Nonlinear Analysis: Modelling and Control, Vol. 29, No. 2, 365–378
https://doi.org/10.15388/namc.2024.29.34669

Comparative analysis of classical and stochastic Maccari system of nonlinear equations*

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Received: June 4, 2023 / **Revised:** November 26, 2023 / **Published online:** February 23, 2024

Abstract. In this paper, the exact solutions of classical and stochastic Maccari system is constructed. The exact comparative solutions are examined and plotted. Interesting results in the case of multiplicative noise are formulated and graphically elaborated. The applications of the stochastic

*This work was funded by Deanship of Scientific Research, Taif University, Saudi Arabia.
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Maccari system are added for the physical purpose. The existence of results for the real part of underlying system are discussed first time for a priori estimates. The perturbations, which disturbed the formation of Langmuir waves, are geometrically expressed in this article. Due to the presence of multiplicative noise term, our system brings a real flavor to the dynamics of the problem.

Keywords: stochastic partial differential equation, Langmuir soliton, MEDA method.

1 Introduction

The major contribution of partial differential equations (PDEs) is in the field of analysis and geometry. The behavior of electromagnetic radiation, the dynamical motion of photons and electrons in the molecules, propagation of sound, heat flow, waves of any type, etc. are all expressed with the help of PDEs. PDEs also play a vital role in general relativity, Riemannian geometry, differential geometry, etc. [19]. The significant importance of PDEs is in the fields of physics, applied mathematics, and engineering. In short, PDEs are very helpful in describing different phenomena such as fluid dynamics, elasticity, and quantum mechanics. Due to its wide variety of applications in physics and mathematical analysis, numerous methods have been presented to study the solution and physical behavior of nonlinear wave equations [16]. In the study of nonlinear physical phenomena for various fields of engineering, science, mathematical physics, biophysics, the propagation of shallow water waves, high-energy physics, fluid dynamics, plasma, optical fibers, and so on, the research of exact traveling wave solutions to nonlinear evolution equations (NLEEs) is crucial [7]. Numerous techniques, such as the inverse scattering transform [1], Hirota's bilinear method [8], the Darboux transformation method [6], etc., have been devised to determine the explicit solution to NLEEs.

The bell-shaped sech solutions and kink-shaped tanh solutions physically demonstrate the behavior of nonlinear waves especially observed in the fluid dynamics, plasma, and optical fibers. Recently, researchers have shown more interest in the exploration of different kinds of exact solutions of NLEEs like periodic, quasiperiodic, rational, cuspon, complexiton, peakon, negaton, soliton solutions, etc. [12]. Exact solutions to nonlinear partial differential equations play an essential position in nonlinear physical science because they bring forth deep information about the physical aspect of many problems and their applications. In short, recently distinct methods for attaining the explicit solutions of solitary and traveling wave solutions of NLEEs have been presented such as homogenous balance method [25], solitary wave ansatz method [14], Jacobi elliptic function expansion method [5], the tanh-function method [15], F-expansion method [13], projective Ricatti equation method [26], (G'/G)-expansion method [17], and so on.

In this manuscript, we examine the exact solution of a more general coupled nonlinear Maccari system by applying the technique of the modified extended direct algebraic (MEDA) method. The objective of this article is to acquire the exact traveling wave solution of a coupled stochastic Maccari system by applying the technique of MEDA method [4, 23]. One example of the nonlinear evolution equation is the Maccari system, which expresses the behavior of the motion of the isolated waves observed in a small part of space in many fields such as plasma physics, hydrodynamics, nonlinear optics, and so on. The exact solutions of the Maccari system have been discussed by Zhang [30] by applying the technique of exp-function method.

2 Statement of problems

This paper deals with optimum existence for the solutions and extraction of soliton solutions for stochastic Maccari system (MS) described by

$$\imath\psi_t + \psi_{xx} + \psi v = \dot{\beta}\psi, \tag{1}$$

$$v_t + v_y + \left(|\psi|^2\right)_x = 0 \tag{2}$$

with initial conditions

$$\psi(x, y, 0) = \psi(0) = \psi_0, \qquad v(x, y, 0) = v(0) = v_0,$$

where the complex-valued scalar function ψ and real-valued scalar function v are the functions of independent spatial variables x, y and the temporal variable $t > 0$. Also, they respectively describe the high-frequency waves and the potential transferred through such waves. The Maccari system (MS) was introduced in a little portion of space to describe the propagation of the different isolated waves in numerous fields of natural science. This system is important to show the nature of sonic Langmuir solitons in plasma physics.

The construction and, more generally, the generator of the dynamical system in the classical sense are basically the stochastic differential equations. The solutions of these types of equations using the given initial values encode information about the Markov process. At a specific time, the solutions define a random diffeomorphism, and the family of these diffeomorphisms highlighted the closed cochain over the Wiener space [2]. Providing the exact solitons solutions, we also expressed the existence of these solutions by using the fixed point theory. The next section yields the directions about the continuous interval for the existence of solutions. Section 3 expresses the detailed steps for the algorithm of the MEDA method, and Section 4 brings forth the application of the MEDA method. Section 5 demonstrates the solutions plots. Section 6 underlines the physical interpretation, and Section 7 debates over the conclusion.

3 Existence of solutions

In this section, we discuss the existence and continuity of solutions of stochastic Maccari system (1), (2) whose integral representation is

$$\psi(x, y, t) = \psi_0 + \int_0^t \left(|\psi_{xx}| + |\psi||v| + |\dot{\beta}||\psi|\right) \mathrm{d}\tau, \tag{3}$$

$$v(x, y, t) = v_0 + \int_0^t \left(v_y + \left(|\psi|^2\right)_x\right) \mathrm{d}\tau. \tag{4}$$

The construction of Eqs. (3) and (4) guides us to rewrite it in the form of fixed operator, so we have

$$\Psi = \Psi\bigl(\psi(x,y,t)\bigr) = \psi_0 + \int_0^t \bigl(|\psi_{xx}| + |\psi||v| + E|\dot{\beta}||\psi|\bigr)\,\mathrm{d}\tau, \tag{5}$$

$$V = V\bigl(v(x,y,t)\bigr) = v_0 + \int_0^t \bigl(v_y + \bigl(|\psi|^2\bigr)_x\bigr)\,\mathrm{d}\tau. \tag{6}$$

The operator equation is a fixed point representation of the problem for $\psi(x,y,t)$ and $v(x,y,t)$. Hence, any fixed functions $\psi^*(x,y,t)$ and $v^*(x,y,t)$ of $\Psi(x,y,t)$ and $V(x,y,t)$ will turn out to be a solution of not only of Eqs. (1) and (2) but also of Eqs. (3), 4. Before proceeding for the construction of fixed points of Eqs. (5) and (6), we must look for the topological spaces. Here we choose the space of continuous functions C equipped with supremum norm, i.e., $m^* \in C[0,\rho] \Rightarrow \|m^*\|_C = \max_{[0,\rho]} |m^*|$.

Assuming that the functions ψ, v, ψ_0, v_0, ψ_{xx}, v_y, $(|\psi|^2)_x$, β are continuous and, consequently, locally bounded,

$$\|v\| \leqslant r, \qquad \|\psi\| \leqslant r, \qquad \|\psi_{xx}\| \leqslant k_1, \qquad \beta \leqslant k_2,$$
$$\bigl(\|\psi\|^2\bigr)_x \leqslant k_3, \qquad \|v_y\| \leqslant k_4, \qquad \|v_0\| \leqslant k_5, \qquad \|\psi_0\| \leqslant k_6.$$

Here as $r, k_1, k_2, k_3, k_4, k_5, k_6 > 0$.

For the analysis of the existence of the solutions to the underlying system, we shall apply an important and basic fixed point theorem in the Banach space of all continuous functions known as Schauder fixed point theorem [10], which is a generalization of Brouwer's fixed point theorem for infinite dimensional space; see [11]. There are two important steps in Schauder's fixed point theorem, which are the implications of the statement of the theorem.

Theorem 1. (*See* [18].) *Suppose B is a closed, convex, and bounded subset of Banach space C. The functions $\Psi(x,y,t)$ and $V(x,y,t)$ are continuous operators, which map the set B into itself, and the mappings $\Psi(B): B \to B$ and $V(B): B \to B$ are relatively compact or precompact. Then the operators $\Psi(x,y,t)$ and $V(x,y,t)$ must have at least one fixed point x^* in B.*

In view of the above result, the following three conditions must be verified:

(i) $\Psi(x,y,t): B \to B$,
(ii) $V(x,y,t): B \to B$,
(iii) $\Psi(B)$ and $V(B)$ are relatively compact.

The set B is closed, convex, and bounded subset of $C[0,\rho]$, and it is defined by

$$B_r(\Theta) = \bigl\{\psi, v \in C[0,\rho]\colon \|\psi\| = \|v\| \leqslant r\bigr\},$$

where the capital theta symbol Θ denotes the zero element of function space C, r is the radius of the ball and define the complete length of continuity of the solution.

To verify conditions (i) and (ii), we shall use $B = B_r(\Theta)$. For this purpose, we apply the norm on both sides of Eqs. (5) and (6), and further simplification leads to

$$E\|\Psi\| \leqslant k_6 + \left(r^2 + E\|\dot{\beta}\|r + k_1\right)\rho,$$
$$\|V\| \leqslant k_5 + \left(k_4 + k_3\right)\rho,$$

where $\rho = (t - 0)$, length of continuity. For mapping the ball into itself, we have the following conditions:

$$\rho \leqslant \frac{r - k_6}{r^2 + k_2 r + k_1}, \qquad \rho \leqslant \frac{r - k_5}{k_4 + k_3}.$$

This will hold true if $r > k_6$ and $r > k_5$.

Now this step is essential for the accomplishment of the existence of a solution by Schauder's well-known result for the existence in an infinite dimensional setting. For this, we again consider Eqs. (5) and (6). The equicontinuity of Eq. (5) is checked at two points t and t^*. Then following the same pattern, we checked the equicontinuity of Eq. (6). Therefore, we have the following equation:

$$\Psi_i(x,y,t) - \Psi_i(x,y,t^*) = \psi_{i_0} + \int_0^t \left[\left|\frac{\partial^2 \psi_i}{\partial x^2}\right| + |\psi_i||v| + |\dot{\beta}||\psi_i|\right] \mathrm{d}\tau - \psi_{i_0} + \int_0^{t^*} \left[\left|\frac{\partial^2 \psi_i}{\partial x^2}\right| + |\psi_i||v| + |\dot{\beta}||\psi_i|\right] \mathrm{d}\tau.$$

After applying the norm and using the triangular inequality, we have

$$E\left\|\Psi_i(x,y,t) - \Psi_i(x,y,t^*)\right\| \leqslant \left(r^2 + E\|\dot{\beta}\|r + k_1\right)|t - t^*|.$$

The equicontinuity of Eq. (6) gives us the final result

$$E\left\|V_i(x,y,t) - V_i(x,y,t^*)\right\| \leqslant (k_4 + k_3)|t - t^*|.$$

Clearly, $E\|\Psi_i(x,y,t) - \Psi_i(x,y,t^*)\| \to 0$ as $t \to t^*$. Similarly, $\|V_i(x,y,t) - V_i(x,y,t^*)\| \to 0$ as $t \to t^*$, and even for a special pair (x^*, y^*, t^*), $E\|\Psi_i(x,y,t) - \Psi_i(x^*,y^*,t^*)\|$, $\|V_i(x,y,t) - V_i(x,y,t^*)\| \to 0$ as $(x,y,t) \to (x^*,y^*,t^*)$. So, the families $\Psi_i(x,y,t)$ and $V_i(x,y,t)$ are turn out to be equicontinuous, so, Arzelà–Ascoli theorem [9] is applicable. By Arzelà–Ascoli there exist subsequences $\Psi_{i_j}(x,y,t)$ and $V_{i_j}(x,y,t)$ of Ψ_i and V_i such that Ψ_{i_j} and $V_{i,j}$ are uniformly convergent. So, the operators $\Psi(x,y,t)$ and $V(x,y,t)$ turn out to be relatively compact operators. So, Schauder theorem is applicable [9, 11, 24], and there exists at least one solution to the stochastic Maccari system, provided the noise term made bounded.

4 Specification for modified extended direct algebraic method

This portion describes an effective technique for finding exact solutions of the system, namely, the modified extended direct algebraic method (MEDA) [3, 21, 22, 27, 28, 31]. This strategy is more advanced and modified compared to exact traveling wave solutions for $(2+1)$-dimensional Konopelchenko–Dubrovsky equation by using the hyperbolic trigonometric functions methods. New analytical solutions and modulation instability analysis for the nonlinear $(1+1)$-dimensional phi-four model [20]. The modified extended direct algebraic method (MEDA) is the latest and implemented on nonlinear space-time systems. This technique enables the researchers to get fresh and wide-ranging closed-form soliton solutions compared to others.

For a detailed description of the MEDA method, see [29]. We consider the following PDE in two independent variables:

$$G(w, w_x, w_t, w_{xx}, \dots) = 0. \tag{7}$$

Using the transformation $w(x,t) = w(\delta)$, $\delta = \imath(x - \omega t)$, Eq. (7) is transformed to an ODE

$$I(w, \imath w', -\imath\omega w', \dots) = 0, \tag{8}$$

where $w' = \mathrm{d}w/\mathrm{d}\delta$. The following ansatz will be used in order to find the solution of Eq. (8).

So,

$$w(\delta) = b_0 + \sum_{i=1}^{N} \left(b_i u^i + a_i u^{-i}\right), \tag{9}$$

$$u' = a + u^2, \tag{10}$$

where a is the parameter to be determined, and $u = u(\delta)$, $u' = \mathrm{d}u/\mathrm{d}\delta$. The value of N can be found by using the balancing principle. By using Eqs. (8)–(10) we construct a system of algebraic equations with respect to b_i, a_i, a, and ω. After collecting all the same-order term of u and equating them equal to zero, we determine the values of b_0, b_i, a_i, a, and ω. The general solution of Eq. (10) is as follows.

Case 1: $a < 0$.

$$u = -\sqrt{-a}\tanh(\sqrt{-a}\,\delta) \quad \text{or} \quad u = -\sqrt{-a}\coth(\sqrt{-a}\,\delta).$$

It depends on initial conditions.

Case 2: $a > 0$.

$$u = \sqrt{a}\tan(\sqrt{a}\,\delta) \quad \text{or} \quad u = -\sqrt{a}\cot(\sqrt{a}\,\delta).$$

It depends on initial conditions.

Case 3: $a = 0$.

$$u = -\frac{1}{\delta}.$$

From backward substituting these results in Eq. (9), the exact travelling wave solutions of Eq. (7) can be obtained.

5 Application of modified extended direct algebraic method

In this section, MEDA method has been used to find the exact solutions of stochastic Maccari system (1), (2) with the dependent variables $\psi = \psi(x, y, t)$ and $v = v(x, y, t)$ specifying the complex and real scalar fields. x, y, and t are the spatial and temporal independent variables, respectively, and $\beta = \beta(x, y, t)$ is the noise term.

Using the transformation

$$\psi(x,y,t) = U(\lambda)\mathrm{e}^{\imath\phi}, \qquad v(x,y,t) = V(\lambda), \qquad \beta(x,y,t) = \beta(\lambda),$$
$$\lambda = \lambda_1 x + \lambda_2 y + \lambda_3 t, \qquad \phi = \phi_1 x + \phi_2 y + \phi_3 t,$$

Eqs. (1) and (2) become

$$\imath\lambda_3 U' - U\phi_3 + \lambda_1^2 U'' + 2\imath U'\lambda_1\phi_1 - U\phi_1^2 + UV - \beta U = 0, \tag{11}$$
$$V'(\lambda_3 + \lambda_2) + \lambda_1\big(U^2\big)' = 0. \tag{12}$$

Integrating Eq. (12) and for the sake of simplicity, we are taking constant of integration equal to zero. So, we have

$$V = -\frac{\lambda_1 U^2}{\lambda_3 + \lambda_2}. \tag{13}$$

Using Eq. (13), Eq. (11) becomes

$$\imath\lambda_3 U' - U\phi_3 + \lambda_1^2 U'' + 2\imath U'\lambda_1\phi_1 - U\phi_1^2 + U\left(-\frac{\lambda_1 U^2}{\lambda_3 + \lambda_2}\right) - \beta U = 0.$$

Separating the real and imaginary parts, we have

$$\imath(\lambda_3 + 2\lambda_1\phi_1)U' = 0,$$
$$\lambda_1^2 U'' - \frac{\lambda_1 U^3}{\lambda_3 + \lambda_2} - \big(\phi_1^2 + \phi_3 + \beta\big)U = 0. \tag{14}$$

Further simplification leads us to rewrite Eq. (14) as follows:

$$\eta_1 U'' + \eta_2 U^3 + \eta_3 U = 0, \tag{15}$$

where

$$\eta_1 = \lambda_1^2, \qquad \eta_2 = -\frac{\lambda_1}{\lambda_3 + \lambda_2}, \qquad \eta_3 = -\big(\phi_1^2 + \phi_3 + \beta\big)$$

as $N = 1$ (by using the homogenous balance in Eq. (15)). So, its solution will be of the form

$$U(\lambda) = \alpha_0 + \alpha_1 P + \gamma_1 P^{-1}, \qquad P' = \gamma + P^2, \tag{16}$$

where α_0, α_1, γ_1, γ are the arbitrary constants, and their values can be determined by Eqs. (15) and (16). We get the following system of algebraic equations:

$$\begin{aligned}
&2\alpha_1\eta_1 + \eta_2\alpha_1^3 = 0, \qquad 3\alpha_0\alpha_1^2\eta_2 = 0,\\
&2\alpha_1\eta_1\gamma + 3\eta_2\alpha_0^2\alpha_1 + 3\eta_2\alpha_1^2 + \eta_3\alpha_1 = 0,\\
&\eta_2\alpha_0^3 + 6\eta_2\alpha_0\alpha_1\gamma_1 + \eta_3\alpha_0 = 0,\\
&2\eta_1\gamma_1\gamma + 3\eta_2\alpha_0^2\gamma_1 + 3\eta_2\gamma_1^2\alpha_1 + \eta_3\gamma_1 = 0,\\
&3\gamma_1^2\eta_2\alpha_0 = 0, \qquad 2\eta_1\gamma_1\gamma^2 + \eta_2\gamma_1^3 = 0.
\end{aligned}$$

Further simplification leads us to write

$$\alpha_0 = 0, \quad \alpha_1 = \imath\sqrt{\frac{2\eta_1}{\eta_2}}, \qquad \gamma = -\frac{3\eta_2\gamma_1\alpha_1 + \eta_3}{2\eta_1}, \quad \gamma_1 = \imath\gamma\sqrt{\frac{2\eta_1}{\eta_2}}.$$

So, the exact solution of Eqs. (1) and (2) are given as follows.

Case 1: $\gamma < 0$.

$$\begin{aligned}
\psi(x,y,t) &= \left(\alpha_0 + \alpha_1\left[\sqrt{-\gamma}\tanh\left(\sqrt{-\gamma}\lambda\right)\right] + \gamma_1\left[\sqrt{-\gamma}\tanh\left(\sqrt{-\gamma}\lambda\right)\right]^{-1}\right)\mathrm{e}^{\imath\phi},\\
v(x,y,t) &= -\frac{\lambda_1}{\lambda_3+\lambda_2}\left(\alpha_0 + \alpha_1\left[\sqrt{-\gamma}\tanh\left(\sqrt{-\gamma}\lambda\right)\right] + \gamma_1\left[\sqrt{-\gamma}\tanh\left(\sqrt{-\gamma}\lambda\right)\right]^{-1}\right)^2.
\end{aligned}$$

Case 2: $\gamma > 0$.

$$\begin{aligned}
\psi(x,y,t) &= \left(\alpha_0 + \alpha_1\left[\sqrt{\gamma}\tan\left(\sqrt{\gamma}\lambda\right)\right] + \gamma_1\left[\sqrt{\gamma}\tan\left(\sqrt{\gamma}\lambda\right)\right]^{-1}\right)\mathrm{e}^{\imath\phi},\\
v(x,y,t) &= -\frac{\lambda_1}{\lambda_3+\lambda_2}\left(\alpha_0 + \alpha_1\left[\sqrt{\gamma}\tan\left(\sqrt{\gamma}\lambda\right)\right] + \gamma_1\left[\sqrt{\gamma}\tan\left(\sqrt{\gamma}\lambda\right)\right]^{-1}\right)^2.
\end{aligned}$$

Case 3: $\gamma = 0$.

$$\psi(x,y,t) = \left(\alpha_0 - \frac{\alpha_1}{\lambda} - \gamma_1\lambda\right)\mathrm{e}^{\imath\phi}, \qquad v(x,y,t) = -\frac{\lambda_1}{\lambda_3+\lambda_2}\left(\alpha_0 - \frac{\alpha_1}{\lambda} - \gamma_1\lambda\right)^2.$$

6 Comparisons of solutions plots

We present the surface plots for different parameter values in Figs. 1–6.

Case 1: $\phi_1 = 3$, $\phi_2 = 20$, $\phi_3 = 2$, $\lambda_1 = 6$, $\lambda_2 = 7$, $\lambda_3 = 2$ for both solution ψ and v.
Case 2: $\phi_1 = 3$, $\phi_2 = 20$, $\phi_3 = 2$, $\lambda_1 = 6$, $\lambda_2 = 7$, $\lambda_3 = 4$ for ψ and $\lambda_3 = 0.2$ for v.
Case 3: $\phi_1 = 3$, $\phi_2 = 20$, $\phi_3 = 2$, $\lambda_1 = 6$, $\lambda_2 = 7$, $\lambda_3 = 4$ for ψ and $\phi_1 = 0.3$, $\phi_2 = 0.20$, $\phi_3 = 10$, $\lambda_1 = 0.6$, $\lambda_2 = 0.7$, $\lambda_3 = 10$ for v.

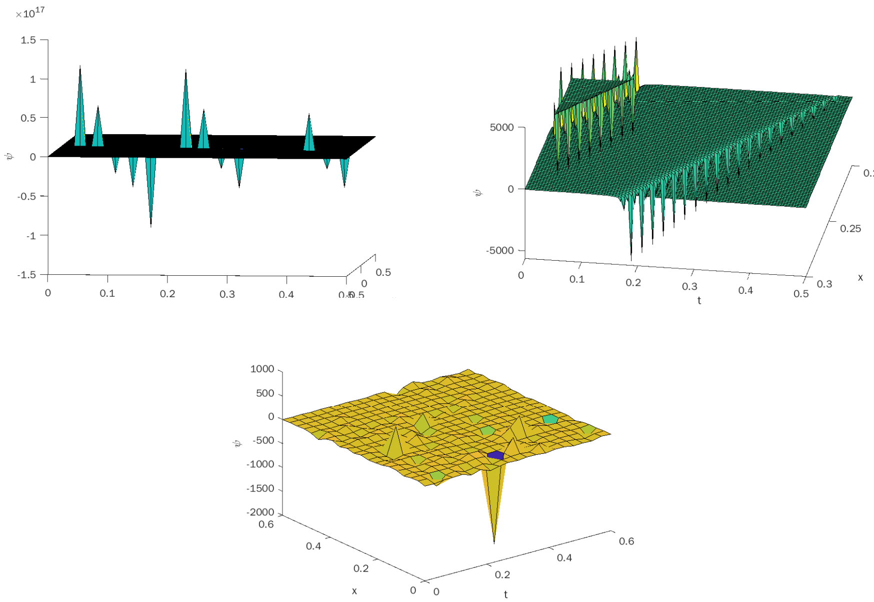

Figure 1. Surface plot of solution ψ for case 1.

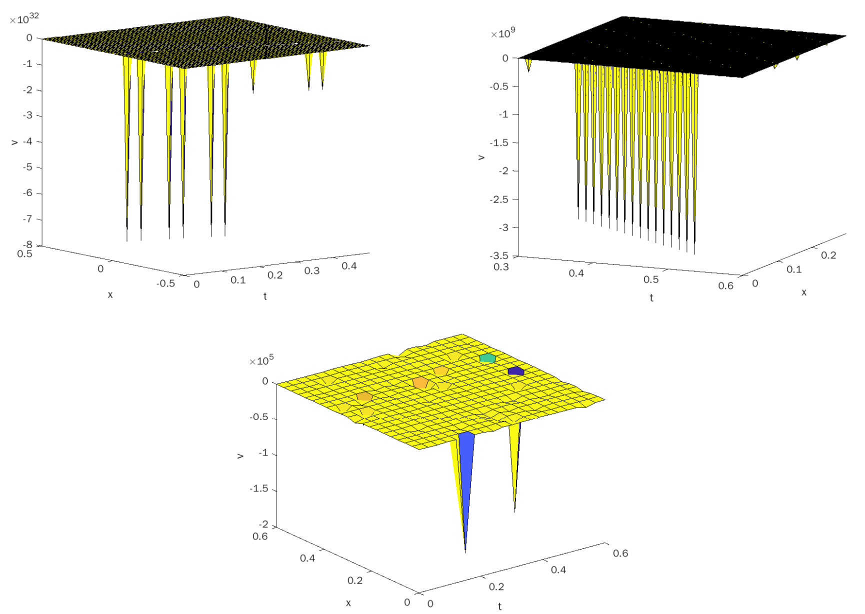

Figure 2. Surface plot of solution v for case 1.

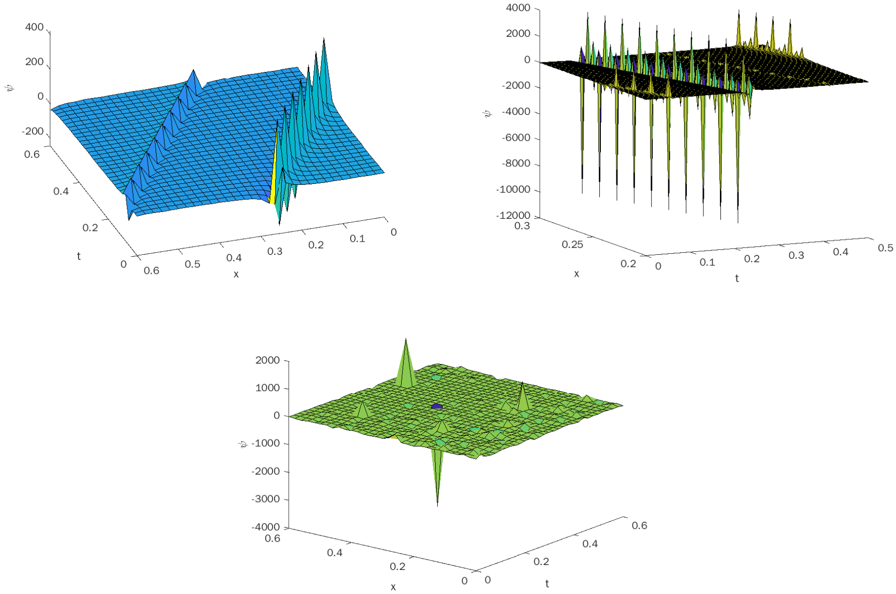

Figure 3. Surface plot of solution ψ for case 2.

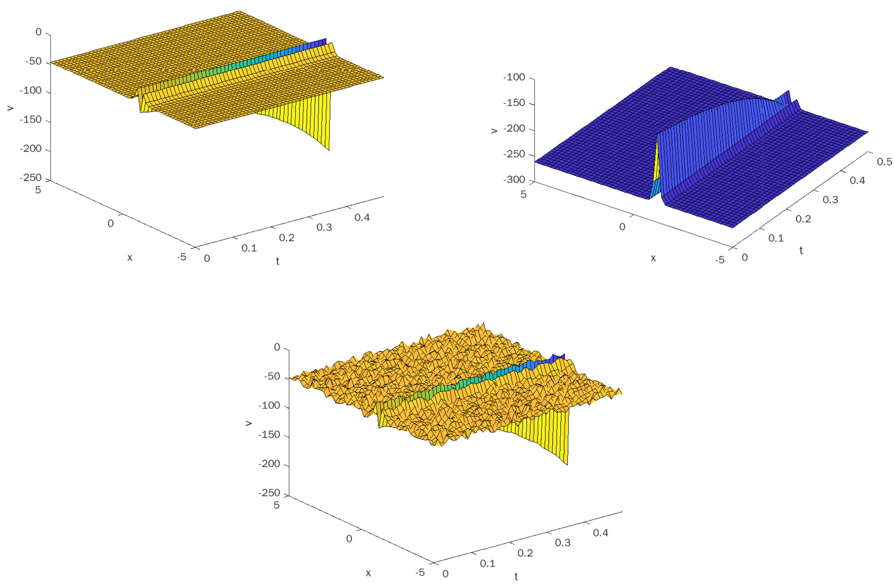

Figure 4. Surface plot of solution v for case 2.

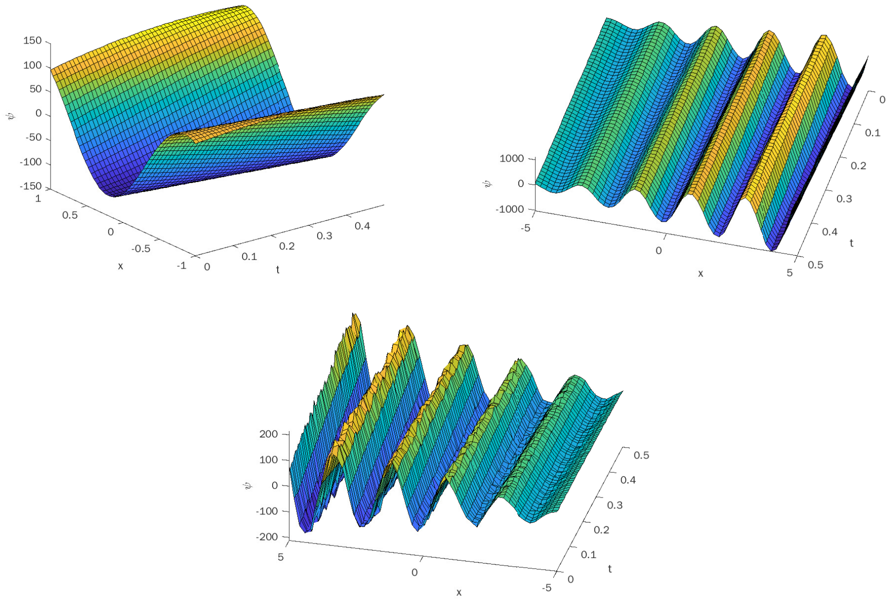

Figure 5. Surface plot of solution ψ for case 3.

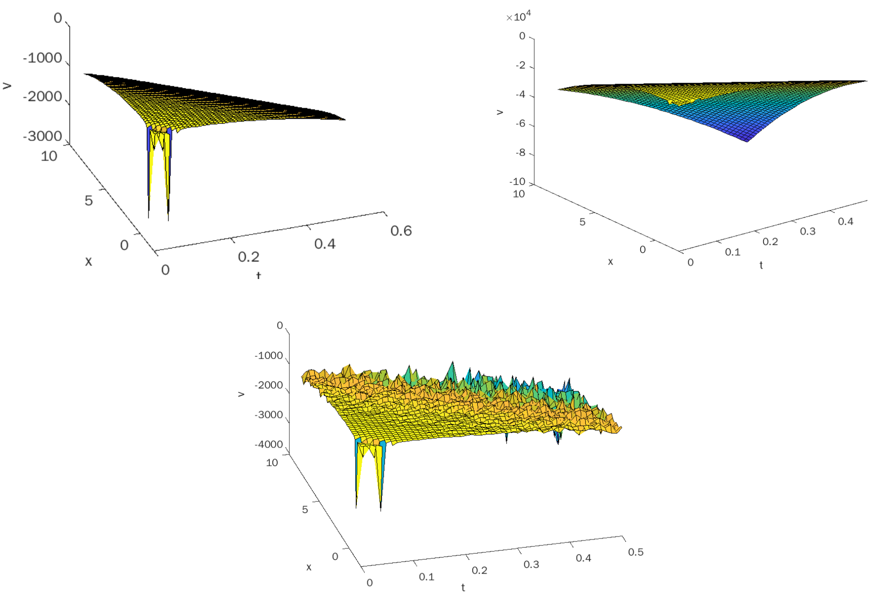

Figure 6. Surface plot of solution v for case 3.

7 Graphical interpretation

In all the surface plots the first figure represents the plot of classical Maccari system, and the second and third figures physically represent the Maccari system in the presence of noise term using the Gaussian white noise and random variable in the MATLAB. The classical Maccari system demonstrated the behavior of Langmuir solitons, which are basically the high-frequency waves in the field of quantum mechanics, plasma physics, nonlinear optics, hydrodynamics, etc. Langmuir solitons are also called Langmuir oscillations. In short, they are the energy-dissipated oscillations. Due to the high frequency and the motion of isolated waves in the form of Langmuir soliton, the perturbation factor is described by the choice of noise term as clearly shown in all surface plots. Also, if the noise term approaches zero, then we again come back to our classical model. The smoothness of the solitons wave from Figs. 1–6 instructed us that our system is of classical nature in the absence of noise term, where no fluctuation is observed in the wave formation. However, due to the presence of noise terms, the inevitable perturbation and spikes are observed due to the interaction of surroundings. Due to the presence of multiplicative noise terms in the system, the spatiotemporal perturbation in the formation of continuum regularization of Langmuir solitons is observed.

8 Conclusion

We consider the more generic form of the Maccari system. Due to the presence of multiplicative noise terms, the nature of our system is nonstationary. The stochastic model of the system provides us with deep insight into the structural and physical properties of the system in the presence of random fluctuations. Also, the existence theory provides us ample information about the existence of solution and their equicontinuity.

Conflicts of interest. The authors declare no conflicts of interest.

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