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Stability analysis and stabilization control of discrete-time impulsive switched time-delay systems with all unstable subsystems*

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Abstract. In this paper, stability analysis and stabilization control of discrete-time impulsive switched time-delay systems with all unstable subsystems are discussed. By utilizing a switching time-varying Lyapunov–Krasovskii functional and the mode-dependent interval dwell-time switching rule, we derive some more general stability theorems for the considered time-delay system with all subsystems being unstable. Moreover, we design a time-varying state feedback controller to ensure the stabilization of the resulting closed-loop system. Eventually, the theoretical findings are demonstrated utilizing numerical examples.

Keywords: discrete-time switched delay system, switching time-varying Lyapunov–Krasovskii functional, impulse, stability, stabilization.

1 Introduction

Switched systems, a significant type of hybrid dynamical systems, are made up of a set of subsystems defined by difference or differential equations and a switching signal that specifies the switching between subsystems. The studies on switched systems derive from the fact that such systems may be adopted to simulate actual-world systems like electric power systems, motion control systems, and so on. Consequently, many results concerning switched systems have been published in [9, 14, 17]. The primary issues with switched systems are stability and stabilization. Lyapunov function methods have been successfully used to address stability and stabilization problems for switched systems. Additionally, the H_∞ performance study for switched systems has generated a lot of interest recently on account of the fact that disturbances are regularly encountered in

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practical situations [4, 8]. However, due to abrupt jumps at specific points in time of the dynamical process, some switched systems show impulsive dynamical behaviors. These systems may be described as impulsive switched systems if they exhibit these behaviors [5]. Over the last ten years, impulsive switched systems have been thoroughly studied [10, 11, 25].

Typically, the behavior of hybrid dynamical systems under consideration is merely dependent on their current state. Many phenomena, however, cannot be adequately explained by the particular limitations resulting from the current state. Accordingly, it is preferable to take into account that the behavior of the system also contains data about previous states. This property is known as time delay. Time delay is a source of instability and chaos. Time delay is regularly encountered in the discrete state and internal functioning of each subsystem in various hybrid systems, such as electrical systems. Therefore, more recently, many studies have concentrated on the analysis of switched delay systems. There exist two main Lyapunov methods for time-delay systems: the Lyapunov–Razumikhin function methods and the Lyapunov–Krasovskii functional methods. Finding a Lyapunov function that declines under the Razumikhin condition or a Lyapunov functional that reduces over the whole state space is the fundamental idea of Lyapunov approaches. Generally speaking, time-delay switched systems exhibit more complex behavior than switched systems without delay. These two Lyapunov approaches have been effectively employed for the analysis of stability and stabilization of time-delay systems [16, 19, 22, 27, 29, 31].

In general, a system with unstable subsystems performs differently than a system with all subsystems stable. We can conclude from the majority of the aforementioned research articles that all subsystems are stable. Therefore, the stability of systems containing stable and unstable subsystems is studied. There have been considerable breakthroughs in discrete-time switched systems with unstable subsystems [3, 6, 13, 28]. It should be noted that the literature mentioned above does not consider how time delay affects the system. In [23], the authors introduced a sufficient criterion for the first time to maintain the uniformly asymptomatic stability of nonlinear switched time-delay systems containing unstable modes. Through asynchronous switching, some stability criteria of general nonlinear switched time-delay systems with unstable modes were proposed in [24]. In [21], the authors used the new inequality method and average dwell-time technique to examine the stability of switched nonlinear delay systems made up of unstable modes and stable modes. In [18], the author studied the robust stability of switched positive delay systems containing unstable modes by developing appropriate Lyapunov–Krasovskii functionals for time scheduling and applying mode-dependent dwell-time switching technique. The aforementioned literature, however, focused mostly on continuous-time switched systems and ignored discrete cases. In [12], the authors considered the stabilization for discrete-time positive switched delay systems that are entirely made up of unstable subsystems via dwell-time switching by introducing discretized copositive functionals. Nevertheless, in the aforementioned studies, impulses were not taken into account. This paper is motivated by the fact that no research has been done on the issue of stability and stabilization of discrete-time switched systems made up of all unstable subsystems with time delay and impulse.

Stability analysis and stabilization control of discrete-time impulsive switched time-delay systems are presented in this paper. Firstly, by introducing a switching time-varying Lyapunov–Krasovskii functional, stability theorems for discrete-time switched delay systems made up of all unstable subsystems with impulse via mode-dependent interval dwell-time switching are derived. The functional we constructed makes the stability criteria less conservative. Following that, we create a time-varying controller to enable stabilization of the resulting closed-loop system.

The remaining part of this paper proceeds as follows. The problem is formulated in Section 2. On the basis of a new Lyapunov–Krasovskii functional and mode-dependent interval dwell-time approach, Section 3 deals with stability and stabilization for discrete-time impulsive switched time-delay systems. The theoretical results are illustrated with numerical examples in Section 4. Ultimately, Section 5 provides this paper's conclusions.

2 Problem formulation

In this paper, $\mathbb{N}$ and $\mathbb{N}_0$ denote the set of positive integers and nonnegative integers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ represent the n-dimensional real vector space and the $n\times n$-dimensional real matrices, respectively. For $V\in\mathbb{R}^{n\times n}$, V^{T} denotes the transpose of V, and $V>0$ means that V is a symmetric positive definite matrix. For $\omega(k)\in\mathbb{R}^n$, $\|\omega(k)\| = (\sum_{k=0}^{\infty}\omega^{\mathrm{T}}(k)\omega(k))^{1/2}$ stands for the L_2-norm of $\omega(k)$. For $m\in\mathbb{N}$, let $\langle m\rangle = \{1,2,\dots,m\}$. For integers p and q satisfying $p\leqslant q$, let $I[p,q]=\{p,p+1,\dots,q\}$.

Take into account the following discrete-time impulsive switched time-delay system:

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}\tilde{x}(k) + B_{\sigma(k)}\tilde{x}(k-d) + C_{\sigma(k)}\omega(k) + L_{\sigma(k)}u(k), \quad k\in\mathbb{N}_0,\\ z(k) &= A_{z\sigma(k)}\tilde{x}(k) + B_{z\sigma(k)}\tilde{x}(k-d) + C_{z\sigma(k)}\omega(k) + M_{\sigma(k)}u(k), \quad k\in\mathbb{N}_0,\\ x(\theta) &= \phi(\theta), \quad \theta=-d,-d+1,\dots,0,\\ \tilde{x}(k) &= \begin{cases} x(k), & k\neq k_s,\ s\in\mathbb{N},\\ F_{\sigma(k)}x(k), & k=k_s,\ s\in\mathbb{N}, \end{cases} \end{aligned} \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the system state, and $\tilde{x}(k)\in\mathbb{R}^n$ is the impulse state, $\omega(k)\in\mathbb{R}^l$ is the L_2-norm-bounded disturbance, $u(k)\in\mathbb{R}^v$ is the control input, $z(k)\in\mathbb{R}^n$ is the controlled output. $\sigma(k):\mathbb{N}_0\to\langle m\rangle$ is the switching signal, and $m>1$ is the total quantity of subsystems. $d>0$ denotes the constant time delay. For a switching sequence $0=k_0<k_1<\dots<k_s<k_{s+1}<\cdots$, when $k_s\leqslant k<k_{s+1}$, $\sigma(k_s)$th subsystem is activated. $A_i, B_i, C_i, L_i, A_{zi}, B_{zi}, C_{zi}, M_i$, $i\in\langle m\rangle$, are given matrices with appropriate dimensions. $F_i\in\mathbb{R}^{n\times n}$, $i\in\langle m\rangle$, are the impulse matrices. $\phi(\theta)$ represents the initial condition.

Definition 1. For a switching instant $k_s\in\mathbb{N}_0$ and a switching signal $\sigma(k)$, if there are positive integers τ_{i1} and τ_{i2} for $i\in\langle m\rangle$ satisfying

$$\tau_{i1}\leqslant k_{s+1}-k_s\leqslant\tau_{i2},$$

where $\sigma(k_s)=i$, such a switching signal is defined as mode-dependent interval dwell-time (MDIDT) switching.

Remark 1. Compared with the general IDT, MDIDT means that each subsystem has its own running time, and the running time range of different subsystems can be different, as long as it is within the range, which can reduce conservatism.

Definition 2. System (1) with $\omega(k) = u(k) = 0$ is said to be exponentially stable via the MDIDT switching if there exist positive scalars $\beta < 1$ and c such that for any initial state $\phi(\theta) \in \mathbb{R}^n$ and any MDIDT switching signal,

$$\|x(k)\| \leqslant c\beta^k \|\phi\|_d, \quad k \in \mathbb{N}_0,$$

where $\|\phi\|_d = \sup_{-d \leqslant \theta \leqslant 0} \|\phi(\theta)\|$.

Definition 3. Given $\gamma > 0$, system (1) with $u(k) = 0$ is said to be asymptotically stable with H_∞ performance γ via the MDIDT switching if it satisfies the below conditions:

(i) When $\omega(k) = u(k) = 0$, system (1) is asymptotically stable under the MDIDT switching;

(ii) When $\phi(\theta) = 0$, $\theta \in I[-d, 0]$, it holds that

$$\sum_{k=0}^{\infty} \|z(k)\|^2 \leqslant \gamma^2 \sum_{k=0}^{\infty} \|\omega(k)\|^2$$

for each $\omega(k) \in L_2[0, \infty)$.

Definition 4. System (1) with $\omega(k) = 0$ is said to be exponentially stabilizable via the MDIDT switching if there exists a state feedback controller in the form of $u(k) = K_{\sigma(k)}(k)\tilde{x}(k)$ such that the closed-loop system (1) with $\omega(k) = 0$ is exponentially stable via the MDIDT switching.

Definition 5. System (1) is said to be asymptotically stabilizable with H_∞ performance γ via the MDIDT switching if there is a controller $u(k) = K_{\sigma(k)}(k)\tilde{x}(k)$ such that the closed-loop system (1) with $\omega(k) = 0$ is asymptotically stable via the MDIDT switching, and for any $\omega(k) \in L_2[0, \infty)$ and $\phi(\theta) = 0$, $\theta \in I[-d, 0]$, the response $z(k)$ satisfies

$$\sum_{k=0}^{\infty} \|z(k)\|^2 \leqslant \gamma^2 \sum_{k=0}^{\infty} \|\omega(k)\|^2.$$

Lemma 1. (*See Schur complement* [2].) *For a given matrix*

$$V = \begin{bmatrix} V_{11} & V_{12} \\ * & V_{22} \end{bmatrix}$$

with $V_{11} = V_{11}^{\mathrm{T}}$, $V_{22} = V_{22}^{\mathrm{T}}$, *the following conditions are equivalent*:

(i) $V < 0$,

(ii) $V_{22} < 0$, $V_{11} - V_{12}V_{22}^{-1}V_{12}^{\mathrm{T}} < 0$,

(iii) $V_{11} < 0$, $V_{22} - V_{12}^{\mathrm{T}}V_{11}^{-1}V_{12} < 0$.

3 Main results

Firstly, we consider the case $u(k) = 0$, that is,

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}\tilde{x}(k) + B_{\sigma(k)}\tilde{x}(k-d) + C_{\sigma(k)}\omega(k), \quad k \in \mathbb{N}_0, \\ z(k) &= A_{z\sigma(k)}\tilde{x}(k) + B_{z\sigma(k)}\tilde{x}(k-d) + C_{z\sigma(k)}\omega(k), \quad k \in \mathbb{N}_0, \\ x(\theta) &= \phi(\theta), \quad \theta = -d, -d+1, \dots, 0, \\ \tilde{x}(k) &= \begin{cases} x(k), & k \neq k_s,\ s \in \mathbb{N}, \\ F_{\sigma(k)}x(k), & k = k_s,\ s \in \mathbb{N}. \end{cases} \end{aligned} \tag{2}$$

Theorem 1. *If there are scalars $0 < \lambda < 1$, $\mu \geqslant 1$ and $n \times n$-dimensional matrices $P_i^{(v)} > 0$, $v \in \langle \tau_{i2} + 1\rangle$, $Q_i^{(w)} > 0$, $w \in I[-d, \tau_{i2} - 1]$, $i \in \langle m\rangle$, such that for $p \in \langle \tau_{i2}\rangle$, $q \in I[\tau_{i1} + 1, \tau_{i2} + 1]$, $g \in I[-d, -1]$, $i, j \in \langle m\rangle$, $j \neq i$,*

$$\begin{bmatrix} A_i^{\mathrm{T}} P_i^{(p+1)} A_i - \lambda P_i^{(p)} + Q_i^{(p-1)} & A_i^{\mathrm{T}} P_i^{(p+1)} B_i \\ * & B_i^{\mathrm{T}} P_i^{(p+1)} B_i - \lambda^d Q_i^{(p-1-d)} \end{bmatrix} < 0, \tag{3}$$

$$F_j^{\mathrm{T}} P_j^{(1)} F_j < \mu P_i^{(q)}, \tag{4}$$

$$Q_j^{(g)} < \mu Q_i^{(q+g-1)}, \tag{5}$$

and $\tau_0 > -\ln\mu/\ln\lambda$, where $\tau_0 = \min_{i\in\langle m\rangle}\{\tau_{i1}\}$, then system (2) with $\omega(k) = 0$ is exponentially stable via the MDIDT switching.

Proof. Select a switching time-varying Lyapunov–Krasovskii functional as follows:

$$V_{\sigma(k_s)}\big(k, x(k)\big) = x^{\mathrm{T}}(k) P_{\sigma(k_s)}^{(k-k_s+1)} x(k) + \sum_{l=k-d}^{k-1} \lambda^{k-l-1} x^{\mathrm{T}}(l) Q_{\sigma(k_s)}^{(l-k_s)} x(l) \tag{6}$$

for $k \in I[k_s, k_{s+1}]$, where $x(k_s) = \tilde{x}(k_s)$, $P_{\sigma(k_s)}^{(k-k_s+1)} > 0$, and $Q_{\sigma(k_s)}^{(l-k_s)} > 0$. Let $\sigma(k_s) = i$, $\sigma(k_{s+1}) = j$. When $k \in I[k_s, k_{s+1} - 1]$, denote $k - k_s + 1 = p$ for $p \in \langle \tau_{i2}\rangle$. Applying (6), we get

$$\begin{aligned} &V_i\big(k+1, x(k+1)\big) - \lambda V_i\big(k, x(k)\big) \\ &\quad = x^{\mathrm{T}}(k+1) P_i^{(k-k_s+2)} x(k+1) + \sum_{l=k+1-d}^{k} \lambda^{k-l} x^{\mathrm{T}}(l) Q_i^{(l-k_s)} x(l) \\ &\qquad - \lambda x^{\mathrm{T}}(k) P_i^{(k-k_s+1)} x(k) - \sum_{l=k-d}^{k-1} \lambda^{k-l} x^{\mathrm{T}}(l) Q_i^{(l-k_s)} x(l) \\ &\quad = x^{\mathrm{T}}(k) A_i^{\mathrm{T}} P_i^{(p+1)} A_i x(k) - \lambda x^{\mathrm{T}}(k) P_i^{(p)} x(k) + x^{\mathrm{T}}(k) Q_i^{(p-1)} x(k) \\ &\qquad + x^{\mathrm{T}}(k) A_i^{\mathrm{T}} P_i^{(p+1)} B_i x(k-d) + x^{\mathrm{T}}(k-d) B_i^{\mathrm{T}} P_i^{(p+1)} A_i x(k) \\ &\qquad + x^{\mathrm{T}}(k-d) B_i^{\mathrm{T}} P_i^{(p+1)} B_i x(k-d) - \lambda^d x^{\mathrm{T}}(k-d) Q_i^{(p-1-d)} x(k-d). \end{aligned} \tag{7}$$

Let $\psi^{\mathrm{T}}(k) = [x^{\mathrm{T}}(k)\ x^{\mathrm{T}}(k-d)]$. It is obvious that (7) can be rewritten as

$$\begin{aligned} &V_i\big(k+1,\, x(k+1)\big) - \lambda V_i\big(k, x(k)\big) \\ &\quad = \psi^{\mathrm{T}}(k) \begin{bmatrix} A_i^{\mathrm{T}} P_i^{(p+1)} A_i - \lambda P_i^{(p)} + Q_i^{(p-1)} & A_i^{\mathrm{T}} P_i^{(p+1)} B_i \\ * & B_i^{\mathrm{T}} P_i^{(p+1)} B_i - \lambda^d Q_i^{(p-1-d)} \end{bmatrix} \psi(k). \end{aligned} \tag{8}$$

It follows from (3) and (8) that

$$V_i\big(k+1,\, x(k+1)\big) < \lambda^{k-k_s+1} V_i\big(k_s, x(k_s)\big), \quad k \in I[k_s,\, k_{s+1}-1]. \tag{9}$$

When $k = k_{s+1}$, let $k_{s+1} - k_s + 1 = q$, $q \in I[\tau_{i1}+1,\, \tau_{i2}+1]$, $l - k_{s+1} = g$, $g \in I[-d, -1]$. From (4) and (5) it can be seen that

$$\begin{aligned} &V_j(k_{s+1}, x(k_{s+1})) - \mu V_i(k_{s+1}, x(k_{s+1})) \\ &\quad = x^{\mathrm{T}}(k_{s+1}) P_j^{(1)} x(k_{s+1}) + \sum_{l=k_{s+1}-d}^{k_{s+1}-1} \lambda^{k_{s+1}-l-1} x^{\mathrm{T}}(l) Q_j^{(l-k_{s+1})} x(l) \\ &\qquad - \mu x^{\mathrm{T}}(k_{s+1}) P_i^{(k_{s+1}-k_s+1)} x(k_{s+1}) - \mu \sum_{l=k_{s+1}-d}^{k_{s+1}-1} \lambda^{k_{s+1}-l-1} x^{\mathrm{T}}(l) Q_i^{(l-k_s)} x(l) \\ &\quad = \tilde{x}^{\mathrm{T}}(k_{s+1}) P_j^{(1)} \tilde{x}(k_{s+1}) + \sum_{l=k_{s+1}-d}^{k_{s+1}-1} \lambda^{k_{s+1}-l-1} x^{\mathrm{T}}(l) Q_j^{(l-k_{s+1})} x(l) \\ &\qquad - \mu x^{\mathrm{T}}(k_{s+1}) P_i^{(k_{s+1}-k_s+1)} x(k_{s+1}) - \mu \sum_{l=k_{s+1}-d}^{k_{s+1}-1} \lambda^{k_{s+1}-l-1} x^{\mathrm{T}}(l) Q_i^{(l-k_s)} x(l) \\ &\quad = x^{\mathrm{T}}(k_{s+1}) \big[F_j^{\mathrm{T}} P_j^{(1)} F_j - \mu P_i^{(q)}\big] x(k_{s+1}) \\ &\qquad + \sum_{l=k_{s+1}-d}^{k_{s+1}-1} \lambda^{k_{s+1}-l-1} x^{\mathrm{T}}(l) \big[Q_j^{(l-k_{s+1})} - \mu Q_i^{(l-k_s)}\big] x(l) < 0. \end{aligned} \tag{10}$$

Based on (9) and (10), we can get

$$\begin{aligned} V_{\sigma(k_s)}\big(k, x(k)\big) &\leqslant \lambda^{k-k_s} V_{\sigma(k_s)}\big(k_s, x(k_s)\big) \leqslant \mu \lambda^{k-k_s} V_{\sigma(k_{s-1})}\big(k_s, x(k_s)\big) \\ &\leqslant \mu \lambda^{k-k_{s-1}} V_{\sigma(k_{s-1})}\big(k_{s-1}, x(k_{s-1})\big) \leqslant \cdots \\ &\leqslant \mu^s \lambda^k V_{\sigma(0)}\big(0, x(0)\big) \leqslant \mu^{k/\tau_0} \lambda^k V_{\sigma(0)}\big(0, x(0)\big) \\ &\leqslant \mathrm{e}^{(\ln \mu/\tau_0 + \ln \lambda)k} V_{\sigma(0)}\big(0, x(0)\big), \quad s \in \mathbb{N}_0. \end{aligned}$$

So, there exist scalars ν_1 and ν_2, which makes the following inequality hold:

$$\begin{aligned} \nu_1^2 \big\|x(k)\big\|^2 &\leqslant V_{\sigma(k_s)}\big(k, x(k)\big) \leqslant \mathrm{e}^{(\ln \mu/\tau_0 + \ln \lambda)k} V_{\sigma(0)}\big(0, x(0)\big) \\ &\leqslant \nu_2^2 \mathrm{e}^{(\ln \mu/\tau_0 + \ln \lambda)k} \|\phi\|_d^2, \quad s \in \mathbb{N}_0, \end{aligned}$$

namely,

$$\|x(k)\| \leqslant c\beta^k \|\phi\|_d, \quad k \in \mathbb{N}_0,$$

where $c = \nu_2/\nu_1$, $\beta = \mathrm{e}^{\ln\mu/\tau_0 + \ln\lambda}$. By Definition 2, system (2) with $\omega(k) = 0$ is exponentially stable. The proof is complete. □

Remark 2. In Theorem 1, the asymptotical stability via the MDIDT switching of system (2) with $\omega(k) = 0$ can be achieved by setting $\lambda = 1$.

Remark 3. When $Q_{\sigma(k_s)}^{(l-k_s)} \equiv Q_{\sigma(k_s)} > 0$, the Lyapunov–Krasovskii functional (6) can be written as

$$\begin{aligned} V_{\sigma(k_s)}\big(k, x(k)\big) &= x^{\mathrm{T}}(k) P_{\sigma(k_s)}^{(k-k_s+1)} x(k) \\ &\quad + \sum_{l=k-d}^{k-1} \lambda^{k-l-1} x^{\mathrm{T}}(l) Q_{\sigma(k_s)} x(l) \end{aligned} \tag{11}$$

for $k \in I[k_s, k_{s+1}]$. According to Theorem 1, we can derive Corollary 1.

Corollary 1. *If there are scalars $0 < \lambda < 1$, $\mu \geqslant 1$ and $n \times n$-dimensional matrices $P_i^{(v)} > 0$, $v \in \langle \tau_{i2} + 1\rangle$, $Q_i > 0$, $i \in \langle m\rangle$, such that* (4) *holds,*

$$\begin{bmatrix} A_i^{\mathrm{T}} P_i^{(p+1)} A_i - \lambda P_i^{(p)} + Q_i & A_i^{\mathrm{T}} P_i^{(p+1)} B_i \\ * & B_i^{\mathrm{T}} P_i^{(p+1)} B_i - \lambda^d Q_i \end{bmatrix} < 0, \quad p \in \langle \tau_{i2}\rangle,\ i \in \langle m\rangle,$$

$$Q_j < \mu Q_i, \quad i, j \in \langle m\rangle,\ j \neq i,$$

and $\tau_0 > -\ln\mu/\ln\lambda$, where $\tau_0 = \min_{i\in\langle m\rangle}\{\tau_{i1}\}$, then system (2) *with $\omega(k) = 0$ is exponentially stable via the MDIDT switching.*

Theorem 2. *For a given scalar $\gamma > 0$, if there exist $n\times n$-dimensional matrices $P_i^{(v)} > 0$, $v \in \langle \tau_{i2} + 1\rangle$, and $Q_i^{(w)} > 0$, $w \in I[-d, \tau_{i2} - 1]$, $i \in \langle m\rangle$, such that for $p \in \langle \tau_{i2}\rangle$, $q \in I[\tau_{i1} + 1, \tau_{i2} + 1]$, $g \in I[-d, -1]$, $i, j \in \langle m\rangle$, $j \neq i$,*

$$\begin{bmatrix} -P_i^{(p+1)} & 0 & P_i^{(p+1)} A_i & P_i^{(p+1)} B_i & P_i^{(p+1)} C_i \\ * & -I & A_{zi} & B_{zi} & C_{zi} \\ * & * & -P_i^{(p)} + Q_i^{(p-1)} & 0 & 0 \\ * & * & * & -Q_i^{(p-1-d)} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{12}$$

$$F_j^{\mathrm{T}} P_j^{(1)} F_j < P_i^{(q)}, \tag{13}$$

$$Q_j^{(g)} < Q_i^{(q+g-1)}, \tag{14}$$

then system (2) *is asymptotically stable with an H_∞ performance index γ via the MDIDT switching.*

Proof. Select a switching time-varying Lyapunov–Krasovskii functional as follows:

$$V_{\sigma(k_s)}\big(k, x(k)\big) = x^{\mathrm{T}}(k)P_{\sigma(k_s)}^{(k-k_s+1)}x(k) + \sum_{l=k-d}^{k-1} x^{\mathrm{T}}(l)Q_{\sigma(k_s)}^{(l-k_s)}x(l), \quad k \in I[k_s, k_{s+1}],$$

where $x(k_s) = \tilde{x}(k_s)$, $P_{\sigma(k_s)}^{(k-k_s+1)} > 0$, and $Q_{\sigma(k_s)}^{(l-k_s)} > 0$. Similar to the proof of Theorem 1, when $k \in I[k_s,\, k_{s+1} - 1]$, we have that

$$H_i(k) = \Delta V_i\big(k, x(k)\big) + z^{\mathrm{T}}(k)z(k) - \gamma^2\omega^{\mathrm{T}}(k)\omega(k) = \eta^{\mathrm{T}}(k)\Theta_i\eta(k),$$

where $\eta^{\mathrm{T}}(k) = [x^{\mathrm{T}}(k)\ x^{\mathrm{T}}(k-d)\ \omega^{\mathrm{T}}(k)]$ and

$$\Theta_i = \begin{bmatrix} \Theta_{1i} & \Theta_{2i} & \Theta_{3i} \\ * & \Theta_{4i} & \Theta_{5i} \\ * & * & \Theta_{6i} \end{bmatrix},$$

where

$$\begin{aligned}
\Theta_{1i} &= A_i^{\mathrm{T}}P_i^{(p+1)}A_i + A_{zi}^{\mathrm{T}}A_{zi} - P_i^{(p)} + Q_i^{(p-1)},\\
\Theta_{2i} &= A_i^{\mathrm{T}}P_i^{(p+1)}B_i + A_{zi}^{\mathrm{T}}B_{zi},\\
\Theta_{3i} &= A_i^{\mathrm{T}}P_i^{(p+1)}C_i + A_{zi}^{\mathrm{T}}C_{zi},\\
\Theta_{4i} &= B_i^{\mathrm{T}}P_i^{(p+1)}B_i + B_{zi}^{\mathrm{T}}B_{zi} - Q_i^{(p-1-d)},\\
\Theta_{5i} &= B_i^{\mathrm{T}}P_i^{(p+1)}C_i + B_{zi}^{\mathrm{T}}C_{zi},\\
\Theta_{6i} &= C_i^{\mathrm{T}}P_i^{(p+1)}C_i + C_{zi}^{\mathrm{T}}C_{zi} - \gamma^2 I.
\end{aligned}$$

In order to prove $H_i(k) < 0$, it suffices to show that $\Theta_i < 0$ for $k \in I[k_s,\, k_{s+1} - 1]$. Applying Lemma 1, we have that $\Theta_i < 0$ is equivalent to (12). Therefore, we have $H_i(k) < 0$ for $k \in I[k_s,\, k_{s+1} - 1]$. Then we can easily derive

$$V_i\big(k_{s+1}, x(k_{s+1})\big) - V_i\big(k_s, x(k_s)\big) + \sum_{k=k_s}^{k_{s+1}-1} z^{\mathrm{T}}(k)z(k) - \gamma^2 \sum_{k=k_s}^{k_{s+1}-1} \omega^{\mathrm{T}}(k)\omega(k) < 0. \tag{15}$$

When $k = k_{s+1}$, it follows from (13) and (14) that

$$V_j\big(k_{s+1}, x(k_{s+1})\big) < V_i\big(k_{s+1}, x(k_{s+1})\big). \tag{16}$$

Combining (15) and (16), we have

$$V_j\big(k_{s+1}, x(k_{s+1})\big) - V_i\big(k_s, x(k_s)\big) + \sum_{k=k_s}^{k_{s+1}-1} z^{\mathrm{T}}(k)z(k) - \gamma^2 \sum_{k=k_s}^{k_{s+1}-1} \omega^{\mathrm{T}}(k)\omega(k) < 0. \tag{17}$$

From $H_i(k) < 0$ we derive

$$V_i\big(k+1, x(k+1)\big) < V_i\big(k_s, x(k_s)\big), \quad k \in \{k_s, k_s+1, \ldots, k_{s+1}-1\} \tag{18}$$

for $\omega(k) = 0$. (16) and (18) mean that system (2) with $\omega(k) = 0$ is asymptotically stable. Additionally, in accordance with (17), there is that

$$\sum_{k=k_s}^{k_{s+1}-1} z^{\mathrm{T}}(k)z(k) - \gamma^2 \sum_{k=k_s}^{k_{s+1}-1} \omega^{\mathrm{T}}(k)\omega(k) \leqslant V_i\big(k_s, x(k_s)\big) - V_j\big(k_{s+1}, x(k_{s+1})\big).$$

Then we can obtain

$$\sum_{k=0}^{\infty} z^{\mathrm{T}}(k)z(k) - \gamma^2 \sum_{k=0}^{\infty} \omega^{\mathrm{T}}(k)\omega(k) \leqslant V_{\sigma(0)}\big(0, x(0)\big) = 0,$$

i.e.,

$$\sum_{k=0}^{\infty} \big\|z(k)\big\|^2 \leqslant \gamma^2 \sum_{k=0}^{\infty} \big\|\omega(k)\big\|^2.$$

Consequently, system (2) is asymptotically stable with an H_∞ performance index γ via the MDIDT switching. The proof is complete. $\square$

Subsequently, we will design a state feedback controller in the form of

$$u(k) = K_{\sigma(k)}(k)\tilde{x}(k). \tag{19}$$

When $k \in I[k_s, k_{s+1}-1]$, define $K_{\sigma(k_s)}(k) = K_{\sigma(k_s)}^{(k-k_s+1)}$, $s \in \mathbb{N}_0$.

Theorem 3. *If there are scalars* $0 < \lambda < 1$, $\mu \geqslant 1$ *and* $n \times n$*-dimensional matrices* $X_i^{(v)} > 0$, $v \in \langle \tau_{i2}+1 \rangle$, $Y_i^{(w)} > 0$, $w \in I[-d, \tau_{i2}-1]$, $\hat{K}_i^{(l)}$, $l \in \langle \tau_{i2} \rangle$, $i \in \langle m \rangle$, *such that for* $p \in \langle \tau_{i2} \rangle$, $q \in I[\tau_{i1}+1, \tau_{i2}+1]$, $g \in I[-d, -1]$, $i, j \in \langle m \rangle$, $j \neq i$,

$$\begin{bmatrix} -X_i^{(p+1)} & A_i X_i^{(p)} + L_i \hat{K}_i^{(p)} & B_i Y_i^{(p-1-d)} & 0 \\ * & -\lambda X_i^{(p)} & 0 & X_i^{(p)} \\ * & * & -\lambda^d Y_i^{(p-1-d)} & 0 \\ * & * & * & -Y_i^{(p-1)} \end{bmatrix} < 0, \tag{20}$$

$$F_j X_i^{(q)} F_j^{\mathrm{T}} - \mu X_j^{(1)} < 0, \tag{21}$$

$$Y_i^{(q+g-1)} - \mu Y_j^{(g)} < 0, \tag{22}$$

and $\tau_0 > -\ln\mu/\ln\lambda$, *where* $\tau_0 = \min_{i\in\langle m \rangle}\{\tau_{i1}\}$, *then system* (1) *with* $\omega(k) = 0$ *is exponentially stabilizable via the MDIDT switching with controller in the form of* (19), *where* $K_i^{(p)} = \hat{K}_i^{(p)}[X_i^{(p)}]^{-1}$, $p \in \langle \tau_{i2} \rangle$.

Proof. Replacing A_i in (3) by $A_i + L_i K_i^{(p)}$ and applying Lemma 1, we have

$$\begin{bmatrix} -P_i^{(p+1)} & P_i^{(p+1)}(A_i + L_i K_i^{(p)}) & P_i^{(p+1)} B_i & 0 \\ * & -\lambda P_i^{(p)} & 0 & I \\ * & * & -\lambda^d Q_i^{(p-1-d)} & 0 \\ * & * & * & -[Q_i^{(p-1)}]^{-1} \end{bmatrix} < 0. \tag{23}$$

Set $X_i^{(p)} = [P_i^{(p)}]^{-1}$, $Y_i^{(p-1)} = [Q_i^{(p-1)}]^{-1}$, and $\hat{K}_i^{(p)} = K_i^{(p)} X_i^{(p)}$. Multiplying (23) by $\mathrm{diag}\{X_i^{(p+1)}, X_i^{(p)}, Y_i^{(p-1-d)}, I\}$ on the right- and left-hand side, we have (20). In addition, using Lemma 1, (4) can be rewritten as

$$\begin{bmatrix} -P_j^{(1)} & P_j^{(1)} F_j \\ * & -\mu P_i^{(q)} \end{bmatrix} < 0. \tag{24}$$

Multiplying (24) by $\mathrm{diag}\{X_j^{(1)}, X_i^{(q)}\}$ on the right- and left-hand side and applying Lemma 1, we can get (21). Similarly, we can know that (5) is equivalent to (22). The proof is complete. □

Theorem 4. *For a given scalar $\gamma > 0$, if there are $n \times n$-dimensional matrices $X_i^{(v)} > 0$, $v \in \langle \tau_{i2} + 1 \rangle$, and $Y_i^{(w)} > 0$, $w \in I[-d, \tau_{i2} - 1]$, $\hat{K}_i^{(l)}$, $l \in \langle \tau_{i2} \rangle$, $i \in \langle m \rangle$, such that for $p \in \langle \tau_{i2} \rangle$, $q \in I[\tau_{i1} + 1, \tau_{i2} + 1]$, $g \in I[-d, -1]$, $i, j \in \langle m \rangle$, $j \neq i$,*

$$\begin{bmatrix} -X_i^{(p+1)} & 0 & A_i X_i^{(p)} + L_i \hat{K}_i^{(p)} & B_i Y_i^{(p-1-d)} & C_i & 0 \\ * & -I & A_{zi} X_i^{(p)} + M_i \hat{K}_i^{(p)} & B_{zi} Y_i^{(p-1-d)} & C_{zi} & 0 \\ * & * & -X_i^{(p)} & 0 & 0 & X_i^{(p)} \\ * & * & * & -Y_i^{(p-1-d)} & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -Y_i^{(p-1)} \end{bmatrix} < 0, \tag{25}$$

$$F_j X_i^{(q)} F_j^{\mathrm{T}} - X_j^{(1)} < 0, \tag{26}$$

$$Y_i^{(q+g-1)} - Y_j^{(g)} < 0, \tag{27}$$

then system (1) *is asymptotically stabilizable with an H_∞ performance index γ via the MDIDT switching.*

Proof. Replacing A_i in (12) by $A_i + L_i K_i^{(p)}$ and A_{zi} in (12) by $A_{zi} + M_i K_i^{(p)}$, we can get

$$\begin{bmatrix} -P_i^{(p+1)} & 0 & P_i^{(p+1)}(A_i + L_i K_i^{(p)}) & P_i^{(p+1)} B_i & P_i^{(p+1)} C_i \\ * & -I & A_{zi} + M_i K_i^{(p)} & B_{zi} & C_{zi} \\ * & * & -P_i^{(p)} + Q_i^{(p-1)} & 0 & 0 \\ * & * & * & -Q_i^{(p-1-d)} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0,$$

i.e.,

$$\begin{bmatrix} -P_i^{(p+1)} & 0 & P_i^{(p+1)}(A_i + L_i K_i^{(p)}) & P_i^{(p+1)} B_i & P_i^{(p+1)} C_i & 0 \\ * & -I & A_{zi} + M_i K_i^{(p)} & B_{zi} & C_{zi} & 0 \\ * & * & -P_i^{(p)} & 0 & 0 & I \\ * & * & * & -Q_i^{(p-1-d)} & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -[Q_i^{(p-1)}]^{-1} \end{bmatrix} < 0, \quad (28)$$

where $i \in \langle m \rangle$, $p \in \langle \tau_{i2} \rangle$. Set $X_i^{(p)} = [P_i^{(p)}]^{-1}$, $Y_i^{(p-1)} = [Q_i^{(p-1)}]^{-1}$, and $\hat{K}_i^{(p)} = K_i^{(p)} X_i^{(p)}$. Multiplying (28) by $\operatorname{diag}\{X_i^{(p+1)}, I, X_i^{(p)}, Y_i^{(p-1-d)}, I, I\}$ on the right- and left-hand side, we have (25). In addition, using Lemma 1, (13) can be rewritten as

$$\begin{bmatrix} -P_j^{(1)} & P_j^{(1)} F_j \\ * & -P_i^{(q)} \end{bmatrix} < 0. \quad (29)$$

Multiplying (29) by $\operatorname{diag}\{X_j^{(1)}, X_i^{(q)}\}$ on the right- and left-hand side and applying Lemma 1, we can get (26). Similarly, we can know that (14) is equivalent to (27). The proof is complete. □

Remark 4. This paper introduces a switching time-varying Lyapunov–Krasovskii functional (6), which can be used to prove the stability of the system when all subsystems are unstable. The classical Lyapunov–Krasovskii functional

$$V_{\sigma(k_s)}\big(k, x(k)\big) = x^{\mathrm{T}}(k) P_{\sigma(k_s)} x(k) + \sum_{l=k-d}^{k-1} \lambda^{k-l-1} x^{\mathrm{T}}(l) Q_{\sigma(k_s)} x(l)$$

is applicable only in the case where the subsystems are stable. However, due to the introduction of time-varying functional, the amount of calculation increases. In other words, the number of definite matrices $P_{\sigma(k_s)}^{(k-k_s+1)}$ and $Q_{\sigma(k_s)}^{(k-k_s)}$ depends not only on the number of subsystems, but also on the time delay and the upper bound of the MDIDT, which increases the difficulty of calculation.

4 Numerical examples

Example 1. Consider system (2) with $\omega(k) = 0$, where

$$A_1 = \begin{bmatrix} -0.14 & 0.11 \\ 0.13 & 1.12 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.07 & -0.04 \\ -0.06 & 0.05 \end{bmatrix}, \qquad F_1 = \begin{bmatrix} 1.02 & 0.13 \\ -0.21 & 1.03 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1.11 & 0.09 \\ -0.18 & 0.11 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.05 & -0.03 \\ -0.02 & 0.06 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 1.03 & 0.21 \\ -0.13 & 1.04 \end{bmatrix}.$$

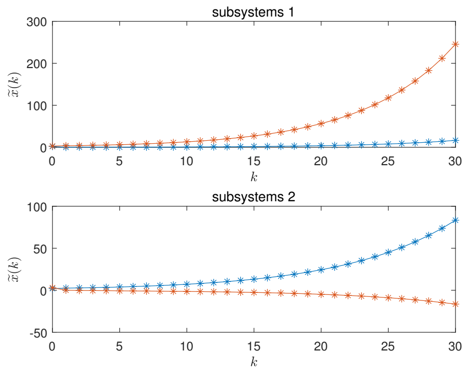

Figure 1. State trajectories of the two subsystems.

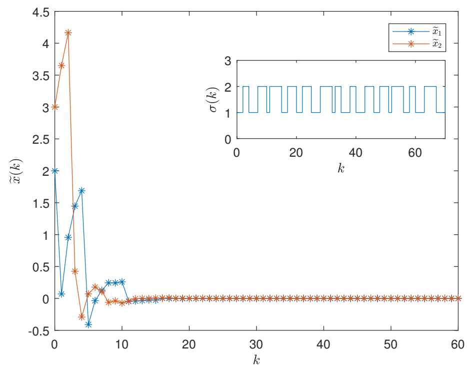

Figure 2. State trajectories of system (2) with $\omega(k) = 0$ via the MDIDT switching rule.

Select time delay $d = 3$, $\tau_{11} = 1$, $\tau_{12} = 3$, $\tau_{21} = 2$, $\tau_{22} = 4$. Given the initial states $x(\theta) = (2, 3)^{\mathrm{T}}$, $\theta = -3, -2, -1, 0$. Figure 1 illustrates two subsystems are unstable. The feasible solution for Theorem 1 can be obtain through solving linear matrix inequalities. Consequently, system (2) with $\omega(k) = 0$ is exponentially stable via the given MDIDT switching. Figure 2 illustrates the state trajectories of system (2) with $\omega(k) = 0$ via the given MDIDT switching rule. In addition, we can easily know that the result of Corollary 1 is not applicable to this example, which shows that the constructed Lyapunov–Krasovskii functional (6) is less conservative than (11). However, due to the impulse, the results in [7, 15, 20, 26] cannot be achieved in this example either. Even if we do not consider the impulse, these results cannot get the exponential stability of system (2) with $\omega(k) = 0$.

Example 2. Consider system (2) and the following parameters:

$$A_1 = \begin{bmatrix} -0.21 & 0.08 \\ 0.29 & 1.01 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -0.02 & 0.41 \\ 0.05 & -0.09 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.13 & 0.34 \\ -0.55 & 0.23 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1.03 & 0.05 \\ -0.09 & 0.08 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.03 & -0.05 \\ -0.01 & 0.32 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} -0.12 & 0.17 \\ -0.46 & 0.03 \end{bmatrix},$$

$$A_{z1} = \begin{bmatrix} -0.04 & 0.08 \\ 0.07 & 0.02 \end{bmatrix}, \qquad B_{z1} = \begin{bmatrix} 0.12 & -0.31 \\ -0.05 & 0.07 \end{bmatrix}, \qquad C_{z1} = \begin{bmatrix} 0.15 & 0.02 \\ -0.03 & -0.09 \end{bmatrix},$$

$$A_{z2} = \begin{bmatrix} 0.11 & 0.04 \\ -0.05 & 0.06 \end{bmatrix}, \qquad B_{z2} = \begin{bmatrix} 0.03 & -0.05 \\ -0.01 & 0.02 \\ & \end{bmatrix}, \qquad C_{z2} = \begin{bmatrix} -0.02 & 0.03 \\ 0.19 & -0.05 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} -1.01 & 0.09 \\ 0.12 & 1.04 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} -1.03 & 0.13 \\ 0.44 & 1.02 \end{bmatrix}, \qquad \omega(k) = 0.1 \begin{bmatrix} |\sin(k)| \\ |\cos(k)| \end{bmatrix}.$$

Select time delay $d = 2$ and the MDIDT switching signal with $\tau_{11} = 1$, $\tau_{12} = 2$ and $\tau_{21} = 2$, $\tau_{22} = 3$ and H_∞ performance index $\gamma = 0.9$. Given the initial states

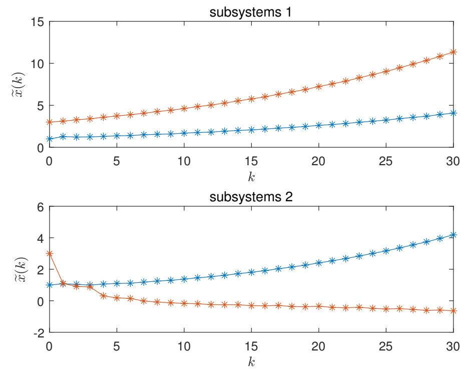

Figure 3. State trajectories of the two subsystems.

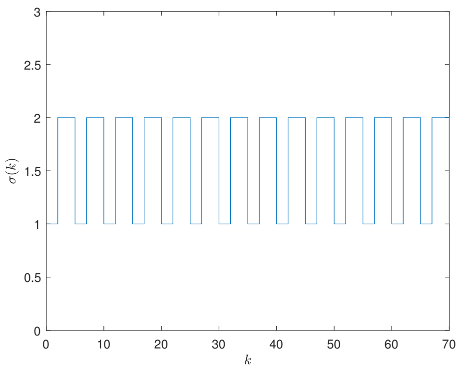

Figure 4. The given MDIDT switching signal.

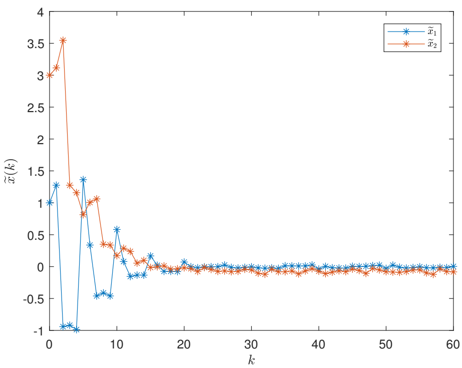

Figure 5. State trajectories of system (2).

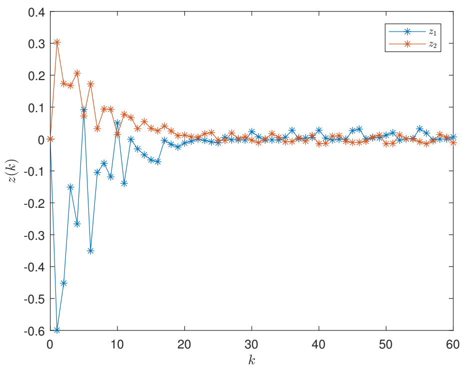

Figure 6. Output state trajectories of system (2).

$x(\theta) = (1, 3)^{\mathrm{T}}$, $\theta = -2, -1, 0$. Figure 3 illustrates two subsystems are unstable. The feasible solution for Theorem 2 can be easily obtained by solving linear matrix inequalities. Consequently, system (2) is asymptotically stabilizable with an H_∞ performance index γ via the MDIDT switching. Figure 4 illustrates the given MDIDT switching signal. Figures 5 and 6 illustrate the state trajectories and the output state trajectories of system (2), respectively.

Example 3. Consider system (1) with $\omega(k) = 0$, where

$$A_1 = \begin{bmatrix} -1.04 & 0.19 \\ 0.25 & 1.12 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.13 & -0.52 \\ -0.25 & 0.16 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.23 & 0.24 \\ -0.3 & 1.05 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 0.4 \\ -0.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1.12 & 0.26 \\ -0.31 & 1.06 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.14 & -0.51 \\ -0.23 & 0.17 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.24 & 0.25 \\ -0.2 & 1.06 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.2 \\ 0.7 \end{bmatrix}.$$

Select time delay $d = 1$ and the MDIDT switching signal with $\tau_{11} = 2$, $\tau_{12} = 3$, $\tau_{21} = 4$, $\tau_{22} = 5$. Given the initial states $x(-1) = (1, 2)^{\mathrm{T}}$ and $x(0) = (1, 3)^{\mathrm{T}}$.

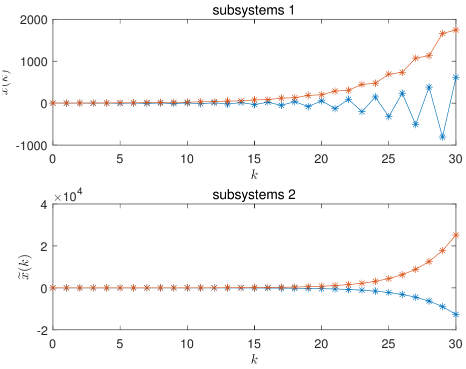

Figure 7. State trajectories of the two subsystems.

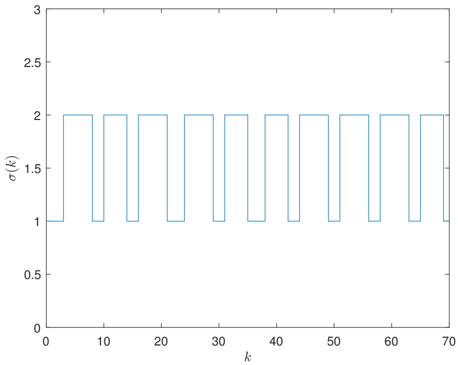

Figure 8. The given MDIDT switching signal.

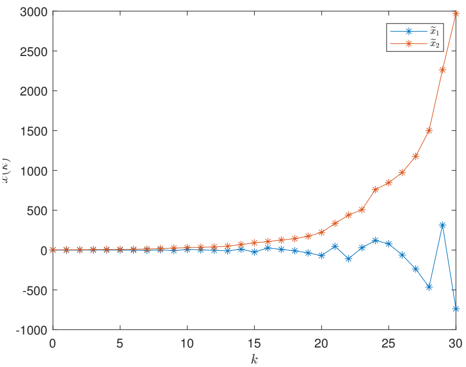

Figure 9. State trajectories of the open-loop system (1).

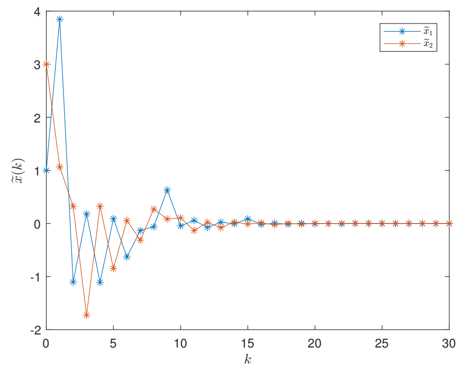

Figure 10. State trajectories of the closed-loop system (1).

Figure 7 illustrates two subsystems are unstable. It is easy to obtain the feasible solution for Theorem 3 through solving linear matrix inequalities. Consequently, system (1) with $\omega(k) = 0$ is exponentially stabilizable via the given MDIDT switching with controller in the form of (19), where

$$\begin{aligned}
K_1^{(1)} &= \begin{bmatrix}1.6485 & 3.8068\end{bmatrix}, & K_1^{(2)} &= \begin{bmatrix}1.7412 & 3.3896\end{bmatrix},\\
K_1^{(3)} &= \begin{bmatrix}0.6590 & 8.2594\end{bmatrix}, & K_2^{(1)} &= \begin{bmatrix}0.4088 & -1.5329\end{bmatrix},\\
K_2^{(2)} &= \begin{bmatrix}1.2280 & -1.0858\end{bmatrix}, & K_2^{(3)} &= \begin{bmatrix}1.4225 & -0.9797\end{bmatrix},\\
K_2^{(4)} &= \begin{bmatrix}0.7944 & -1.3225\end{bmatrix}, & K_2^{(5)} &= \begin{bmatrix}0.3564 & -1.5622\end{bmatrix}.
\end{aligned}$$

Figure 8 illustrates the given MDIDT switching signal. Figures 9 and 10 illustrate the state trajectories of the open-loop system (1) and the closed-loop system (1), respectively. Furthermore, in this example, if we adopt the traditional state feedback controller

$u(k) = K_{\sigma(k)}x(k)$, which is the form in [1, 30], we cannot get the corresponding state feedback matrices. In other words, the time-varying state feedback controller (19) is applicable to more situations.

5 Conclusion

In this paper, we analyze the stability and stabilization of discrete-time switched delay systems with impulse in the case of all subsystems being unstable. Through establishing a switching time-varying Lyapunov–Krasovskii functional, we obtain some new stability theorems for the time-delay system via the MDIDT switching. Moreover, we design a state feedback controller to ensure the stabilization of the resulting closed-loop system. Subsequently, numerical examples demonstrate our theoretical findings. We only consider the influence of impulse and constant time delay on the stability and stabilization of discrete-time switched systems, and we will endeavor to extend the results to stability and stabilization of discrete-time switched systems with time-varying delay, impulse and parameter uncertainties in the future.

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