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Global exponential convergence of delayed inertial Cohen–Grossberg neural networks*

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Abstract. In this paper, the exponential convergence of delayed inertial Cohen–Grossberg neural networks (CGNNs) is studied. Two methods are adopted to discuss the inertial CGNNs, one is expressed as two first-order differential equations by selecting a variable substitution, and the other does not change the order of the system based on the nonreduced-order method. By establishing appropriate Lyapunov function and using inequality techniques, sufficient conditions are obtained to ensure that the discussed model converges exponentially to a ball with the prespecified convergence rate. Finally, two simulation examples are proposed to illustrate the validity of the theorem results.

Keywords: inertial Cohen–Grossberg neural networks, time-varying delays, exponential convergence, convergence rate.

1 Introduction

In the past few decades, neural networks have been widely studied due to their practical application in combinatorial optimization, associative memory, pattern recognition and other fields. In the meantime, the dynamic properties of neural networks, such as equilibrium state, stability, attractor, attractor domain, periodic solution, bifurcation problem

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and chaos problem, have been explored (see [3–5, 15, 20, 28, 31–33]). For instance, Xiong et al. [28] researched the robust convergence for CGNNs with delay. Scholars discussed global asymptotic stability of delayed complex-valued CGNNs (see [31, 32]). Peng et al. [20] presented finite-time synchronization of CGNNs via designing a suitable controller. Cao et al. [4] discussed the finite-time stabilization of leakage delay on complex-valued BAM neural networks, and the event-based state estimation problem for discrete-time recurrent delayed semi-Markovian neural networks was investigated in [3].

From the view of both theory and application, it is necessary to consider the multistability characteristics of systems. When the neural networks is dedicated to the function of associative memory, the networks should have multiple equilibrium points (see [7, 18]). In addition, the boundedness of solutions for the networks is an important characteristic, and scholars study the stability of the networks. In some cases, they do not need to care about the information of equilibrium points. For example, Li and Jian presented the stability of the system without considering the existence and uniqueness of equilibrium points [16]. Under different delay types, Jian and Wan discussed global exponential convergence of generalized chaotic systems in [11].

Many of the proposed researches focused on the neural networks, whose states are the first-order derivatives, and the neural networks, whose states are the second-order derivatives, have better performance in dynamic behavior research. Second-order neural networks have more complex dynamic behavior and clear biological background. For example, an equivalent circuit, including inductance in animals semicircular canal, was proposed to realize the hair cell membrane (see [1, 2]). In 1997, Wheeler and Schieve [27] firstly proposed the inertial neural networks and studied its stability, bifurcation and chaos phenomenon. Since then, He et al. [10] used the inertia item as a critical tool and added the inertial term to the neuron system that can produce bifurcation and chaos. Up to now, many existing literatures have discussed the dynamic behavior of inertial neural networks (see [6, 9, 13, 14, 17, 21, 23–26, 29, 30]). For example, in [23, 26], the authors investigated global exponential convergence of impulsive inertial neural networks. Global exponential stabilization for inertial neural networks have been explored (see [21, 25]), and scholars have researched the synchronization of drive-response systems by designing different control strategies (see [6, 9]). In general, the exponential convergence of inertial CGNN has not attracted much attention, which is the purpose of this study.

Although scholars have studied the stability of inertial CGNN, few work focused on solving the exponential convergence problem of nonreduced-order inertial CGNN. From what has been discussed above, it is meaningful to further explore the exponential convergence of inertial CGNNs, regardless of the existence and uniqueness of the equilibrium point. This paper mainly has the following two aspects of contributions:

(i) By variable substitutions, the inertial system is converted into two first-order systems (see [12]). Different from [12], both reduced order and nonreduced order are adopted to discuss the inertial CGNNs model in this paper.

(ii) Compared with the networks model in [23, 26], the proposed inertial CGNNs in this paper are more general, the exponential convergence rate and the specific estimation problem of the convergence ball for the networks are given out.

The arrangement of this paper is as follows. Section 2 gives the description of the model, definition and lemmas. By setting up a suitable Lyapunov function and using inequality techniques, Section 3 discusses the global exponential convergence of the model. Section 4 verifies the validity of the results by two examples, and Section 5 draws the conclusion.

Notations. $\mathbb{R}$, $\mathbb{R}^m$, $\mathbb{R}^{m\times n}$ refer to the set of real numbers, m-dimensional real space, $m\times n$ real matrices, respectively. Denote $\|\ell\| = (\sum_{i=1}^n \ell_i^2)^{1/2}$ for any $\ell = (\ell_1, \ell_2, \dots, \ell_n)^{\mathrm{T}} \in \mathbb{R}^n$. I represents the identity matrix of the appropriate dimension. $\lambda_{\min}(\mathcal{Q})$ and $\lambda_{\max}(\mathcal{Q})$ stand for the minimal and maximum eigenvalues of matrix $\mathcal{Q} \in \mathbb{R}^{m\times m}$, respectively. $\mathcal{Q} < 0$ shows that the matrix $\mathcal{Q}$ is negative definite. Let $\mathcal{N} = 1, 2, \dots, n$.

2 Preliminaries

Consider the inertial CGNNs as follows:

$$\begin{aligned}\frac{\mathrm{d}^2 y_k(t)}{\mathrm{d}t^2} = -\beta_k \frac{\mathrm{d}y_k(t)}{\mathrm{d}t} - \alpha_k\big(y_k(t)\big)\Bigg[f_k\big(y_k(t)\big) - \sum_{m=1}^n c_{km} g_m\big(y_m(t)\big)\\ - \sum_{m=1}^n d_{km} g_m\big(y_m\big(t - \varrho_m(t)\big)\big) - U_k(t)\Bigg],\end{aligned} \tag{1}$$

where $k, m \in \mathcal{N}$, $y_k(t)$ represents the kth state variable of the networks at time t, the second derivative of $y_k(t)$ is called as the inertial term, $\beta_k > 0$ is the damping coefficient, $\alpha_k(\cdot)$ denotes amplification function, $f_k(\cdot)$ is an appropriately behaved function such that the solutions of system (1) remain bounded, and $g_m(\cdot)$ is the activation function, which shows how neurons respond to each other. c_{km} and d_{km} represent the weight coefficients, the external input is $U_k(t)$. The transmission delay $\varrho_m(t)$ is a continuous bounded function with $0 \leqslant \varrho_m(t) \leqslant \varrho$.

The initial conditions of (1) are

$$y_k(s) = \widetilde{\phi}_k(s), \quad \dot{y}_k(s) = \widetilde{\varphi}_k(s), \quad s \in [-\varrho, 0],\ k \in \mathcal{N},$$

where $\widetilde{\phi}_k(s)$ and $\widetilde{\varphi}_k(s)$ are continuous and bounded functions.

Remark 1. The inertial system considered in [12] is without time-varying delays. The authors considered CGNNs only with the first-order derivative of the states in [22, 28]. The presented system here is more general than the systems discussed in [12, 22, 28].

The following assumptions will be used in this article:

(H1) $\alpha_k(y_k(t))$ is a bounded function, and there exist two positive constants $\underline{\alpha_k}$ and $\overline{\alpha_k}$ satisfying the inequality $\underline{\alpha_k} < \alpha_k(y_k(t)) < \overline{\alpha_k}$. Let $\underline{\alpha} = \min\{\underline{\alpha_1}, \underline{\alpha_2}, \dots, \underline{\alpha_n}\}$, $\overline{\alpha} = \max\{\overline{\alpha_1}, \overline{\alpha_2}, \dots, \overline{\alpha_n}\}$.

(H2) There exist some nonnegative constants l_k to satisfy the following inequalities:

$$\left|g_k(\breve{a}) - g_k(\breve{b})\right| \leqslant l_k|\breve{a} - \breve{b}|, \quad g_k(0) = 0, \quad k \in \mathcal{N},\ \breve{a}, \breve{b} \in \mathbb{R}.$$

Let $\widetilde{l} = \max\{l_1, l_2, \dots, l_n\}$.

(H3) The external input $U_k(t)$ is bounded with $\max\{|U_1(t)|, \dots, |U_n(t)|\} \leqslant \widetilde{U}$.

Definition 1. (See [23].) If for all $\delta \geqslant \sup_{-\varrho \leqslant s \leqslant 0} \|\varPsi(s)\|$, there exists a number $\varGamma = \varGamma(\delta)$ such that

$$\left\|y(t)\right\| \leqslant \gamma + \varGamma e^{-\sigma t}, \quad t \geqslant 0,$$

where $\varPsi(s) = (\widetilde{\phi}_1(s), \widetilde{\phi}_2(s), \dots, \widetilde{\phi}_n(s), \widetilde{\varphi}_1(s), \widetilde{\varphi}_2(s), \dots, \widetilde{\varphi}_n(s))^{\mathrm{T}}$, $s \in [-\varrho, 0]$, then system (1) is said to be globally uniformly exponentially convergent (GUEC) to the ball $\breve{b}(r) = \{y(t) \in \mathbb{R}^n \mid \|y(t)\| \leqslant \gamma\}$ with a rate $\sigma > 0$.

Lemma 1. (*See* [8].) *If there exists* $\varepsilon > 0$, *then the positive-definite Hermitian matrix* $\mathcal{M} \in \mathbb{R}^{n\times n}$ *satisfies the following inequality*:

$$2\breve{a}^{\mathrm{T}}\mathcal{M}\breve{b} \leqslant \varepsilon\breve{a}^{\mathrm{T}}\mathcal{M}\breve{a} + \varepsilon^{-1}\breve{b}^{\mathrm{T}}\mathcal{M}\breve{b} \quad \forall \breve{a}, \breve{b} \in \mathbb{R}^n.$$

Lemma 2. (*See* [19].) *Let* $h(t)$ *be a continuous function,* $h(t) \geqslant 0$ *holds for all* $t \geqslant -\varrho$, *and let*

$$\dot{h}(t) \leqslant -\breve{a}h(t) + \breve{b} \sup_{t-\varrho \leqslant s \leqslant t} h(s) + \breve{c} \quad \forall t \geqslant 0,$$

where $\sup_{-\varrho \leqslant s \leqslant 0} h(s) \leqslant \zeta$, $\breve{a}, \breve{b}, \breve{c} > 0$. *If* $0 < \breve{b} < \breve{a}$, *then*

$$h(t) \leqslant \mathfrak{m} + \zeta e^{-\sigma t} \quad \forall t \geqslant 0,$$

where $\mathfrak{m} = \breve{c}/(\breve{a} - \breve{b})$, *and* σ *is the unique solution to* $\sigma = \breve{a} - \breve{b}e^{\sigma\varrho}$.

3 Main results

The purpose of this section is to obtain some sufficient conditions to guarantee that the state variable of the network (1) is GUEC to a ball.

Theorem 1. *Based on assumptions* (H1)–(H3), *suppose* $\alpha_k(y_k(t))f_k(y_k(t)) = \xi_k y_k(t)$. *For given positive constants* μ_k, ν_k, ξ_k, *if the state variable* $y_k(t)$ *satisfies* $\|y(t)\|^2 \leqslant 2n\widetilde{U}^2/(\omega - \varpi) + \zeta e^{-rt}$, $t \geqslant 0$, *system* (1) *is GUEC. As the same time, network* (1) *is globally exponentially convergent to a ball*

$$\mathfrak{B} = \left\{y(t) \mid \left\|y(t)\right\|^2 \leqslant \frac{2n\widetilde{U}^2}{\omega - \varpi}\right\}$$

with the convergence rate $r/2$, *where* $\sup_{-\varrho \leqslant s \leqslant 0} V(s) \leqslant \zeta$, $\mu_k = 2\xi_k - 3\overline{\alpha_k}^2 - 2\sum_{m=1}^n \sum_{s=1}^n (c_{ms}l_s)^2$, $\nu_k = 2\beta_k - 2 - 3\overline{\alpha_k}^2 > 0$, $\eta_k = \beta_k + \xi_k - 2$, $\omega = \min_{1\leqslant k\leqslant n}\{\mu_k/2, \nu_k, \eta_k\}$, $\varpi = 2\sum_{k=1}^n \sum_{m=1}^n (d_{mk}l_k)^2$ *and* $\omega > \varpi$, $r = \omega - \varpi e^{r\varrho} > 0$.

Proof. Consider the following Lyapunov function $V(t)$:

$$V(t) = \sum_{k=1}^{n} y_k^2(t) + \sum_{k=1}^{n} \bigl(y_k(t) + \dot{y}_k(t)\bigr)^2.$$

Compute the derivative of $V(t)$ along the network (1), one can obtain

$$\begin{aligned}\dot{V}(t)\big|_{(1)} \leqslant{}& 2\sum_{k=1}^{n} y_k(t)\dot{y}_k(t) \\ &+ 2\sum_{k=1}^{n}\Biggl[\bigl(y_k(t)+\dot{y}_k(t)\bigr)\Biggl((1-\beta_k)\dot{y}_k(t) - \alpha_k\bigl(y_k(t)\bigr)\Biggl(f_k\bigl(y_k(t)\bigr) \\ &- \sum_{m=1}^{n} c_{km} g_m\bigl(y_m(t)\bigr) - \sum_{m=1}^{n} d_{km} g_m\bigl(y_m\bigl(t-\varrho_m(t)\bigr)\bigr) - U_k(t)\Biggr)\Biggr)\Biggr].\end{aligned}$$

Combine (H1), (H3) and $2\mathfrak{a}\mathfrak{b} \leqslant \mathfrak{a}^2 + \mathfrak{b}^2$, for all $\mathfrak{a}, \mathfrak{b} \in \mathbb{R}$, one gets

$$\begin{aligned}\dot{V}(t) \leqslant{}& \sum_{k=1}^{n}\Biggl[\bigl(-2\xi_k + 3\overline{\alpha_k}^2\bigr)y_k^2(t) + \bigl(2 - 2\beta_k + 3\overline{\alpha_k}^2\bigr)\dot{y}_k^2(t) \\ &+ 2y_k(t)(2-\beta_k-\xi_k)\dot{y}_k(t) + 2\Biggl(\sum_{m=1}^{n}|c_{km}|\bigl|g_m\bigl(y_m(t)\bigr)\bigr|\Biggr)^2 \\ &+ 2\Biggl(\sum_{m=1}^{n}|d_{km}|\bigl|g_m\bigl(y_m\bigl(t-\varrho_m(t)\bigr)\bigr)\bigr|\Biggr)^2 + 2U_k^2(t)\Biggr].\end{aligned} \tag{2}$$

Based on (H2) and the Cauchy inequality

$$\sum_{i=1}^{n} \mathfrak{a}_i\mathfrak{b}_i \leqslant \Biggl(\sum_{i=1}^{n}\mathfrak{a}_i^2\Biggr)^{1/2}\Biggl(\sum_{i=1}^{n}\mathfrak{b}_i^2\Biggr)^{1/2},$$

for all $\mathfrak{a}_i, \mathfrak{b}_i \in \mathbb{R}$, the following inequalities hold:

$$\begin{aligned}\Biggl(\sum_{m=1}^{n}|c_{km}|\bigl|g_m\bigl(y_m(t)\bigr)\bigr|\Biggr)^2 &\leqslant \Biggl(\sum_{m=1}^{n}|c_{km}l_m|\bigl|y_m(t)\bigr|\Biggr)^2 \\ &\leqslant \sum_{m=1}^{n}(c_{km}l_m)^2\sum_{m=1}^{n}\bigl(y_m(t)\bigr)^2.\end{aligned} \tag{3}$$

$$\Biggl(\sum_{m=1}^{n}|d_{km}|\bigl|g_m\bigl(y_m\bigl(t-\varrho_m(t)\bigr)\bigr)\bigr|\Biggr)^2 \leqslant \sum_{m=1}^{n}(d_{km}l_m)^2\sum_{m=1}^{n}\bigl(y_m\bigl(t-\varrho_m(t)\bigr)\bigr)^2. \tag{4}$$

By (2)–(4) one has

$$\begin{aligned}
\bigl|\dot V(t)\bigr| &\leqslant \sum_{k=1}^{n}\Biggl[-2\xi_k + 3\overline{\alpha_k}^2 + 2\sum_{m=1}^{n}\sum_{s=1}^{n}(c_{ms}l_s)^2 y_k^2(t) \\
&\quad + \bigl(2 - 2\beta_k + 3\overline{\alpha_k}^2\bigr)\dot y_k^2(t) + 2(2-\beta_k-\xi_k)y_k(t)\dot y_k(t)\Biggr] \\
&\quad + 2\sum_{k=1}^{n}\sum_{m=1}^{n}(d_{mk}l_k)^2\sum_{k=1}^{n} y_k^2\bigl(t-\varrho_k(t)\bigr) + 2n\widetilde{U}^2 \\
&\leqslant \sum_{k=1}^{n}\bigl(-\mu_k y_k^2(t) - \nu_k \dot y_k^2(t) - 2\eta_k y_k(t)\dot y_k(t)\bigr) \\
&\quad + \varpi\sum_{k=1}^{n} y_k^2\bigl(t-\varrho_k(t)\bigr) + 2n\widetilde{U}^2 \\
&\leqslant \sum_{k=1}^{n}\biggl(-\frac{\mu_k}{2}y_k^2(t) - \nu_k\dot y_k^2(t) - 2\eta_k y_k(t)\dot y_k(t)\biggr) \\
&\quad - \sum_{k=1}^{n}\frac{\mu_k}{2}y_k^2(t) + \varpi\sum_{k=1}^{n} y_k^2\bigl(t-\varrho_k(t)\bigr) + 2n\widetilde{U}^2 \\
&\leqslant -\omega\sum_{k=1}^{n}\bigl(y_k^2(t) + \dot y_k^2(t) + 2y_k(t)\dot y_k(t)\bigr) - \sum_{k=1}^{n}\frac{\mu_k}{2}y_k^2(t) \\
&\quad + \varpi\sum_{k=1}^{n} y_k^2\bigl(t-\varrho_k(t)\bigr) + 2n\widetilde{U}^2 \\
&\leqslant -\omega V(t) + \varpi \sup_{-\varrho\leqslant s\leqslant 0} V(s) + 2n\widetilde{U}^2.
\end{aligned} \tag{5}$$

Based on (5) and Lemma 2, one obtains

$$\sum_{k=1}^{n} y_k^2(t) \leqslant V(t) \leqslant \frac{2n\widetilde{U}^2}{\omega-\varpi} + \zeta \mathrm{e}^{-rt}, \quad t \geqslant 0,$$

i.e.,

$$\bigl\|y(t)\bigr\|^2 \leqslant \frac{2n\widetilde{U}^2}{\omega-\varpi} + \zeta\mathrm{e}^{-rt}, \quad t\geqslant 0. \tag{6}$$

According to Definition 1 and (6), network (1) is globally exponentially convergent. So, the proof is completed. □

Remark 2. Ke and Li have studied exponential synchronization for inertial CGNNs with time delays [12]. Tang and Jian have discussed the exponential convergence of inertial complex-valued neural network [23]. Kong et al. investigated the fixed-time stabilization and finite-time stabilization for discontinuous inertial CGNNs with delays in [13, 14], respectively. The reduced-order method is used in [12–14, 23], while the nonreduced-order method is developed to explore the global exponential convergence of inertial CGNNs in Theorem 1.

For some selected scalar λ_k, let variable transformation $z_k(t) = \mathrm{d}y_k(t)/\mathrm{d}t + \lambda_k y_k(t)$, then system (1) can be expressed as

$$\begin{aligned}\frac{\mathrm{d}y_k(t)}{\mathrm{d}t} &= -\lambda_k y_k(t) + z_k(t),\\ \frac{\mathrm{d}z_k(t)}{\mathrm{d}t} &= -\lambda_k(\lambda_k - \beta_k)y_k(t) - (\beta_k - \lambda_k)z_k(t)\\ &\quad - \alpha_k\big(y_k(t)\big)\Bigg[f_k\big(y_k(t)\big) - \sum_{m=1}^{n} c_{km} g_m\big(y_m(t)\big)\\ &\quad - \sum_{m=1}^{n} d_{km} g_m\big(y_m\big(t - \varrho_m(t)\big)\big) - U_k(t)\Bigg].\end{aligned} \tag{7}$$

System (7) can be represented in a compact form

$$\begin{aligned}\frac{\mathrm{d}y(t)}{\mathrm{d}t} &= -\varLambda y(t) + z(t),\\ \frac{\mathrm{d}z(t)}{\mathrm{d}t} &= -\mathcal{A}y(t) - \mathcal{B}z(t)\\ &\quad - \alpha\big(y(t)\big)\big[f\big(y(t)\big) - \mathcal{C}g\big(y(t)\big) - \mathcal{D}g\big(y\big(t - \varrho(t)\big)\big) - \mathcal{U}(t)\big],\end{aligned} \tag{8}$$

where

$$\begin{gathered} y(t) = \big(y_1(t), y_2(t), \dots, y_n(t)\big)^{\mathrm{T}}, \qquad z(t) = \big(z_1(t), z_2(t), \dots, z_n(t)\big)^{\mathrm{T}},\\ \varLambda = \mathrm{diag}(\lambda_1, \lambda_2, \dots, \lambda_n), \qquad \mathcal{B} = \mathrm{diag}(\beta_1 - \lambda_1,\ \beta_2 - \lambda_2,\ \dots,\ \beta_n - \lambda_n),\\ \mathcal{A} = \mathrm{diag}\big(\lambda_1(\lambda_1 - \beta_1),\ \lambda_2(\lambda_2 - \beta_2),\ \dots,\ \lambda_n(\lambda_n - \beta_n)\big),\\ f\big(y(t)\big) = \big(f_1\big(y_1(t)\big), f_2\big(y_2(t)\big),\ \dots,\ f_n\big(y_n(t)\big)\big)^{\mathrm{T}}, \qquad \mathcal{C} = (c_{km})_{n\times n},\\ g\big(y(t)\big) = \big(g_1\big(y_1(t)\big), g_2\big(y_2(t)\big), \dots, g_n\big(y_n(t)\big)\big)^{\mathrm{T}}, \qquad \mathcal{D} = (d_{km})_{n\times n},\\ g\big(y\big(t - \varrho(t)\big)\big) = \big(g_1\big(y_1\big(t - \varrho_1(t)\big)\big),\ g_2\big(y_2\big(t - \varrho_2(t)\big)\big),\ \dots,\ g_n\big(y_n\big(t - \varrho_n(t)\big)\big)\big)^{\mathrm{T}},\\ \alpha\big(y(t)\big) = \mathrm{diag}\big(\alpha_1\big(y_1(t)\big), \alpha_2\big(y_2(t)\big), \dots, \alpha_n\big(y_n(t)\big)\big),\\ \mathcal{U}(t) = \big(U_1(t), U_2(t), \dots, U_n(t)\big)^{\mathrm{T}}.\end{gathered}$$

Theorem 2. *Let assumptions* (H1)–(H3) *and* $f_k(y_k(t)) = h_k(y_k(t))z_k(t)$ *hold, where* $h_k(\cdot)$ *is a continuous bounded function, and for given positive constants* ε, μ, ν, $\omega > \varpi$, *there exists a definite Hermitian matrix* Q *such that*

(i) $-2\varLambda + \varepsilon^{-1}I + \varepsilon^{-1}\overline{\alpha}^2\widetilde{l}^2\mathcal{C}^{\mathrm{T}}\mathcal{C} \leqslant -\mu Q$,
(ii) $-2\mathcal{B} - 2\underline{\alpha}\hbar I + 4\varepsilon I - 2\varepsilon\mathcal{A} + \varepsilon\mathcal{A}^2 \leqslant -\nu I$.

Then system (8) *is GUEC,*

$$\|y(t)\|^2 + \|z(t)\|^2 \leqslant \frac{n\varepsilon^{-1}\overline{\alpha}^2\widetilde{U}^2}{\omega - \varpi} + \zeta \mathrm{e}^{-r(t-t_0)}, \quad t \geqslant 0.$$

Furthermore, networks (8) *is globally exponentially convergent to a ball*

$$\mathfrak{B} = \left\{ y(t), z(t) \;\middle|\; \|y(t)\|^2 + \|z(t)\|^2 \leqslant \frac{n\varepsilon^{-1}\overline{\alpha}^2\widetilde{U}^2}{\omega - \varpi} \right\}$$

with a convergence rate $r/2$*, where* $\varpi = \varepsilon^{-1}\overline{\alpha}^2\widetilde{l}^2\lambda_{\max}(\mathcal{D}^{\mathrm{T}}\mathcal{D})$, $\omega = \min\{\mu\lambda_{\min}(Q), \nu\}$, $\sup_{-\varrho\leqslant s\leqslant 0}(y(s)^{\mathrm{T}}y(s) + z(s)^{\mathrm{T}}z(s)) \leqslant \zeta$, $r = \omega - \varpi\mathrm{e}^{r\varrho} > 0$, $h(y(t)) = \min(h_1(y_1(t)), h_2(y_2(t)), \dots, h_n(y_n(t))) \geqslant \hbar$.

Proof. From (8) one gets

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\bigl(y(t)^{\mathrm{T}}y(t) + z(t)^{\mathrm{T}}z(t)\bigr) \\
&\quad = -2y^{\mathrm{T}}(t)\varLambda y(t) + 2y^{\mathrm{T}}(t)(I - \mathcal{A})z(t) - 2z^{\mathrm{T}}(t)\mathcal{B}z(t) \\
&\qquad - 2z^{\mathrm{T}}(t)\alpha\bigl(y(t)\bigr)f\bigl(y(t)\bigr) + 2z^{\mathrm{T}}(t)\alpha\bigl(y(t)\bigr)\mathcal{C}g\bigl(y(t)\bigr) \\
&\qquad + 2z^{\mathrm{T}}(t)\alpha\bigl(y(t)\bigr)\mathcal{D}g\bigl(y\bigl(t - \varrho(t)\bigr)\bigr) + 2z^{\mathrm{T}}(t)\alpha\bigl(y(t)\bigr)\mathcal{U}(t).
\end{aligned}$$

According to (H1)–(H3) and Lemma 1, the following inequalities hold:

$$\begin{aligned}
-2z^{\mathrm{T}}(t)\alpha\bigl(y(t)\bigr)f\bigl(y(t)\bigr) &\leqslant -2z^{\mathrm{T}}(t)\underline{\alpha}\hbar z(t), \\
2y^{\mathrm{T}}(t)(I - \mathcal{A})z(t) &\leqslant \varepsilon^{-1}y^{\mathrm{T}}(t)y(t) + \varepsilon z^{\mathrm{T}}(t)(I - \mathcal{A})(I - \mathcal{A})z(t), \\
2z^{\mathrm{T}}(t)\alpha\bigl(y(t)\bigr)\mathcal{C}g\bigl(y(t)\bigr) &\leqslant \varepsilon z^{\mathrm{T}}(t)z(t) + \varepsilon^{-1}\overline{\alpha}^2\widetilde{l}^2 y^{\mathrm{T}}(t)\mathcal{C}^{\mathrm{T}}\mathcal{C}y(t), \\
2z^{\mathrm{T}}(t)\alpha\bigl(y(t)\bigr)\mathcal{D}g\bigl(y\bigl(t - \varrho(t)\bigr)\bigr) &\leqslant \varepsilon z^{\mathrm{T}}(t)z(t) \\
&\quad + \varepsilon^{-1}\overline{\alpha}^2\widetilde{l}^2 y^{\mathrm{T}}\bigl(t - \varrho(t)\bigr)\mathcal{D}^{\mathrm{T}}\mathcal{D}y\bigl(t - \varrho(t)\bigr), \\
2z^{\mathrm{T}}(t)\alpha\bigl(y(t)\bigr)\mathcal{U}(t) &\leqslant \varepsilon z^{\mathrm{T}}(t)z(t) + \varepsilon^{-1}\mathcal{U}^{\mathrm{T}}(t)\overline{\alpha}^2\mathcal{U}(t) \\
&\leqslant \varepsilon z^{\mathrm{T}}(t)z(t) + n\varepsilon^{-1}\overline{\alpha}^2\widetilde{U}^2.
\end{aligned} \tag{9}$$

In view of (9), one has

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\bigl(y(t)^{\mathrm{T}}y(t) + z(t)^{\mathrm{T}}z(t)\bigr) \\
&\quad \leqslant y^{\mathrm{T}}(t)\bigl(-2\varLambda + \varepsilon^{-1}I + \varepsilon^{-1}\overline{\alpha}^2\widetilde{l}^2\mathcal{C}^{\mathrm{T}}\mathcal{C}\bigr)y(t) \\
&\qquad + z^{\mathrm{T}}(t)\bigl(-2\mathcal{B} - 2\underline{\alpha}\hbar I + 4\varepsilon I - 2\varepsilon\mathcal{A} + \varepsilon\mathcal{A}^2\bigr)z(t) \\
&\qquad + \varepsilon^{-1}\overline{\alpha}^2\widetilde{l}^2 y^{\mathrm{T}}\bigl(t - \varrho(t)\bigr)\mathcal{D}^{\mathrm{T}}\mathcal{D}y\bigl(t - \varrho(t)\bigr) + n\varepsilon^{-1}\overline{\alpha}^2\widetilde{U}^2 \\
&\quad \leqslant -\mu y^{\mathrm{T}}(t)Qy(t) - \nu z^{\mathrm{T}}(t)z(t) + \varpi y^{\mathrm{T}}\bigl(t - \varrho(t)\bigr)y\bigl(t - \varrho(t)\bigr) + n\varepsilon^{-1}\overline{\alpha}^2\widetilde{U}^2 \\
&\quad \leqslant -\omega\bigl(y^{\mathrm{T}}(t)y(t) + z^{\mathrm{T}}(t)z(t)\bigr) + \varpi \sup_{-\varrho\leqslant s\leqslant 0}\bigl(y^{\mathrm{T}}(t)y(t) + z^{\mathrm{T}}(t)z(t)\bigr) \\
&\qquad + n\varepsilon^{-1}\overline{\alpha}^2\widetilde{U}^2.
\end{aligned} \tag{10}$$

Based on (10) and Lemma 2, one yields

$$y^{\mathrm{T}}(t)y(t) + z^{\mathrm{T}}(t)z(t) \leqslant \frac{n\varepsilon^{-1}\overline{\alpha}^2\widetilde{U}^2}{\omega - \varpi} + \zeta \mathrm{e}^{-rt}, \quad t \geqslant 0,$$

i.e.,

$$\left\|y(t)\right\|^2 + \left\|z(t)\right\|^2 \leqslant \frac{n\varepsilon^{-1}\overline{\alpha}^2\widetilde{U}^2}{\omega - \varpi} + \zeta \mathrm{e}^{-rt}, \quad t \geqslant 0. \tag{11}$$

By virtue of condition (11) and Definition 1, one can get that the network model (8) is globally exponentially convergent. This completes the proof. □

Remark 3. Shi et al. [21] have investigated global exponential stabilization of inertial neural networks, and Zhang et al. [29] have explored the dissipativity for delayed memristor-based inertial neural networks. However, the derivatives of time-varying delay in [21,29] are required to be no greater than one. It should be pointed out that the restricted conditions are removed in this paper.

4 Illustrative examples

Example 1. Consider the following 2-dimensional inertial CGNNs:

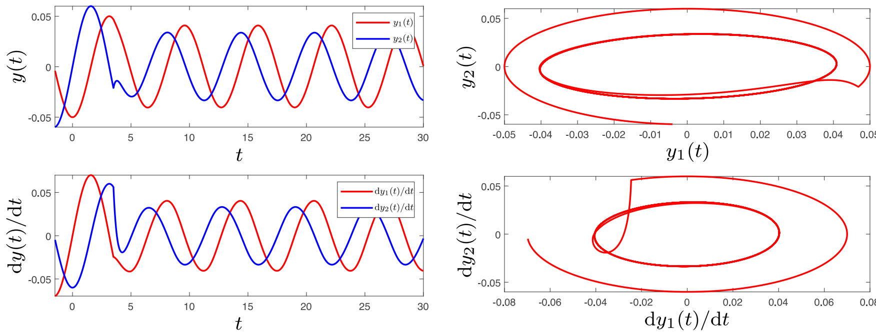
$$\begin{aligned}
\frac{\mathrm{d}^2 y_1(t)}{\mathrm{d}t^2} &= -3.5\frac{dy_1(t)}{\mathrm{d}t} - 2.5y_1(t) \\
&\quad + \left(\frac{1}{8}\sin\bigl(y_1(t)\bigr) + \frac{3}{8}\right)\bigl[0.3g_1\bigl(y_1(t)\bigr) - 0.2g_2\bigl(y_2(t)\bigr) \\
&\quad - 0.4g_1\bigl(y_1\bigl(t-\varrho_1(t)\bigr)\bigr) + 0.3g_2\bigl(y_2\bigl(t-\varrho_2(t)\bigr)\bigr) + U_1(t)\bigr], \\
\frac{\mathrm{d}^2 y_2(t)}{\mathrm{d}t^2} &= -4\frac{\mathrm{d}y_2(t)}{\mathrm{d}t} - 3y_2(t) \\
&\quad + \left(\frac{1}{8}\sin\bigl(y_2(t)\bigr) + \frac{3}{8}\right)\bigl[0.2g_1\bigl(y_1(t)\bigr) - 0.3g_2\bigl(y_2(t)\bigr) \\
&\quad - 0.3g_1\bigl(y_1\bigl(t-\varrho_1(t)\bigr)\bigr) - 0.35g_2\bigl(y_2\bigl(t-\varrho_2(t)\bigr)\bigr) + U_2(t)\bigr],
\end{aligned} \tag{12}$$

where $g_k(y_k) = 0.5(|y_k+1| - |y_k-1|)$, $k = 1,2$, $U_1(t) = 0.8\sin t$, $U_2(t) = 0.8\cos t$. So, one has $|g_k(a) - g_k(b)| \leqslant |a-b|$ for all $a,b \in \mathbb{R}$, then $l_k = 1$.

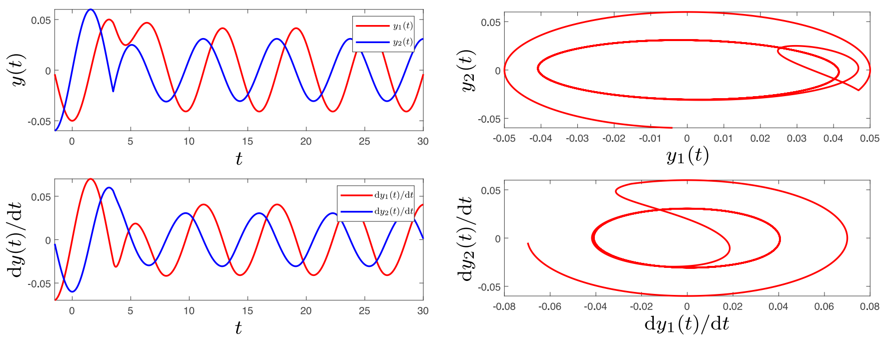
From Theorem 1 one can calculate that $\mu_1 = 3.73$, $\mu_2 = 4.73$, $\nu_1 = 4.25$, $\nu_2 = 5.25$, $\eta_1 = 4$, $\eta_2 = 5$, $\omega = 1.865 > \varpi = 0.925$, system (12) with a convergence rate $r/2 = 0.1677$ is globally exponentially convergent to the ball

$$\mathfrak{B} = \left\{ y(t) \;\middle|\; \left\|y(t)\right\|^2 \leqslant \frac{2n\widetilde{U}^2}{\omega - \varpi} = 2.7234 \right\}.$$

Choose the initial values of system (12) as $\widetilde{\phi}_1(s) = -0.05\cos s$, $\widetilde{\phi}_2(s) = 0.06\sin s$, $\widetilde{\varphi}_1(s) = 0.07\sin s$, $\widetilde{\varphi}_2(s) = 0.07\sin s$. Figure 1(a) depicts the state trajectories and the



(a) $\varrho_k(t) = 1.5 \sin^2 t$



(b) $\varrho_k(t) = 1.5 \cos^2 t$

Figure 1. The state trajectory and phase graph of system (12).

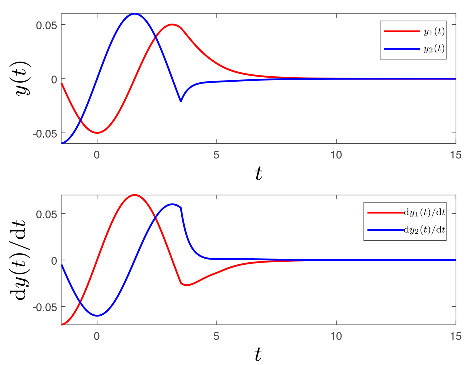


Figure 2. The state trajectory of system (12) with $\varrho_k(t) = 1.5 \sin^2 t$ and $U_k(t) = 0$.

phrase trajectories of system (12) with $\varrho_k(t) = 1.5 \sin^2 t$, $k = 1, 2$. Figure 1(b) gives the states trajectories and phrase trajectories of system (12) with $\varrho_k(t) = 1.5 \cos^2 t$, $k = 1, 2$, respectively. Combining Figs. 1(a) and 1(b), one can get that the presented criteria are independent of time-varying delay. Figure 2 shows the state trajectories with $U_1(t) = U_2(t) = 0$, and it can be further concluded that system (12) is globally exponential stable.

Example 2. Consider the inertial CGNNs as follows:

$$\begin{aligned}\frac{\mathrm{d}^2 y_1(t)}{\mathrm{d}t^2} &= -1.8\frac{\mathrm{d}y_1(t)}{\mathrm{d}t} - \alpha_1\big(y_1(t)\big)\big[f_1\big(y_1(t)\big) + 0.2g_1\big(y_1(t)\big) + 0.3g_2\big(y_2(t)\big) \\ &\quad - 0.3g_1\big(y_1\big(t-\varrho_1(t)\big)\big) + 0.2g_2\big(y_2\big(t-\varrho_2(t)\big)\big) + \cos t\big], \\ \frac{\mathrm{d}^2 y_2(t)}{\mathrm{d}t^2} &= -1.7\frac{\mathrm{d}y_2(t)}{\mathrm{d}t} - \alpha_2\big(y_2(t)\big)\big[f_1\big(y_1(t)\big) - 0.4g_1\big(y_1(t)\big) + 0.2g_2\big(y_2(t)\big) \\ &\quad + 0.4g_1\big(y_1\big(t-\varrho_1(t)\big)\big) - 0.3g_2(y_2\big(t-\varrho_2(t)\big)\big) + \sin t\big],\end{aligned} \tag{13}$$

where $\alpha_k(y_k(t)) = 1/(10(1+|y_k(t)|)) + 1/2$, $g_k(y_k(t)) = 0.8\tanh(y_k(t))$, $\varrho_k(t) = \sin t + 1$, $k = 1, 2$. So, one can obtain

$$\begin{aligned}\big|g_k(a) - g_k(b)\big| &= \big|\tanh(\xi)'(a-b)\big| = \big|\big(1 - \tanh^2(\xi)\big)(a-b)\big| \\ &\leqslant 0.8|a-b| \quad \forall a, b \in \mathbb{R},\end{aligned}$$

where $\xi \in [\min\{a,b\}, \max\{a,b\}]$, then $l_k = 0.8$.

Choose $\lambda_1 = \lambda_2 = 0.9$ and $f_k(y_k(t)) = (3.5 - y_k(t)/(y_k(t)^2 + 1))z_k(t)$, $k = 1, 2$. Rewriting system (13) into (8) form, one has the corresponding matrix

$$\Lambda = \mathrm{diag}(0.9,\ 0.9), \qquad \mathcal{A} = \mathrm{diag}(-0.81,\ -0.72), \qquad \mathcal{B} = \mathrm{diag}(0.9,\ 0.8),$$
$$\mathcal{C} = \begin{bmatrix} 0.2 & 0.3 \\ -0.4 & 0.2 \end{bmatrix}, \qquad \mathcal{D} = \begin{bmatrix} -0.3 & 0.2 \\ 0.4 & -0.3 \end{bmatrix}, \qquad \mathcal{U}(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}.$$

Take $\mu = 0.3$, $\nu = 0.35$, $\varepsilon = 0.7$. Using LMI, the Q that satisfies the condition is as follows:

$$Q = \begin{bmatrix} 2.0151 & 0.1292 \\ 0.1292 & 2.2729 \end{bmatrix}.$$

Based on Theorem 2, one can compute the $\varpi = 0.1250$ and $\omega = \min\{0.5885, 0.3500\} = 0.3500$, then the conditions in Theorem 2 hold. At the same time, network (13), with a convergence rate $\lambda/2 = 0.0866$, is globally exponentially convergent to the ball

$$\mathfrak{B} = \left\{ y(t), z(t) \;\middle|\; \big\|y(t)\big\|^2 + \big\|z(t)\big\|^2 \leqslant \frac{n\varepsilon^{-1}\overline{\alpha}^2\widetilde{U}^2}{\omega - \varpi} \right\} = 4.5714.$$

Let the initial conditions be $\widetilde{\phi}_1(s) = -0.015\sin s$, $\widetilde{\phi}_2(s) = 0.017\sin s$, $\widetilde{\varphi}_1(s) = 0.017\sin s$, $\widetilde{\varphi}_2(s) = -0.013\sin s$ for $s \in [-2, 0]$. Figure 3 depicts the state trajectories and the phrase trajectories of system (13). Figure 4 shows the state trajectories with $U(t) = 0$. Besides, when $U(t) = 0$, system (13), with convergence rate 0.1105, is globally exponential stable.

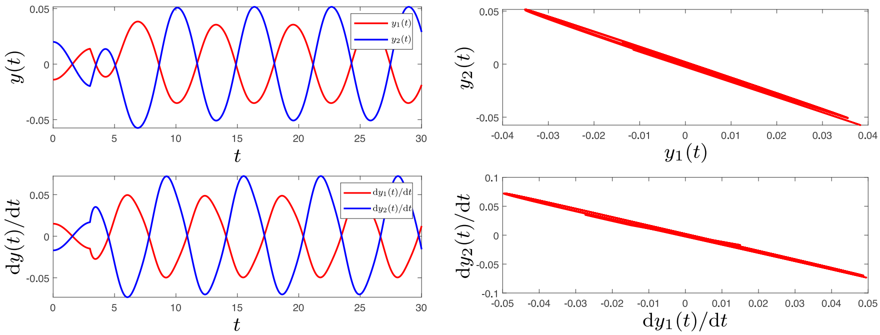


Figure 3. The state trajectories and the phrase trajectories of system (13).

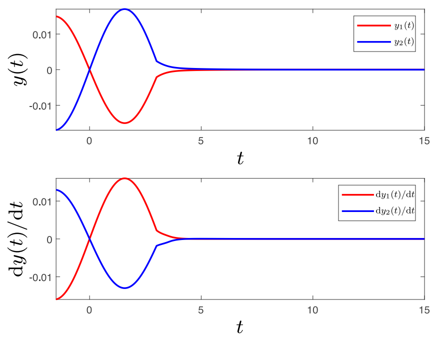


Figure 4. The state trajectory of system (13) with $U(t) = 0$.

5 Conclusions

This paper presents some delay-dependent sufficient conditions, which ensure the inertial CGNNs to globally exponentially converge to a ball with a prespecified convergence rate. The proposed results here do not require the derivative of time-varying delayed to be less than one. In addition, based on the reduce-order method and nonreduced-order method, this paper presents the exponential convergence for inertial CGNNs without discussing the equilibrium point. Finally, two examples illustrate the validity of the results. In the future work, we will explore the finite-time state estimation problem of delayed inertial CGNNs via the nonreduced-order method.

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