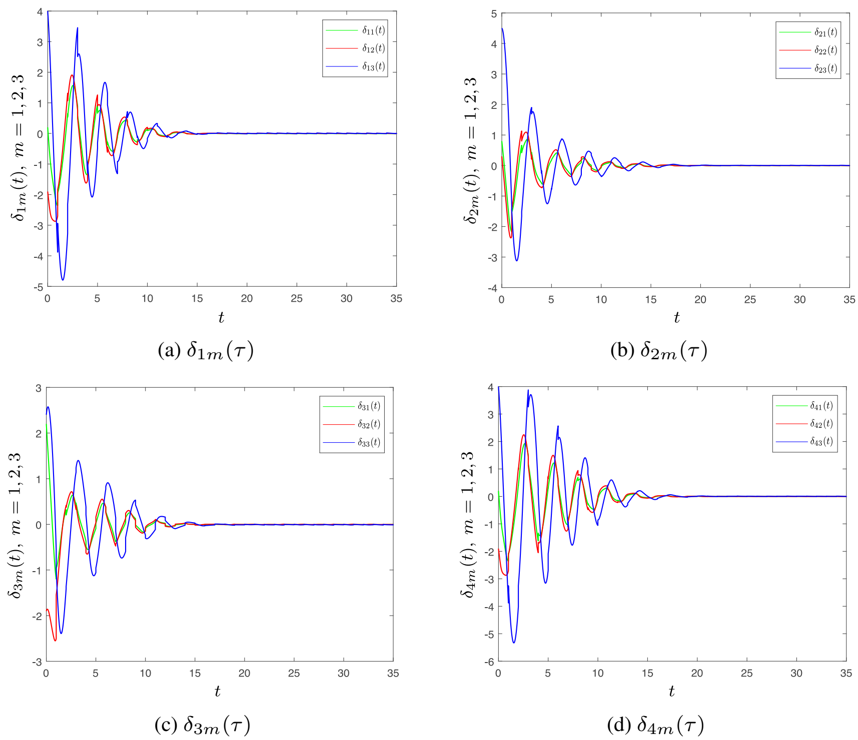


Figure 4. The trajectory diagram of the first component of the four agents.

Based on Theorem 1, one obtains $u = 0.954$ and $\upsilon_\sigma = 1.99$, then the error boundary is 0.00411. After verification, all conditions of Theorem 1 are satisfied. Figure 2 stands for the jumping mode of Markov chain $\sigma(\tau)$. Figure 3 describes the trajectory diagram of the leader. Figure 4 shows the trajectory diagram of the first variables of the four followers, the trajectories of the other two vector components are the same as those of the first component. Figure 5 stands for the trajectory diagram of the error systems. According to the simulation results, the bounded identical consensus of NMJMASs under DAs can be achieved. Therefore, it is proved that the result of Theorem 1 is correct and the method is effective.



(a) $\delta_{1m}(\tau)$ (b) $\delta_{2m}(\tau)$ (c) $\delta_{3m}(\tau)$ (d) $\delta_{4m}(\tau)$

Figure 5. The trajectory diagram of the error systems.

Example 2. Consider that NMJMASs have one leader (node 0) as well as four followers (nodes 1, 2, 3, and 4). The network communication topology of multi-agent is shown in Fig. 6. At the same time, the topology diagram is a directed topology diagram. Suppose $\mathbb{M} = \{1, 2\}$. Let the constant coefficient matrices A_1, A_2, B_1, and B_2 be as follows:

$$A_1 = \begin{bmatrix} -1.1 & 2 & 0 \\ 2 & -1.3 & 0 \\ 4 & 0 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1.14 & -1.4 & -1.5 \\ -1.4 & 1.16 & -1.0 \\ -1.1 & 2.0 & 1.16 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.71 & -1.9 & 0.03 \\ 2.1 & -0.9 & 0.01 \\ 3.5 & 0.04 & -1.5 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -1.65 & -1.8 & 2.8 \\ -1.16 & 1.1 & -0.3 \\ 0.7 & -2.4 & 1.2 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} -0.4 & 0.3 & 0.1 \\ 0.1 & -0.5 & 0.4 \\ 0.4 & 0.2 & -0.6 \end{bmatrix}.$$

Assume that the state dimension of each agent is 3, then $x_i(\tau) = [x_{i1}, x_{i2}, x_{i3}]^{\mathrm{T}} \in \mathbb{R}^3$, $i = 1, 2, 3, 4$. The nonlinear function $f_i(x(\tau)) = (\tanh(x_{i1}(\tau)), \tanh(x_{i2}(\tau)),$

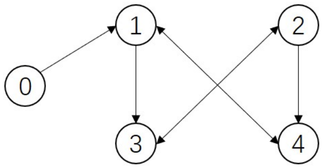


Figure 6. The communication topological structure of NMJMASs under DoS attacks.

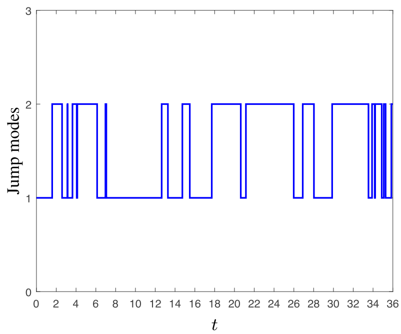


Figure 7. The jumping mode of Markov chain $\sigma(\tau)$.

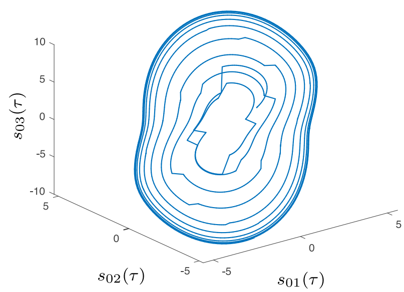


Figure 8. The trajectory diagram of the leader.

$\tanh(x_{i3}(\tau)))^{\mathrm{T}}$. It is assumed that the initial states are described as follows:

$$s_0(0) = [0.2, -0.1, 0.1]^{\mathrm{T}},$$

$$x_1(0) = [1.2, -0.7, 2.5]^{\mathrm{T}}, \qquad x_2(0) = [1, 1.2, -1.5]^{\mathrm{T}},$$

$$x_3(0) = [0.4, -1, -2]^{\mathrm{T}}, \qquad x_4(0) = [-1, 1.2, 0.1]^{\mathrm{T}}.$$

From the communications topology diagram in Fig. 6 it can be obtained that $D = \operatorname{diag}([1, 0, 0, 0])$ and the corresponding Laplacian matrix

$$L = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & -1 & 0 & 2 \end{bmatrix}.$$

With other parameters $c = 0.1$, $\varepsilon = 7.5$, $\xi = 0.981$, $\beta = 10.21$, and $\rho = 5.8$, according to (33) in Theorem 2, one obtains $T_b < 0.002$. Based on (32), one has

$$\frac{T_a}{\tau} \leqslant \frac{\varepsilon + (-\beta + \frac{\ln \xi}{T_b})}{\frac{\ln \xi}{T_b}} \leqslant 0.7.$$

After verification, all conditions of Theorem 2 are satisfied. Figure 7 stands for the jumping mode of Markov chain $\sigma(\tau)$. Figure 8 shows the trajectory diagram of the leader. Figure 9 describes the trajectory diagram of the first variables of the four followers,

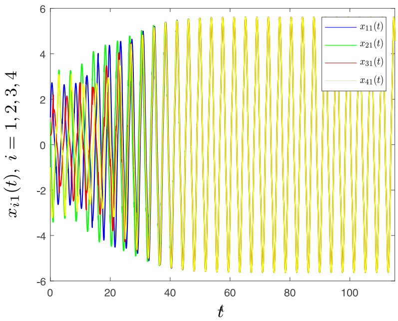
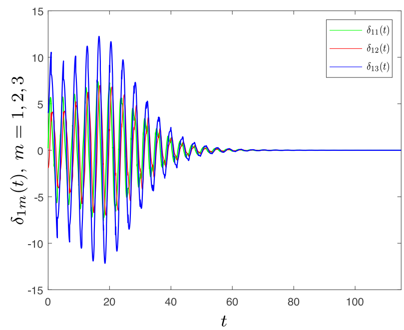
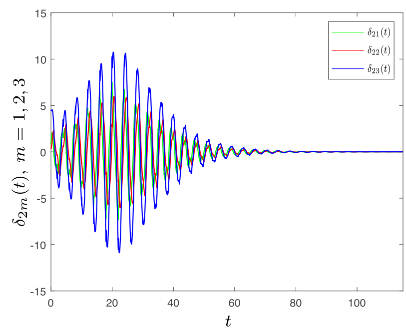


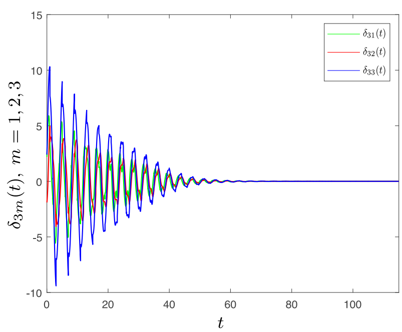
Figure 9. The trajectory diagram of the first component of the four agents.



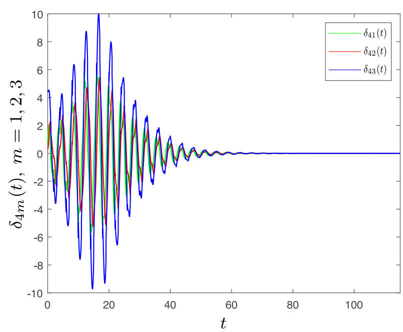
(a) $\delta_{1m}(\tau)$



(b) $\delta_{2m}(\tau)$



(c) $\delta_{3m}(\tau)$



(d) $\delta_{4m}(\tau)$

Figure 10. The trajectory diagram of the error systems.

the trajectories of the other two vector components are the same as those of the first component. Figure 10 shows that the trajectory diagram of the error systems tends to zero within both the leader and followers, so the NMJMASs under DoS attacks can achieve identical consensus. Thus, it is shown that the result of Theorem 2 is correct and the method is valid.

5 Conclusion

The issue of leader-following identical consensus for NMJMASs with impulse under DAs or DoS attacks has been addressed. Based on the Lyapunov stability theory as well as stochastic analysis method, sufficient conditions of the identical consensus for MASs have been obtained by using the impulsive control strategy, which generalizes the results of some existing literatures. At last, two numerical simulations have been given to prove the correctness and validity of the results. We will be concerned with the leader-following identical consensus for NMASs with random communication topology and subjected to DAs or DoS attacks. Furthermore, the partial component consensus and lag consensus of NMASS will be studied.

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