Nonlinear Analysis: Modelling and Control, Vol. 29, No. 5, 983–1002
https://doi.org/10.15388/namc.2024.29.36103

Vilnius University Press

# Diverse exact solutions to Davey–Stewartson model using modified extended mapping method

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**Received:** October 30, 2023 / **Revised:** June 1, 2024 / **Published online:** July 30, 2024

**Abstract.** In this study, we obtain solitary wave solutions and other exact wave solutions for Davey–Stewartson equation (DSE), which explains how waves move through water with a finite depth while being affected by gravity and surface tension. The study is conducted with the aid of the modified extended mapping method (MEMM). A variety of distinct traveling wave solutions are furnished. The obtained solutions comprise dark, bright, and singular solitary wave solutions. Additionally, Jacobi elliptic function solutions, exponential wave solutions, singular periodic wave solutions, rational wave solutions, and periodic wave solutions are also offered. To help readers physically grasp the acquired solutions, graphical representations of some of the extracted solutions are provided.

**Keywords:** Davey–Stewartson equation, solitary wave solutions, Jacobi elliptic function solutions, periodic solutions, modified extended mapping method.

# 1 Introduction

Visualizing the fundamental physical phenomena, including fluid mechanics, electromagnetism, optics, magnetohydrodynamics, quantum mechanics, superconductivity, thermodynamics, chemical reactions, finance, neuroscience, and elasticity, is frequently done with the aid of nonlinear partial differential equations (NLPDEs) (see [1, 2, 4, 5, 7–10, 13, 19–27, 29–32]). Numerous researchers have investigated the exact solutions of nonlinear partial differential models to understand the behavior exhibited by physical phenomena (see [3, 6, 33]).

A particular kind of partial differential equation is the Davey–Stewartson equation (DSE), which was developed by Davey and Stewartson and explains how a wave packet develops in three dimensions in shallow water [15].

In the current work, we take into consideration the DSEs for a complex field, which is the wave amplitude, named $U$, and a real field, which is the mean flow, named $V$, defined by the subsequent nonlinear coupled system [23]:

$$\begin{aligned} &\mathrm{i}U_t + \frac{1}{2}\delta^2\big(U_{xx} + \delta^2 U_{yy}\big) + \lambda|U|^2U - UV_x = 0,\\ &V_{xx} - \delta^2 V_{yy} - 2\lambda\big(|U|^2\big)_x = 0, \end{aligned} \tag{1}$$

where $U(x,y,t)$ defines the surface wave packet's amplitude, and $V(x,y,t)$ symbolizes the potential velocity of the mean flow's interaction with the surface's wave [15]. The case of focusing or defocusing is defined by the parameter $\lambda$, besides, $\delta$ is considered a certain constant. In [14], Chow and Lou derived two new exact solutions for the DS system. But Babaoglu [12] studied the long-wave–short-wave interaction equations that come from DSE. Groves et al. [18] focused on the DSEs, which have an explicit brilliant line solitary solution when considering the elliptic–elliptic problem. Many other researchers applied different techniques to the DSE system such as the improved $(\tan(\phi/2))$-expansion method, He's semiinverse variational principle method, and $(G'/G)$-expansion method in [35], Galerkin methods in [17], and many other methods in [11, 16, 28, 34]. For creating the traveling wave solution of NLPDEs, the MEMM is among the most effective direct techniques [8].

In this study, the MEMM is used to handle the suggested DS model. Through this method, we are able to produce many new and novel solutions such as dark solitary, bright solitary, singular solitary, exponential solutions, periodic solutions, rational solutions, singular periodic solutions, and Jacobi elliptic function solutions. The solutions that were extracted confirm the reliability and potency of the current approach. Simulated results in 3D, contour, and 2D are displayed to further highlight the nature of the solutions that have been found.

# 2 The proposed method

In this part, we outline the main steps of the modified extended mapping scheme [8, 10].

We consider the following NLPDE:

$$\mathcal{Z}(\psi, \psi_t, \psi_x, \psi_y, \psi_{xx}, \psi_{xt}, \psi_{xy}, \psi_{xxt}, \dots) = 0, \tag{2}$$

where $\mathcal{Z}$ stands for a function of the polynomial type of $\psi(x, y, t)$ and its corresponding partial derivatives with respect to the two-dimensional space and time.

*Step I.* The solution of Eq. (2) will be accomplished by the ensuing travelling wave transformation

$$\psi(x, y, t) = P(\zeta), \quad \zeta = x + y - \rho t,\ \rho \neq 0, \tag{3}$$

where $\rho$ represents the wave speed that can be evaluated later on.

The subsequent nonlinear ordinary differential equation (NLODE) can be produced by entering the transformation in Eq. (3) into Eq. (2):

$$\mathcal{F}(P, P', P'', P''' \dots) = 0. \tag{4}$$

*Step II.* The general solution for Eq. (4) can be represented as

$$\begin{aligned} P(\zeta) = {} & \sum_{j=0}^{M} \alpha_j \mathcal{W}^j(\zeta) + \sum_{j=-1}^{-M} \beta_{-j} \mathcal{W}^j(\zeta) \\ & + \sum_{j=2}^{M} c_j \mathcal{W}^{j-2}(\zeta)\mathcal{W}'(\zeta) + \sum_{j=-1}^{-M} \theta_{-j} \mathcal{W}^j(\zeta)\mathcal{W}'(\zeta), \end{aligned} \tag{5}$$

where $\alpha_j$, $\beta_{-j}$, $c_j$, $\theta_{-j}$ are real constants to be estimated, and $\mathcal{W}(\zeta)$ meets the requirements of the auxiliary equation as follows:

$$\mathcal{W}'(\zeta) = \sqrt{\tau_0 + \tau_1\mathcal{W}(\zeta) + \tau_2\mathcal{W}^2(\zeta) + \tau_3\mathcal{W}^3(\zeta) + \tau_4\mathcal{W}^4(\zeta) + \tau_6\mathcal{W}^6(\zeta)}, \tag{6}$$

where $\tau_i$ ($i = 0, 1, 2, 3, 4, 6$) are constants.

*Step III.* The integer $M$ can be calculated with the help of the principle of balance through Eq. (4) between the highest-order derivatives and the highest-order nonlinear terms.

*Step IV.* Plugging the proposed solution in Eq. (5) with using Eq. (6) into Eq. (4), following that, equalizing the coefficients of $\mathcal{W}'^j(\zeta)\mathcal{W}^i(\zeta)$ ($j = 0, 1$; $i = 0, \pm1, \pm2, \dots$) to zero acquires a group of nonlinear algebraic equations (NLAEs) for $\alpha_j$, $\beta_{-j}$, $c_j$, $\theta_{-j}$, and $\rho$, which shall be solved using Mathematica software. Then we are able to estimate the used unknowns $\alpha_j$, $\beta_{-j}$, $c_j$, and $\theta_{-j}$. After that, there are numerous exact solutions to Eq. (2) can be obtained.

## 3 Exact solutions for the proposed system

We assume the following wave transformation in order to obtain the solutions of Eq. (1):

$$U(x, y, t) = P(\zeta)\mathrm{e}^{\mathrm{i}(\kappa x + \ell y + \Omega t)}, \qquad V(x, y, t) = R(\zeta), \tag{7}$$

where $\zeta = x + y - \rho t$, $\rho = \kappa\delta^2 + \ell\delta^4$, and $P(\zeta)$ represents the solution's amplitude, while $\kappa$, $\ell$, and $\Omega$ represent some unknown constants.

Using the transformation (7), Eq. (1) will be transformed into the subsequent ordinary differential equations (ODEs):

$$\delta^2\big(\delta^2+1\big)P'' + 2\lambda P^3 - 2PR' - \big(\kappa^2\delta^2 + 2\Omega + \ell^2\delta^4\big)P = 0, \tag{8}$$

$$\big(\delta^2-1\big)R'' + 4\lambda PP' = 0. \tag{9}$$

Integrating Eq. (9) once with respect to $\zeta$ and neglecting the integral constant, we get

$$R' = \frac{2\lambda}{1-\delta^2}P^2. \tag{10}$$

Substituting Eq. (10) in Eq. (8), we get

$$\delta^2\big(\delta^4-1\big)P'' + 2\lambda\big(\delta^2+1\big)P^3 - \mathcal{M}\big(\delta^2-1\big)P = 0, \tag{11}$$

where $\mathcal{M} = 2\Omega + \kappa^2\delta^2 + \ell^2\delta^4$.

According to the proposed scheme discussed in Section 2, so, the general solution of Eq. (11) will be written as

$$P(\zeta) = \alpha_0 + \alpha_1\mathcal{W}(\zeta) + \beta_1\left(\frac{1}{\mathcal{W}(\zeta)}\right) + \theta_1\left(\frac{\mathcal{W}'(\zeta)}{\mathcal{W}(\zeta)}\right), \tag{12}$$

where $\alpha_i$ $(i = 0, 1)$, $\beta_1$ and $\theta_1$ are unknown constants that can be evaluated under the restrictions $\alpha_1$ or $\beta_1$ or $\theta_1 \neq 0$ simultaneously.

Inserting Eqs. (12) and (6) into Eq. (11), then group coefficients of similar powers and set them all to zero to create a system of NLAEs, which can be solved with the aid of the Mathematica programme to get the outcomes displayed below:

*Case 1.* When $\tau_0 = \tau_1 = \tau_3 = \tau_6 = 0$, these are the sets of solutions that we discovered:

(1.1) $\alpha_0 = \alpha_1 = \beta_1 = 0,\ \theta_1 = \pm\delta\sqrt{1-\delta^2}/\sqrt{\lambda},\ \tau_2 = -\mathcal{M}/(2\delta^2(\delta^2+1))$.

(1.2) $\alpha_0 = \beta_1 = \theta_1 = 0,\ \alpha_1 = \pm\delta\sqrt{-(\delta^2-1)\tau_4}/\sqrt{\lambda},\ \tau_2 = \mathcal{M}/(\delta^2(\delta^2+1))$.

(1.3) $\alpha_0 = \beta_1 = 0,\ \alpha_1 = \pm\delta\sqrt{-(\delta^2-1)\tau_4}/(2\sqrt{\lambda}),\ \theta_1 = \pm\delta\sqrt{1-\delta^2}/(2\sqrt{\lambda})$, $\tau_2 = -2\mathcal{M}/(\delta^2(\delta^2+1))$.

According to the set of solutions (1.1), the solutions of Eq. (1) can be stated as:

(1.1,1) If $\tau_2 > 0$, $\tau_4 < 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$\begin{aligned} U_{1.1,1}(x,y,t) &= \mp\frac{\delta\sqrt{(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\tanh\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{\tau_2}r\big] \\ &\quad\times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \\ V_{1.1,1}(x,y,t) &= 2\delta^2\sqrt{\tau_2}\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{\tau_2} \\ &\quad - \tanh\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{\tau_2}\,\big]\big], \end{aligned} \tag{13}$$

these solutions represent the dark solitary solutions.

(1.1,2) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$\begin{aligned} U_{1.1,2}(x,y,t) &= \pm\frac{\delta\sqrt{-(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\tan\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr] \\ &\quad\times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \\ V_{1.1,2}(x,y,t) &= 2\delta^2\sqrt{-\tau_2}\bigl[\tan\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr] \\ &\quad-\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr], \end{aligned} \tag{14}$$

these solutions represent the singular periodic wave solutions.

(1.1,3) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$\begin{aligned} U_{1.1,3}(x,y,t) &= \mp\frac{\delta\sqrt{-(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\cot\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr] \\ &\quad\times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \\ V_{1.1,3}(x,y,t) &= -2\delta^2\sqrt{-\tau_2}\bigl[\cot\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr] \\ &\quad+\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr], \end{aligned}$$

these solutions represent singular periodic solutions.

According to the set of solutions (1.2), the solutions of Eq. (1) can be stated as:

(1.2,1) If $\tau_2 > 0$, $\tau_4 < 0$, and $\lambda(\delta^2-1) > 0$, then the solutions are:

$$\begin{aligned} U_{1.2,1}(x,y,t) &= \pm\frac{\delta\sqrt{(\delta^2-1)\tau_2}}{\sqrt{\lambda}}\operatorname{sech}\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\tau_2}\,\bigr] \\ &\quad\times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \\ V_{1.2,1}(x,y,t) &= -2\delta^2\sqrt{\tau_2}\tanh\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\tau_2}\,\bigr], \end{aligned} \tag{15}$$

these solutions represent bright and dark solitary solutions, respectively.

(1.2,2) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(\delta^2-1) < 0$, then the solutions are:

$$\begin{aligned} U_{1.2,2}(x,y,t) &= \pm\frac{\delta\sqrt{(\delta^2-1)\tau_2}}{\sqrt{\lambda}}\sec\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr] \\ &\quad\times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \\ V_{1.2,2}(x,y,t) &= 2\delta^2\sqrt{-\tau_2}\tan\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr], \end{aligned}$$

these solutions represent the singular periodic wave solutions.

(1.2,3) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(\delta^2-1) < 0$, then the solutions are:

$$\begin{aligned} U_{1.2,3}(x,y,t) &= \pm\frac{\delta\sqrt{(\delta^2-1)\tau_2}}{\sqrt{\lambda}}\csc\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\bigr] \\ &\quad\times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \\ V_{1.2,3}(x,y,t) &= -2\delta^2\sqrt{-\tau_2}\cot\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr], \end{aligned}$$

these solutions represent the singular periodic wave solutions.

According to the set of solutions (1.3), the solutions of Eq. (1) can be stated as:

(1.3,1) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{1.3,1}(x,y,t) = \pm\frac{\delta\sqrt{-(1-\delta^2)\tau_2}}{2\sqrt{\lambda}}\frac{1+\sin[(x+y-(\kappa\delta^2+\ell\delta^4)t)\sqrt{-\tau_2}\,]}{\cos[(x+y-(\kappa\delta^2+\ell\delta^4)t)\sqrt{-\tau_2}\,]} \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{1.3,1}(x,y,t) = \frac{1}{2}\delta^2\biggl(\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\tau_2 + 2\sqrt{-\tau_2}\frac{1+\sin[(x+y-(\kappa\delta^2+\ell\delta^4)t)\sqrt{-\tau_2}\,]}{\cos[(x+y-(\kappa\delta^2+\ell\delta^4)t)\sqrt{-\tau_2}\,]}\biggr),$$

these solutions represent the periodic wave solutions.

(1.3,2) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{1.3,2}(x,y,t) = \pm\frac{\delta\sqrt{-(1-\delta^2)\tau_2}}{2\sqrt{\lambda}}\tan\left[\frac{1}{2}\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\right] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{1.3,2}(x,y,t) = \frac{1}{2}\delta^2\biggl(\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\tau_2 + 2\sqrt{-\tau_2}\tan\left[\frac{1}{2}\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\right]\biggr),$$

these solutions are singular periodic wave solutions.

*Case 2*. When $\tau_1 = \tau_3 = \tau_6 = 0$, $\tau_0 = \tau_2^2/(4\tau_4)$, these are the sets of solutions that we discovered:

(2.1) $\alpha_0 = \beta_1 = \theta_1 = 0,\ \alpha_1 = \pm\delta\sqrt{-(\delta^2-1)\tau_4}/\sqrt{\lambda},\ \tau_2 = \mathcal{M}/(\delta^4+\delta^2)$.

(2.2) $\alpha_0 = \alpha_1 = \theta_1 = 0,\ \beta_1 = \pm\mathcal{M}\sqrt{-(\delta^2-1)}/(2(\delta^3+\delta)\sqrt{\lambda\tau_4})$, $\tau_2 = \mathcal{M}/(\delta^4+\delta^2)$.

(2.3) $\alpha_0 = \alpha_1 = \beta_1 = 0,\ \theta_1 = \pm\delta\sqrt{1-\delta^2}/\sqrt{\lambda},\ \tau_2 = -\mathcal{M}/(2(\delta^4+\delta^2))$.

(2.4) $\alpha_0 = \theta_1 = 0,\ \alpha_1 = \pm\delta\sqrt{-(\delta^2-1)\tau_4}/\sqrt{\lambda}$, $\beta_1 = \mp\mathcal{M}\sqrt{-(\delta^2-1)}/(4\delta(\delta^2+1)\sqrt{\lambda\tau_4}),\ \tau_2 = -\mathcal{M}/(2(\delta^4+\delta^2))$.

(2.5) $\alpha_0 = 0,\ \alpha_1 = \pm\delta\sqrt{-(\delta^2-1)\tau_4}/(2\sqrt{\lambda})$, $\beta_1 = \mp\mathcal{M}\sqrt{-(\delta^2-1)}/(8\delta(\delta^2+1)\sqrt{\lambda\tau_4}),\ \theta_1 = \pm\delta\sqrt{1-\delta^2}/(2\sqrt{\lambda})$, $\tau_2 = -\mathcal{M}/(2(\delta^4+\delta^2))$.

According to the set of solutions (2.1), the solutions of Eq. (1) can be stated as:

(2.1,1) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(\delta^2-1) < 0$, then the solutions are:

$$U_{2.1,1}(x,y,t) = \pm\frac{\delta\sqrt{(\delta^2-1)\tau_2}}{\sqrt{2\lambda}}\tanh\left[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\frac{\tau_2}{2}}\right] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{2.1,1}(x,y,t) = -\delta^2\biggl(\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\tau_2 + \sqrt{-2\tau_2}\tanh\biggl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\frac{\tau_2}{2}}\biggr]\biggr),$$

these solutions represent the dark solitary solutions.

(2.1,2) If $\tau_2 > 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{2.1,2}(x,y,t) = \pm\frac{\delta\sqrt{(1-\delta^2)\tau_2}}{\sqrt{2\lambda}}\tan\biggl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\frac{\tau_2}{2}}\biggr] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{2.1,2}(x,y,t) = \delta^2\biggl(\sqrt{2\tau_2}\tan\biggl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\frac{\tau_2}{2}}\biggr] - \bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\tau_2\biggr),$$

these solutions represent the singular periodic wave solutions.

According to the set of solutions (2.2), the solutions of Eq. (1) can be stated as:

(2.2,1) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{2.2,1}(x,y,t) = \pm\frac{\mathcal{M}\sqrt{1-\delta^2}}{(\delta^3+\delta)\sqrt{-2\lambda\,\tau_2}}\coth\biggl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\frac{\tau_2}{2}}\biggr] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \tag{16}$$

$$V_{2.2,1}(x,y,t) = \frac{\mathcal{M}^2}{(\delta^3+\delta)^2\tau_2\sqrt{-\tau_2}}\biggl(\sqrt{2}\coth\biggl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\frac{\tau_2}{2}}\biggr] - \bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\biggr), \tag{17}$$

these solutions represent the singular solitary solutions.

(2.2,2) If $\tau_2 > 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{2.2,2}(x,y,t) = \pm\frac{\mathcal{M}\sqrt{1-\delta^2}}{(\delta^3+\delta)\sqrt{2\lambda\tau_2}}\cot\biggl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\frac{\tau_2}{2}}\biggr] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{2.2,2}(x,y,t) = -\frac{\mathcal{M}^2}{(\delta^3+\delta)^2\tau_2\sqrt{\tau_2}}\biggl(\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\tau_2} + \sqrt{2}\cot\biggl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\frac{\tau_2}{2}}\biggr]\biggr),$$

these solutions represent the singular periodic wave solutions.

According to the set of solutions (2.3), the solutions of Eq. (1) can be stated as:

(2.3,1) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{2.3,1}(x,y,t) = \pm\frac{\delta\sqrt{-2(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\operatorname{csch}\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{-2\tau_2}\,\big] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{2.3,1}(x,y,t) = -2\delta^2\sqrt{-2\tau_2}\coth\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{-2\tau_2}\,\big],$$

these solutions represent the singular solitary solutions.

(2.3,2) If $\tau_2 > 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{2.3,2}(x,y,t) = \pm\frac{\delta\sqrt{2(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\csc\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{2\tau_2}\,\big] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{2.3,2}(x,y,t) = -2\delta^2\sqrt{2\tau_2}\cot\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{2\tau_2}\,\big],$$

these solutions represent the singular periodic solutions.

According to the set of solutions (2.4), the solutions of Eq. (1) can be stated as:

(2.4,1) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{2.4,1}(x,y,t) = \pm\frac{\delta\tau_2\sqrt{2(1-\delta^2)}}{\sqrt{-\lambda\tau_2}}\operatorname{csch}\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{-2\tau_2}\,\big] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \tag{18}$$
$$V_{2.4,1}(x,y,t) = -2\delta\sqrt{-2\tau_2}\coth\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{-2\tau_2}\,\big]. \tag{19}$$

Both of Eq. (18) and Eq. (19) represent singular solitary solutions.

(2.4,2) If $\tau_2 > 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{2.4,2}(x,y,t) = \pm\frac{\delta\sqrt{2(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\csc\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{2\tau_2}\,\big] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{2.4,2}(x,y,t) = -2\delta^2\sqrt{2\tau_2}\cot\big[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{2\tau_2}\,\big],$$

these solutions represent singular periodic solutions.

According to the set of solutions (2.5), the solutions of Eq. (1) can be stated as:

(2.5) If $\tau_2 < 0$, $\tau_4 > 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{2.5,1}(x,y,t) = \pm\frac{\delta\sqrt{-(1-\delta^2)\tau_2}}{2\sqrt{2\lambda}}\tanh\left[\big(x+y-\big(\kappa\delta^2+\ell\delta^4\big)t\big)\sqrt{-\frac{\tau_2}{2}}\,\right] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{2.5,1}(x,y,t) = -\frac{1}{4}\delta^2\biggl(\tau_2\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr) + \sqrt{-2\tau_2}\tanh\biggl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\frac{\tau_2}{2}}\biggr]\biggr),$$

these solutions represent dark solitary solutions.

*Case 3*. When $\tau_3 = \tau_4 = \tau_6 = 0$, these are the sets of solutions that we discovered:

(3.1) $\alpha_0 = \alpha_1 = \theta_1 = \tau_1 = 0$, $\beta_1 = \pm\delta\sqrt{-(\delta^2-1)\tau_0}/\sqrt{\lambda}$, $\mathcal{M} = (\delta^4+\delta^2)\tau_2$.

(3.2) $\alpha_0 = \alpha_1 = \beta_1 = \tau_1 = 0$, $\theta_1 = \pm\delta\sqrt{1-\delta^2}/\sqrt{\lambda}$, $\mathcal{M} = -2(\delta^4+\delta^2)\tau_2$.

(3.3) $\alpha_0 = \alpha_1 = 0$, $\beta_1 = \pm\delta\sqrt{-(\delta^2-1)\tau_0}/(2\sqrt{\lambda})$, $\theta_1 = \pm\delta\sqrt{1-\delta^2}/\sqrt{\lambda}$, $\mathcal{M} = -(\delta^4+\delta^2)\tau_2/2$.

According to the set of solutions (3.1), the solutions to Eq. (1) can be stated as:

(3.1,1) If $\tau_0 > 0$, $\tau_2 > 0$, $\tau_1 = 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{3.1,1}(x,y,t) = \pm\frac{\delta\sqrt{(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\operatorname{csch}\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\tau_2}\bigr] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{3.1,1}(x,y,t) = -2\delta^2\sqrt{\tau_2}\coth\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\tau_2}\bigr],$$

these solutions represent the singular solitary solutions.

(3.1,2) If $\tau_0 > 0$, $\tau_2 < 0$, $\tau_1 = 0$, and $\lambda(\delta^2-1) < 0$, then the solutions are:

$$U_{3.1,2}(x,y,t) = \pm\frac{\delta\sqrt{(\delta^2-1)\tau_2}}{\sqrt{\lambda}}\csc\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\bigr] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{3.1,2}(x,y,t) = -2\delta^2\sqrt{-\tau_2}\cot\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\bigr],$$

these solutions represent the singular periodic wave solutions.

According to the set of solutions (3.2), the solutions of Eq. (1) can be stated as:

(3.2,1) If $\tau_0 > 0$, $\tau_2 > 0$, $\tau_1 = 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{3.2,1}(x,y,t) = \pm\frac{\delta\sqrt{(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\coth\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\tau_2}\bigr] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{3.2,1}(x,y,t) = 2\delta^2\bigl(\tau_2\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr) - \sqrt{\tau_2}\coth\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\tau_2}\bigr]\bigr),$$

these solutions represent the singular solitary solutions.

(3.2,2) If $\tau_0 > 0$, $\tau_2 < 0$, $\tau_1 = 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{3.2,2}(x,y,t) = \pm\frac{\delta\sqrt{-(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\cot\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{3.2,2}(x,y,t) = -2\delta^2\sqrt{-\tau_2}\bigl(\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2} + \cot\bigl[\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\,\bigr]\bigr),$$

these solutions represent singular periodic solutions.

According to the set of solutions (3.3), the solutions of Eq. (1) can be stated as:

(3.3,1) If $\tau_0 > 0$, $\tau_2 > 0$, $\tau_1 = 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{3.3,1}(x,y,t) = \pm\frac{\delta\sqrt{(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\coth\left[\frac{1}{2}\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\tau_2}\right] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{3.3,1}(x,y,t) = 2\delta^2\left(\tau_2\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr) - 2\sqrt{\tau_2}\coth\left[\frac{1}{2}\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{\tau_2}\right]\right),$$

these solutions represent the singular solitary solutions.

(3.3,2) If $\tau_0 > 0$, $\tau_2 < 0$, $\tau_1 = 0$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{3.3,2}(x,y,t) = \pm\frac{\delta\sqrt{(\delta^2-1)\tau_2}}{\sqrt{\lambda}}\cot\left[\frac{1}{2}\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\right] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{3.3,1}(x,y,t) = 2\delta^2\left(\tau_2\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr) + 2\sqrt{-\tau_2}\tan\left[\frac{1}{2}\bigl(x+y-\bigl(\kappa\delta 2+\ell\delta^4\bigr)t\bigr)\sqrt{-\tau_2}\right]\right),$$

these solutions represent the singular periodic solutions.

(3.3,3) If $\tau_2 > 0$, $\tau_0 = \tau_1^2/(4\tau_2)$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$U_{3.3,3}(x,y,t) = \pm\frac{\delta\sqrt{(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\,\frac{2\tau_2\mathrm{e}^{(x+y-(\kappa\delta^2+\ell\delta^4)t)\sqrt{\tau_2}}+\tau_1}{2\tau_2\mathrm{e}^{(x+y-(\kappa\delta^2+\ell\delta^4)t)\sqrt{\tau_2}}-\tau_1}\,\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$

$$V_{3.3,3}(x,y,t) = \frac{2\delta^2\tau_2}{\tau_1-2\tau_2\mathrm{e}^{\sqrt{\tau_2}(x+y-(\kappa\delta^2+\ell\delta^4)t)}}\bigl(\tau_1\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr) - 2\bigl(\delta^2-1\bigr)\tau_2\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\mathrm{e}^{(x+y-(\kappa\delta^2+\ell\delta^4)t)\sqrt{\tau_2}}\bigr)$$

these solutions represent the exponential solutions.

*Case 4.* When $\tau_0 = \tau_1 = \tau_2 = \tau_6 = 0$, we found the following set of solution: $\alpha_0 = \beta_1 = \mathcal{M} = 0$, $\alpha_1 = \pm\delta\sqrt{(1-\delta^2)\tau_4}/(2\sqrt{\lambda})$, $\theta_1 = \pm\delta\sqrt{1-\delta^2}/(2\sqrt{\lambda})$. Then the solutions of Eq. (1) are:

$$\begin{aligned} U_4(x,y,t) &= \mp\frac{\delta\sqrt{1-\delta^2}}{2\sqrt{\lambda}}\frac{2\tau_3(\tau_3(x+y-(\kappa\delta^2+\ell\delta^4)t)-2\sqrt{\tau_4})}{\tau_3^2(x+y-(\kappa\delta^2+\ell\delta^4)t)^2-4\tau_4} \\ &\quad\times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \\ V_4(x,y,t) &= -\frac{2\delta^2\tau_3}{\tau_3(x+y-(\kappa\delta^2+\ell\delta^4)t)+2\sqrt{\tau_4}}, \end{aligned}$$

which represent rational wave solutions under the conditions that $\lambda(1-\delta^2) > 0$ and $\tau_4 > 0$.

*Case 5.* When $\tau_0 = \tau_1 = \tau_6 = 0$, this is the set of solutions that we discovered: $\alpha_0 = \beta_1 = 0$, $\alpha_1 = \pm\delta\sqrt{(1-\delta^2)\tau_4}/(2\sqrt{\lambda})$, $\theta_1 = \pm\delta\sqrt{1-\delta^2}/(2\sqrt{\lambda})$, $\mathcal{M} = -(\delta^4+\delta^2)\tau_2/2$. Then the solutions of Eq. (1) are:

(5.1) If $\tau_2 > 0$, $\tau_3^2 = 4\tau_2\tau_4$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$\begin{aligned} U_{5.1,1}(x,y,t) &= \mp\frac{\delta\sqrt{(1-\delta^2)\tau_2}}{2\sqrt{\lambda}}\tanh\left[\frac{1}{2}\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\sqrt{\tau_2}\right] \\ &\quad\times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \\ V_{5.1,1}(x,y,t) &= \frac{1}{2}\delta^2\biggl(\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\tau_2 \\ &\quad - \sqrt{\tau_2}\tanh\left[\frac{1}{2}\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\sqrt{\tau_2}\right]\biggr), \end{aligned}$$

these solutions represent the dark solitary solutions.

(5.2) If $\tau_2 > 0$, $\tau_3^2 = 4\tau_2\tau_4$, and $\lambda(1-\delta^2) > 0$, then the solutions are:

$$\begin{aligned} U_{5.1,2}(x,y,t) &= \mp\frac{\delta\sqrt{(1-\delta^2)\tau_2}}{2\sqrt{\lambda}}\coth\left[\frac{1}{2}(x+y-(\kappa\delta^2+\ell\delta^4)t)\sqrt{\tau_2}\right] \\ &\quad\times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)}, \\ V_{5.1,2}(x,y,t) &= \frac{1}{2}\delta^2\biggl(\tau_2\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right) \\ &\quad - 2\sqrt{\tau_2}\coth\left[\frac{1}{2}\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\sqrt{\tau_2}\right]\biggr), \end{aligned}$$

these solutions represent the singular solitary solutions.

*Case 6.* When $\tau_1 = \tau_3 = 0$, this is the set of solutions that we discovered: $\alpha_0 = \alpha_1 = \beta_1 = \tau_0 = \tau_4 = 0$, $\theta_1 = \pm 2\delta\sqrt{1-\delta^2}/\sqrt{\lambda}$, $\tau_2 = -\mathcal{M}/(8(\delta^4+\delta^2))$. So, Eq. (1) can have the following solutions:

(6.1) If $\tau_2 > 0$ and $\lambda(1-\delta^2) > 0$, so, solutions can be found as

$$U_{6.1}(x,y,t) = \mp\frac{2\delta\sqrt{(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\tanh\left[2\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\sqrt{\tau_2}\right] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{6.1}(x,y,t) = 4\delta^2\left(2\tau_2\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right) - \sqrt{\tau_2}\tanh\left[2\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\sqrt{\tau_2}\right]\right).$$

Both solutions represent dark solitary solutions.

(6.2) If $\tau_2 > 0$ and $\lambda(1-\delta^2) > 0$, so, solutions of the singular periodic wave type can be found as

$$U_{6.2}(x,y,t) = \pm\frac{2\delta\sqrt{(1-\delta^2)\tau_2}}{\sqrt{\lambda}}\tan\left[2\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\sqrt{\tau_2}\right] \times \mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{6.2}(x,y,t) = -4\delta^2\left(2\tau_2\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right) - \sqrt{\tau_2}\tan\left[2\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\sqrt{\tau_2}\right]\right).$$

*Case 7.* When $\tau_1 = \tau_3 = \tau_6 = 0$, these are the sets of solutions that we discovered:
(7.1) $\alpha_0 = \beta_1 = \theta_1 = 0,\ \alpha_1 = \pm\delta\sqrt{(1-\delta^2)\tau_4}/\sqrt{\lambda},\ \mathcal{M} = (\delta^4+\delta^2)\tau_2.$
(7.2) $\alpha_0 = \alpha_1 = \theta_1 = 0,\ \beta_1 = \pm\delta\sqrt{(1-\delta^2)\tau_0}/\sqrt{\lambda},\ \mathcal{M} = (\delta^4+\delta^2)\tau_2.$

According to the set of solutions (7.1), the solutions of Eq. (1) can be stated as

(7.1,1) If $\tau_0 = 1$, $\tau_2 = -m^2-1$, $\tau_4 = m^2$, $\lambda(1-\delta^2) > 0$, and $0 < m \leqslant 1$, then the solutions are:

$$U_{7.1,1}(x,y,t) = \pm\frac{m\,\delta\sqrt{1-\delta^2}}{\sqrt{\lambda}}\operatorname{sn}\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.1,1}(x,y,t) = 2m\,\delta^2\left(-\,\mathrm{JacobiEpsilon}\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right) + \left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\right)$$

or

$$U_{7.1,2}(x,y,t) = \pm\frac{m\delta\sqrt{1-\delta^2}}{\sqrt{\lambda}}\operatorname{cd}\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right)\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.1,2}(x,t) = 2m\delta^2\Bigg(-\,\mathrm{JacobiEpsilon}\left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right) + \left(x+y-\left(\kappa\delta^2+\ell\delta^4\right)t\right) + \frac{m\operatorname{cn}(x+y-(\kappa\delta^2+\ell\delta^4)t)\operatorname{sn}(x+y-(\kappa\delta^2+\ell\delta^4)t)}{\operatorname{dn}(x+y-(\kappa\delta^2+\ell\delta^4)t)}\Bigg),$$

these solutions represent the Jacobi elliptic function (JEF) solutions.

(7.1,2) If $\tau_0 = m^2 - 1$, $\tau_2 = 2 - m^2$, $\tau_4 = -1$, $\lambda(\delta^2 - 1) > 0$, and $0 \leqslant m \leqslant 1$, then the solutions are:

$$U_{7.1,3}(x,y,t) = \pm\frac{\delta\sqrt{\delta^2-1}}{\sqrt{\lambda}}\,\mathrm{dn}\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.1,3}(x,y,t) = -2\delta^2\,\mathrm{JacobiEpsilon}\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr),$$

these solutions represent JEF solutions.

When $m = 1$,

$$U_{7.1,4}(x,y,t) = \pm\frac{\delta\sqrt{\delta^2-1}}{\sqrt{\lambda}}\,\mathrm{sech}\bigl[x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr]\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.1,4}(x,y,t) = -2\delta^2\tanh\bigl[x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr],$$

these solutions represent bright and dark solitary solutions, respectively.

(7.1,3) If $\tau_0 = -m^2$, $\tau_2 = 2m^2 - 1$, $\tau_4 = 1 - m^2$, $\lambda(1 - \delta^2) > 0$, and $0 \leqslant m < 1$, then the solutions are:

$$U_{7.1,4}(x,y,t) = \pm\frac{\delta\sqrt{(1-\delta^2)(1-m^2)}}{\sqrt{\lambda}}\,\mathrm{nc}\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$\begin{aligned}V_{7.1,4}(x,y,t) = {}&-2\delta^2(m+1)\bigl(\mathrm{JacobiEpsilon}\bigl(-t\bigl(\kappa\delta^2+\ell\delta^4\bigr)+x+y\bigr)\\&+(m-1)\bigl(-t\bigl(\kappa\delta^2+\ell\delta^4\bigr)+x+y\bigr)\bigr)\\&-\frac{\mathrm{dn}(x+y-t(\ell\delta^4+\kappa\delta^2))\,\mathrm{sn}(x+y-t(\ell\delta^4+\kappa\delta^2))}{\mathrm{cn}(x+y-t(\ell\delta^4+\kappa\delta^2))},\end{aligned}$$

these solutions represent JEF solutions.

When $m = 0$,

$$U_{7.1,5}(x,y,t) = \pm\frac{\delta\sqrt{1-\delta^2}}{\sqrt{\lambda}}\sec\bigl[x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr]\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.1,5}(x,y,t) = 2\delta^2\tan\bigl[x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr],$$

these solutions represent singular periodic solutions.

(7.1,4) If $\tau_0 = -1$, $\tau_2 = 2 - m^2$, $\tau_4 = m^2 - 1$, $\lambda(\delta^2 - 1) > 0$, and $0 \leqslant m < 1$, then the solutions are:

$$U_{7.1,6}(x,y,t) = \pm\frac{\delta\sqrt{(\delta^2-1)(1-m^2)}}{\sqrt{\lambda}}\,\mathrm{nd}\bigl(x+y-\bigl(\kappa\delta^2+\ell\delta^4\bigr)t\bigr)\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$\begin{aligned}V_{7.1,6}(x,y,t) = {}&2\delta^2(m+1)\Biggl(\frac{m\,\mathrm{cn}(x+y-t(\ell\delta^4+\kappa\delta^2))\,\mathrm{sn}(x+y-t(\ell\delta^4+\kappa\delta^2))}{\mathrm{dn}(x+y-t(\ell\delta^4+\kappa\delta^2))}\\&-\mathrm{JacobiEpsilon}\bigl(-t\bigl(\kappa\delta^2+\ell\delta^4\bigr)+x+y\bigr)\Biggr),\end{aligned}$$

these solutions represent JEF solutions.

(7.1,5) If $\tau_0 = 1/4$, $\tau_2 = (m^2 - 2)/2$, $\tau_4 = m^4/4$, $\lambda(1 - \delta^2) > 0$, and $0 < m \leqslant 1$, then the solutions are:

$$U_{7.1,7}(x,y,t) = \pm\frac{m^2\delta\sqrt{1-\delta^2}}{2\sqrt{\lambda}}\,\frac{\mathrm{sn}(x+y-t(\ell\delta^4+\kappa\delta^2))}{\mathrm{dn}(x+y-t(\ell\delta^4+\kappa\delta^2))+1}\,\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.1,7}(x,y,t) = 2\delta^2(m+1)\biggl(\frac{m\,\mathrm{cn}(x+y-t(\ell\delta^4+\kappa\delta^2))\,\mathrm{sn}(x+y-t(\ell\delta^4+\kappa\delta^2))}{\mathrm{dn}(x+y-t(\ell\delta^4+\kappa\delta^2))} - \mathrm{JacobiEpsilon}\bigl(-t\bigl(\kappa\delta^2+\ell\delta^4\bigr)+x+y\bigr)\biggr),$$

these solutions represent JEF solutions.

According to the set of solutions (7.2), the solutions of Eq. (1) can be stated as:

(7.2,1) If $\tau_0 = 1$, $\tau_2 = -m^2 - 1$, $\tau_4 = m^2$, $\lambda(1 - \delta^2) > 0$, and $0 \leqslant m \leqslant 1$, then the solutions are:

$$U_{7.2,1}(x,y,t) = \pm\frac{\delta\sqrt{1-\delta^2}}{\sqrt{\lambda}}\,\mathrm{ns}\bigl(x+y-t\bigl(\ell\delta^4+\kappa\delta^2\bigr)\bigr)\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$\begin{aligned} V_{7.2,1}(x,y,t) = 2\delta^2\bigl(&- \mathrm{JacobiEpsilon}\bigl(-t\bigl(\kappa\delta^2+\ell\delta^4\bigr)+x+y\bigr) \\ &- \mathrm{cs}\bigl(x+y-t\bigl(\ell\delta^4+\kappa\delta^2\bigr)\bigr)\,\mathrm{dn}\bigl(x+y-t\bigl(\ell\delta^4+\kappa\delta^2\bigr)\bigr) \\ &- t\bigl(\kappa\delta^2+\ell\delta^4\bigr)+x+y\bigr) \end{aligned}$$

or

$$U_{7.2,2}(x,y,t) = \pm\frac{\delta\sqrt{1-\delta^2}}{\sqrt{\lambda}}\,\mathrm{dc}\bigl(x+y-t\bigl(\ell\delta^4+\kappa\delta^2\bigr)\bigr)\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$\begin{aligned} V_{7.2,2}(x,y,t) = 2\delta^2\bigl(&- \mathrm{JacobiEpsilon}\bigl(-t\bigl(\kappa\delta^2+\ell\delta^4\bigr)+x+y,m\bigr) \\ &- t\bigl(\kappa\delta^2+\ell\delta^4\bigr)+x+y\bigr) \\ &+ \frac{\mathrm{dn}(x+y-t(\ell\delta^4+\kappa\delta^2))\,\mathrm{sn}(x+y-t(\ell\delta^4+\kappa\delta^2))}{\mathrm{cn}(x+y-t(\ell\delta^4+\kappa\delta^2))}, \end{aligned}$$

these solutions represent JEF solutions.

When $m = 0$,

$$U_{7.2,3}(x,y,t) = \pm\frac{\delta\sqrt{1-\delta^2}}{\sqrt{\lambda}}\,\csc\bigl[x+y-t\bigl(\ell\delta^4+\kappa\delta^2\bigr)\bigr]\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.2,3}(x,y,t) = -2\delta^2\cot\bigl[x+y-t\bigl(\ell\delta^4+\kappa\delta^2\bigr)\bigr]$$

or

$$U_{7.2,4}(x,y,t) = \pm\frac{\delta\sqrt{1-\delta^2}}{\sqrt{\lambda}}\,\sec\bigl[x+y-t\bigl(\ell\delta^4+\kappa\delta^2\bigr)\bigr]\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.2,4}(x,y,t) = 2\delta^2\tan\bigl[x+y-t\bigl(\ell\delta^4+\kappa\delta^2\bigr)\bigr],$$

these solutions represent the singular periodic solutions.

(7.2,2) If $\tau_0 = m^2 - 1$, $\tau_2 = 2 - m^2$, $\tau_4 = -1$, $\lambda(1-\delta^2) > 0$, and $0 < m < 1$, then the solutions are:

$$U_{7.2,4}(x,y,t) = \pm\frac{\delta\sqrt{-(1-m^2)(1-\delta^2)}}{\sqrt{\lambda}}\,\mathrm{nd}\big(x+y-t\big(\ell\delta^4+\kappa\delta^2\big)\big)\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.2,4}(x,y,t) = 2\delta^2(m+1)\bigg(\frac{m\,\mathrm{cn}(x+y-t(\ell\delta^4+\kappa\delta^2))\,\mathrm{sn}(x+y-t(\ell\delta^4+\kappa\delta^2))}{\mathrm{dn}(x+y-t(\ell\delta^4+\kappa\delta^2))} - \mathrm{JacobiEpsilon}\big(-t\big(\kappa\delta^2+\ell\delta^4\big)+x+y\big)\bigg),$$

these solutions represent JEF solutions.

(7.2,3) If $\tau_0 = -m^2$, $\tau_2 = 2m^2 - 1$, $\tau_4 = 1 - m^2$, $\lambda(\delta^2 - 1) > 0$, and $0 < m \leqslant 1$, then the solutions are:

$$U_{7.2,5}(x,y,t) = \pm\frac{m\delta\sqrt{\delta^2-1}}{\sqrt{\lambda}}\,\mathrm{cn}\big(x+y-t\big(\ell\delta^4+\kappa\delta^2\big)\big)\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.2,5}(x,y,t) = -2m\delta^2\big(\mathrm{JacobiEpsilon}\big(x+y-t\big(\ell\delta^4+\kappa\delta^2\big)\big) + (m-1)\big(x+y-t\big(\ell\delta^4+\kappa\delta^2\big)\big)\big),$$

these solutions represent JEF solutions.

(7.2,4) If $\tau_0 = 1/4$, $\tau_2 = m^2 - 2/2$, $\tau_4 = m^4/4$, $\lambda(\delta^2 - 1) > 0$, and $0 \leqslant m \leqslant 1$, then the solutions are:

$$U_{7.2,6}(x,y,t) = \pm\frac{\delta\sqrt{1-\delta^2}}{2\sqrt{\lambda}}\,\frac{\mathrm{dn}(x+y-t(\ell\delta^4+\kappa\delta^2))+1}{\mathrm{sn}(x+y-t(\ell\delta^4+\kappa\delta^2))}\,\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.2,6}(x,y,t) = -\frac{1}{2}\delta^2\big(2\,\mathrm{JacobiEpsilon}\big(-t\big(\kappa\delta^2+\ell\delta^4\big)+x+y,m\big) + 2\,\mathrm{cs}\big(x+y-t\big(\ell\delta^4+\kappa\delta^2\big)\big)\,\mathrm{dn}\big(x+y-t\big(\ell\delta^4+\kappa\delta^2\big)\big)+1\big) + (m-2)\big(-t\big(\kappa\delta^2+\ell\delta^4\big)+x+y\big),$$

these solutions represent JEF solutions.

When $m = 1$,

$$U_{7.2,7}(x,y,t) = \pm\frac{\delta\sqrt{1-\delta^2}}{2\sqrt{\lambda}}\coth\bigg[\frac{1}{2}\big(x+y-t\big(\ell\delta^4+\kappa\delta^2\big)\big)\bigg]\mathrm{e}^{\mathrm{i}(\kappa x+\ell y+\Omega t)},$$
$$V_{7.2,7}(x,y,t) = -\frac{1}{2}\delta^2\bigg(2\coth\bigg[\frac{1}{2}\big(x+y-t\big(\ell\delta^4+\kappa\delta^2\big)\big)\bigg] - \big(x+y-t\big(\ell\delta^4+\kappa\delta^2\big)\big)\bigg),$$

these solutions represent the singular solitary solutions.

## 4 Numerical simulation of some solutions

By assigning certain values to the parameters, several families of solutions to Eq. (1) were produced such as rational solutions, periodic wave solutions, Jacobi elliptic solutions, exponential solutions, bright solitary, dark solitary, and singular solitary solutions. Thus, a number of original and corrected results that have not before been presented have come from this work. For the purpose of fully comprehending the physical structures of some extracted solutions to be given, the 3D, contour, and 2D figures of some particular solutions are shown.

The solution of Eq. (13), which is dark solitary, is graphed in Fig. 1 using the parameter's values as $\delta = 0.6$, $\lambda = 0.8$, $\Omega = -0.7$, $\ell = 0.6$, $\kappa = 0.7$, $y = 0$, and $-10 < x < 10$.

Equation (14) gives a singular periodic wave solution, which is shown in Fig. 2 when $\delta = 0.7$, $\lambda = 0.55$, $\Omega = 0.6$, $\ell = 0.7$, $\kappa = 0.7$, $y = 0$, and $-10 < x < 10$.

Figure 3 exhibits a bright solitary solution of Eq. (15) with selecting the parameters to be $\delta = 0.9$, $\lambda = -0.8$, $\Omega = 0.5$, $\ell = 0.5$, $\kappa = -0.4$, $y = 0$, and $-10 < x < 10$.

Besides, Eq. (16) is a singular solitary solution that is represented in Fig. 4 with $\delta = 0.7$, $\lambda = 0.6$, $\Omega = -0.8$, $\ell = 0.7$, $\kappa = -0.7$, $y = 0$, and $-10 < x < 10$.

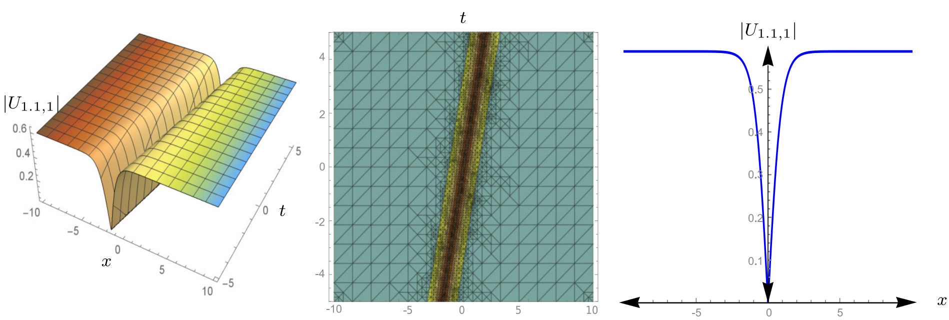


**Figure 1.** The dark solitary wave solution of Eq. (13).

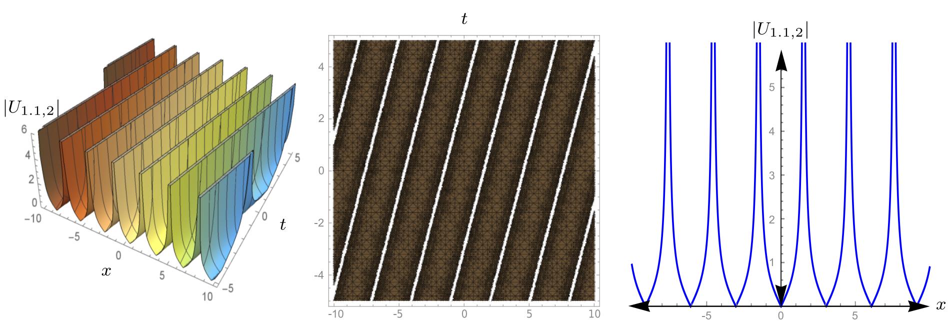


**Figure 2.** The singular periodic wave solution of Eq. (14).

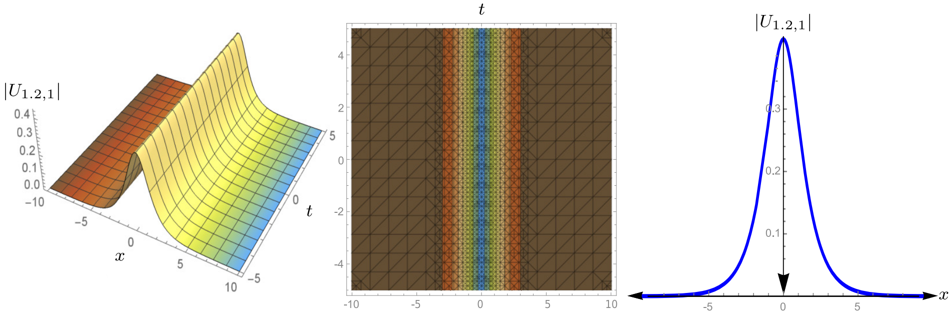


**Figure 3.** The bright solitary wave solution of Eq. (15).

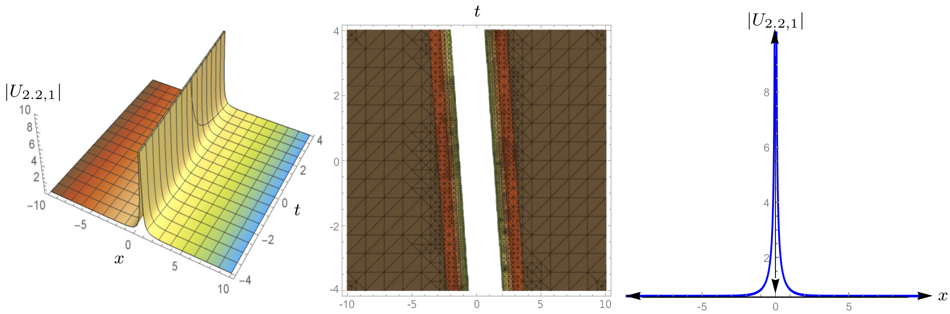


**Figure 4.** The singular solitary wave solution of Eq. (16).

## 5 Conclusion

In this work, the MEMM was successfully implemented to derive various solutions for the DSE model (1), which describes the wave propagation in water of finite depth under the effects of gravity force and surface tension. Dark, bright, and singular solitary solutions were extracted. Other precise solutions, including periodic, singular periodic, exponential, rational, and Jacobi elliptic function solutions, were also derived. Graphical representations for certain solutions were also introduced in order to illustrate their physical character. The solutions we extracted in this research paper are novel, and this model has not been studied via the proposed technique before. The ease, effectiveness, and success with which the approach has been applied further confirm the reliability of the method and its suitability for dealing with NLPDEs.

**Author contributions.** All authors (K.K.A., W.B.R., N.M.B, H.M.A., M.M., and M.E.) have contributed as follows: methodology, K.K.A.; formal analysis, W.B.R. and H.M.A.; software, N.M.B.; validation, M.M.; writing – original draft preparation, M.E. and K.K.A.; writing – review and editing, W.B.R., K.K.A., N.M.B., and H.M.A. All authors have read and approved the published version of the manuscript.

**Conflicts of interest.** The authors declare no conflicts of interest.

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