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Stabilization of chaotic quaternion-valued neutral-type neural networks via sampled-data control with two-sided looped functional approach*

Qiankun Song[a,1], **Qian Wu**[b], **Yurong Liu**[c,d]

[a]Department of Mathematics,
Chongqing Jiaotong University,
Chongqing, China
qiankunsong@163.com

[b]School of Economics and Management,
Chongqing Jiaotong University,
Chongqing, China
qian_wu16@163.com

[c]Department of Mathematics,
Yangzhou University,
Yangzhou 225002, China

[d]School of Mathematics and Physics,
Yancheng Institute of Technology,
Yancheng, 224051, China
yrliu@yzu.edu.cn

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Abstract. The quaternion-valued neutral-type neural networks (QVNTNNs) stability problem through designing sampled-data controller is investigated in this paper. A main stability criterion of the considered neural networks (NNs) is obtained in the form of linear matrix inequalities (LMIs) based on the two-sided looped functional method. The effectiveness of the criterion is shown by a numerical example. It needs to be emphasized that the considered QVNTNNs model in this paper is not broken down into real-valued or complex-valued models in stability analysis, and the acquired criterion holds for both real-valued and complex-valued NNs.

Keywords: quaternion-valued neural networks, neutral delay, stability, sampled-data control, two-sided looped functional.

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[1]Corresponding author.

1 Introduction

NNs have garnered significant interest in recent decades owing to their extensive applications in associative memory, pattern recognition, signal processing, and so on [8, 18]. In such applications, keeping NNs steadily working is a basic requirement [28]. As a result, researching issues related to stability of NNs is crucial [1, 6, 15]. As we know, time delay will inherently exist due to the limitation of amplifier switching speed and communication rate. The emergence of time delays may cause various problems such as poor performance, oscillation, and instability [2]. Thus, many researchers are devoted to the stability of NNs with time delay, and a large number of research results have been published [1, 2, 6, 15, 28]. Numerous control strategies, including state feedback control [29], impulsive control [19], intermittent control [22], and sampled-data control [11], were further developed in the stability research for delayed NNs. Besides, the sampled-data control, as another control strategies, are widely adopted due to its superb digital controller implementation.

For stability problem of sampled-data control systems, the input delay method proposed in [5] was extensively employed in [4, 9, 23], where there was only a typical time-varying delay applied to the signal transfer, and then the classical Lyapunov approach was utilized to determine the stability requirements for sampled-data control systems by constructing Lyapunov functional. However, simple transforming cannot capture the signal's sawtooth structure and requires the functional to be positive definite throughout the sampling interval, such a generated functional may result in excessive conservatism.

To reduce the conservativeness of stability criteria for sampled-data control systems, a looped-functional approach was proposed in [12], which just requiring the created functional to be positive definite at the sampling instants rather than during the entire sampling interval. Subsequently, some meaningful works have been carried out by employing the looped-functional approach [13, 26]. However, there is still room for improvement of the looped-functionals used in [12, 13, 26] since this construction only uses information in $[t_{l-1}, t]$ and does not use information in $[t, t_{l+1}]$. To overcome this insufficient, an improved looped-functional including the information in $[t_{l-1}, t]$ and $[t, t_{l+1}]$, called two-sided looped-functional, was first structured in [25]. In recent years, some stability criteria of NNs with sampled-data control have been established based on the two-sided looped-functional method; see [14, 21, 24, 30] and references therein.

The NNs proposed in [14, 21, 24, 30] are real-valued NNs in which the neuron states, activation functions, connection weights, and outputs are all real values. Although real-valued NNs have gained many applications, they also have certain limitations. In actuality, multidimensional data is commonly encountered, while neurons of real-valued NNs are not perfect at handling all forms of input. It is widely known that the neurons of QVNNs are quaternion values, which manipulate better than complex-valued NNs and real-valued NNs in processing some high-dimensional data, like that color images and body images. Because a quaternion has one real part and three imaginary parts, it can deal with multidimensional data better. For example, in three-dimensional space, the spatial rotation could be described tersely and efficiently with the quaternion. The spatial rotation in 3D geometrical affine transformations – translation, image compression, and color night vision – can all be efficiently and compactly described by quaternion. Therefore, QVNN have received widespread attention, and some stability criteria have been established; see [3, 7, 10, 16, 17] and references therein. To the author's knowledge, there are few research conclusions on stability of QVNN via sampled-data control.

Motivated by the preceding points, this paper aims at the stability of QVNN with sampled-data control by employing the two-sided looped-functional method. The following is a summary of the contributions made to the paper:

(i) The two-sided looped-functional method is first employed to investigate the stability of QVNNs with sampled-data control.
(ii) The considered QVNNs model in this paper is not broken down into real-valued models or complex-valued models.
(iii) The obtained criteria in this paper are in the form of LMIs, and the YALMIP toolbox in Matlab can be applied to calculate it.
(iv) The obtained criteria are valid for both real-valued and complex-valued NNs.

Notations. $\mathbb{Q}$, $\mathbb{Q}^n$, and $\mathbb{Q}^{n\times m}$ symbolize the skew field of quaternion numbers, n-dimensional, and $n\times m$ quaternion-valued vectors and matrices. Suppose $A\in\mathbb{Q}^{n\times m}$, A^* represents its conjugate transpose, and $A>0$ stands for A is positive definite matrix with $A^*=A$. Suppose $z\in\mathbb{Q}^n$, $\|z\|$ denotes its norm.

2 Model description and preliminaries

The following QVNNs with discrete and neutral delays is considered:

$$\dot{\mathfrak{p}}(t) = -D\mathfrak{p}(t) + Af\big(\mathfrak{p}(t)\big) + Bg\big(\mathfrak{p}(t-\varsigma)\big) + C\dot{\mathfrak{p}}(t-\sigma) + u(t), \tag{1}$$

where $\mathfrak{p}(t)\in\mathbb{Q}^n$ represents the state of neuron; $\varsigma>0$ and $\sigma>0$ symbolize the discrete and neutral delay; $f(\cdot), g(\cdot)\in\mathbb{Q}^n$ are activation functions; $D>0$ is a real diagonal matrix; $A, B, C\in\mathbb{Q}^{n\times n}$ are the connection weight matrices; $u(t)\in\mathbb{Q}^n$ is the control input. The corresponding initial condition is

$$\mathfrak{p}(s) = \phi(s), \quad s\in[-\rho, 0], \tag{2}$$

in which $\rho=\max\{\varsigma,\sigma\}$, and $\phi(s)\in\mathbb{Q}^n$ is continuous and bounded in $[-\rho, 0]$.

For system (1) with initial value (2), the sampled-data controller is offered:

$$u(t) = K\mathfrak{p}(t_l), \quad t_l\leqslant t<t_{l+1}, \tag{3}$$

where $l\in\mathbb{N}^+$; $K\in\mathbb{Q}^{n\times n}$ is the state gain matrix, t_l are the sampling instants, where $0=t_0<t_1<\cdots<t_l<\cdots$, and $\lim_{l\to\infty}t_l=+\infty$.

By substituting (3) into (1), there is

$$\dot{\mathfrak{p}}(t) = -D\mathfrak{p}(t) + Af\big(\mathfrak{p}(t)\big) + Bg\big(\mathfrak{p}(t-\varsigma)\big) + C\dot{\mathfrak{p}}(t-\sigma) + K\mathfrak{p}(t_l) \tag{4}$$

for $t_l\leqslant t<t_{l+1}$.

To examine the stability of model (4), the subsequent two presumptions must be met:

(H1) For any $a, b \in \mathbb{Q}$, if f_i and g_i $(i = 1, 2, \dots, n)$ satisfy $f_i(0) = 0$ and $g_i(0) = 0$, there are scalars $\mathcal{F}_i > 0$ and $\mathcal{G}_i > 0$ such that

$$\left|f_i(a) - f_i(b)\right| \leqslant \mathcal{F}_i|a-b|, \qquad \left|g_i(a) - g_i(b)\right| \leqslant \mathcal{G}_i|a-b|,$$

where $\mathcal{F} = \mathrm{diag}\{\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_n\}$ and $\mathcal{G} = \mathrm{diag}\{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_n\}$.

(H2) The variable sampling periods $h_l = t_{l+1} - t_l$ satisfy

$$h_m \leqslant h_l \leqslant h_M,$$

where h_m and h_M are two known positive constants.

The following lemmas are crucial to obtain the stability conclusion of model (4).

Lemma 1. (*See* [3].) *If* $0 < \mathcal{A} \in \mathbb{Q}^{n\times n}$*, the constant* $\varepsilon \geqslant 0$ *always exists for all* $t \geqslant 0$ *and* $\omega(s) \in \mathbb{Q}^n$ *such that*

$$\left(\int_{t-\varepsilon}^{t} \omega(s)\,\mathrm{d}s\right)^* \mathcal{A}\left(\int_{t-\varepsilon}^{t} \omega(s)\,\mathrm{d}s\right) \leqslant h \int_{t-\varepsilon}^{t} \omega^*(s)\mathcal{A}\omega(s)\,\mathrm{d}s.$$

Lemma 2. *Suppose* $\mathfrak{X} \in \mathbb{Q}^{n\times n}$ *exists for given* $0 < A, B \in \mathbb{Q}^{n\times n}$ *such that*

$$\begin{bmatrix} A & \mathfrak{X} \\ \mathfrak{X}^* & B \end{bmatrix} > 0. \tag{5}$$

Then

$$\frac{1}{n(t)}\varpi^* A\varpi + \frac{1}{1-n(t)}\eta^* B\eta \geqslant \begin{bmatrix} \varpi \\ \eta \end{bmatrix}^* \begin{bmatrix} A & \mathfrak{X} \\ \mathfrak{X}^* & B \end{bmatrix} \begin{bmatrix} \varpi \\ \eta \end{bmatrix},$$

where $\varpi, \eta \in \mathbb{Q}^n$ *are arbitrary two vectors,* $n(t) \in (0, 1)$ *is an arbitrary function for* $t \geqslant 0$.

Proof. For arbitrary two vectors $\varpi, \eta \in \mathbb{Q}^n$ and a function $n(t) \in (0, 1)$ with $t \geqslant 0$, we have from (5) that

$$\begin{aligned} 0 &\leqslant \begin{bmatrix} \sqrt{\frac{1-n(t)}{n(t)}}\varpi \\ -\sqrt{\frac{n(t)}{1-n(t)}}\eta \end{bmatrix}^* \begin{bmatrix} A & \mathfrak{X} \\ \mathfrak{X}^* & B \end{bmatrix} \begin{bmatrix} \sqrt{\frac{1-n(t)}{n(t)}}\varpi \\ -\sqrt{\frac{n(t)}{1-n(t)}}\eta \end{bmatrix} \\ &= \frac{1-n(t)}{n(t)}\varpi^* A\varpi - \varpi^*\mathfrak{X}\eta - \eta^*\mathfrak{X}^*\varpi + \frac{n(t)}{1-n(t)}\eta^* B\eta. \end{aligned}$$

That is,

$$\frac{1}{n(t)}\varpi^* A\varpi + \frac{1}{1-n(t)}\eta^* B\eta \geqslant \begin{bmatrix} \varpi \\ \eta \end{bmatrix}^* \begin{bmatrix} A & \mathfrak{X} \\ \mathfrak{X}^* & B \end{bmatrix} \begin{bmatrix} \varpi \\ \eta \end{bmatrix}.$$

The proof is finished. □

3 Main results

Theorem 1. *Suppose there exist five positive definite matrices P_1, P_2, P_3, P_4, P_5, two positive diagonal matrices R_1, R_2, and fifteen matrices $M, S, Q, \mathfrak{X}_{ij}, \mathfrak{L}_{ij}, \mathfrak{Z}_{ij}, N \in \mathbb{Q}^{n\times n}$ $(i, j = 1, 2, \mathfrak{Z}_{21} = \mathfrak{Z}_{12}^*)$ such that*

$$\begin{bmatrix} P_4 & S \\ S^* & P_5 \end{bmatrix} > 0, \tag{6}$$

$$\begin{aligned} \Omega + h_m\Gamma < 0, \qquad \Omega + h_m\Pi < 0, \\ \Omega + h_M\Gamma < 0, \qquad \Omega + h_M\Pi < 0, \end{aligned} \tag{7}$$

where

$$\Omega = (\Omega_{ij})_{8\times 8}, \qquad \Gamma = (\Gamma_{ij})_{8\times 8}, \qquad \Pi = (\Pi_{ij})_{8\times 8}$$

in which

$$\begin{aligned}
\Omega_{11} &= P_2 - P_4 + S + S^* - P_5 - \mathfrak{L}_{11} - \mathfrak{L}_{11}^* - \mathfrak{L}_{12} - \mathfrak{L}_{12}^* \\
&\quad + \mathfrak{L}_{21} + \mathfrak{L}_{21}^* + \mathfrak{L}_{22} + \mathfrak{L}_{22}^* + \mathcal{F}R_1\mathcal{F}, \\
\Omega_{14} &= P_1 - M - M^* - DQ^*, \\
\Omega_{17} &= P_4 - S^* - \mathfrak{X}_{11}^* - \mathfrak{X}_{12}^* + \mathfrak{L}_{11} + \mathfrak{L}_{11}^* + \mathfrak{L}_{12}^* - \mathfrak{L}_{21} - DQ^*, \\
\Omega_{18} &= -S + P_5 - \mathfrak{X}_{21}^* - \mathfrak{X}_{22}^* + \mathfrak{L}_{12} - \mathfrak{L}_{21}^* - \mathfrak{L}_{22} - \mathfrak{L}_{22}^*, \\
\Omega_{22} &= -R_1, \qquad \Omega_{24} = A^*Q^*, \qquad \Omega_{27} = A^*Q^*, \\
\Omega_{33} &= -R_2, \qquad \Omega_{34} = B^*Q^*, \qquad \Omega_{37} = B^*Q^*, \\
\Omega_{44} &= P_3 - Q - Q^*, \qquad \Omega_{46} = QC, \qquad \Omega_{47} = M^* + N - Q^*, \\
\Omega_{48} &= M, \qquad \Omega_{55} = -P_2 + \mathcal{G}R_2\mathcal{G}, \qquad \Omega_{66} = -P_3, \\
\Omega_{67} &= C^*Q^*, \qquad \Omega_{77} = -P_4 + \mathfrak{X}_{11} + \mathfrak{X}_{11}^* - \mathfrak{L}_{11} - \mathfrak{L}_{11}^* + N + N^*, \\
\Omega_{78} &= S + \mathfrak{X}_{12} + \mathfrak{X}_{21}^* - \mathfrak{L}_{12} + \mathfrak{L}_{21}^*, \\
\Omega_{88} &= -P_5 + \mathfrak{X}_{22} + \mathfrak{X}_{22}^* + \mathfrak{L}_{22} + \mathfrak{L}_{22}^*, \\
\Gamma_{14} &= \mathfrak{L}_{11} + \mathfrak{L}_{11}^* - \mathfrak{L}_{21} - \mathfrak{L}_{21}^*, \qquad \Gamma_{44} = h_M P_4, \\
\Gamma_{47} &= \mathfrak{X}_{11}^* - \mathfrak{L}_{11} - \mathfrak{L}_{11}^* + \mathfrak{L}_{21}, \qquad \Gamma_{48} = \mathfrak{X}_{21}^* + \mathfrak{L}_{21}^*, \\
\Gamma_{77} &= \mathfrak{Z}_{11}, \qquad \Gamma_{78} = \mathfrak{Z}_{12}, \qquad \Gamma_{88} = \mathfrak{Z}_{22}; \\
\Pi_{14} &= -\mathfrak{L}_{12} - \mathfrak{L}_{12}^* + \mathfrak{L}_{22} + \mathfrak{L}_{22}^*, \qquad \Pi_{44} = h_M P_5, \\
\Pi_{47} &= -\mathfrak{X}_{12}^* + \mathfrak{L}_{12}^*, \qquad \Pi_{48} = -\mathfrak{X}_{22}^* + \mathfrak{L}_{12} - \mathfrak{L}_{22} - \mathfrak{L}_{22}^*, \\
\Pi_{77} &= -\mathfrak{Z}_{11}, \qquad \Pi_{78} = -\mathfrak{Z}_{12}, \qquad \Pi_{88} = -\mathfrak{Z}_{22}.
\end{aligned}$$

Then model (4) *is globally stable under assumptions* (H1) *and* (H2)*, and the gain matrix of* (3) *is*

$$K = Q^{-1}N. \tag{8}$$

Proof. Let

$$\mathfrak{X} = \begin{bmatrix} \mathfrak{X}_{11} & \mathfrak{X}_{12} \\ \mathfrak{X}_{21} & \mathfrak{X}_{22} \end{bmatrix}, \qquad \mathfrak{L} = \begin{bmatrix} \mathfrak{L}_{11} & \mathfrak{L}_{12} \\ \mathfrak{L}_{21} & \mathfrak{L}_{22} \end{bmatrix}, \qquad \mathfrak{Z} = \begin{bmatrix} \mathfrak{Z}_{11} & \mathfrak{Z}_{12} \\ \mathfrak{Z}_{12}^* & \mathfrak{Z}_{22} \end{bmatrix},$$

$$
\begin{aligned}
\varpi_1 &= \left[\mathfrak{p}^{\mathrm{T}}(t_l), \mathfrak{p}^{\mathrm{T}}(t_{l+1})\right]^{\mathrm{T}}, \\
\varpi_2(t) &= \left[\mathfrak{p}^{\mathrm{T}}(t) - \mathfrak{p}^{\mathrm{T}}(t_l),\ \mathfrak{p}^{\mathrm{T}}(t_{l+1}) - \mathfrak{p}^{\mathrm{T}}(t)\right]^{\mathrm{T}}, \\
\varpi_3(t) &= \left[(t_{l+1} - t)\big(\mathfrak{p}^{\mathrm{T}}(t) - \mathfrak{p}^{\mathrm{T}}(t_l)\big),\ (t - t_l)\big(\mathfrak{p}^{\mathrm{T}}(t_{l+1}) - \mathfrak{p}^{\mathrm{T}}(t)\big)\right]^{\mathrm{T}}.
\end{aligned}
$$

Let

$$W(t) = V(t) + v(t),$$

where

$$V(t) = \mathfrak{p}^*(t)P_1\mathfrak{p}(t) + \int\limits_{t-\varsigma}^{t} \mathfrak{p}(s)P_2\mathfrak{p}(s)\,\mathrm{d}s + \int\limits_{t-\sigma}^{t} \dot{\mathfrak{p}}^*(s)P_3\dot{\mathfrak{p}}(s)\,\mathrm{d}s, \tag{9}$$

$$
\begin{aligned}
v(t) = {}& h_l(t_{l+1} - t)\int\limits_{t_l}^{t} \dot{\mathfrak{p}}^*(s)P_4\dot{\mathfrak{p}}(s)\,\mathrm{d}s - h_l(t - t_l)\int\limits_{t}^{t_{l+1}} \dot{\mathfrak{p}}^*(s)P_5\dot{\mathfrak{p}}(s)\,\mathrm{d}s \\
& + \big(\mathfrak{p}(t) - \mathfrak{p}(t_l)\big)^* M\big(\mathfrak{p}(t_{l+1}) - \mathfrak{p}(t)\big) + \big(\mathfrak{p}(t_{l+1}) - \mathfrak{p}(t)\big)^* M^*\big(\mathfrak{p}(t) - \mathfrak{p}(t_l)\big) \\
& + \varpi_1^*\mathfrak{X}\varpi_3(t) + \varpi_3^*(t)\mathfrak{X}^*\varpi_1 + \varpi_2^*(t)\mathfrak{L}\varpi_3(t) + \varpi_3^*(t)\mathfrak{L}^*\varpi_2(t) \\
& + (t_{l+1} - t)(t - t_l)\varpi_1^*\mathfrak{Z}\varpi_1.
\end{aligned}
$$

Note that $V(t)$ is a Lyapunov functional, and $v(t)$ is a looped functional since $v(t_l) = v(t_{l+1}) = 0$. By doing the straightforward computation for the derivative of $W(t)$ along (4), one obtains that

$$
\begin{aligned}
\dot{V}(t) = {}& \dot{\mathfrak{p}}^*(t)P_1\mathfrak{p}(t) + \mathfrak{p}^*(t)P_1\dot{\mathfrak{p}}(t) + \mathfrak{p}^*(t)P_2\mathfrak{p}(t) - \mathfrak{p}^*(t-\varsigma)P_2\mathfrak{p}(t-\varsigma) \\
& + \dot{\mathfrak{p}}^*(t)P_3\dot{\mathfrak{p}}(t) - \dot{\mathfrak{p}}^*(t-\sigma)P_3\dot{\mathfrak{p}}(t-\sigma),
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\dot{v}(t) = {}& -h_l\int\limits_{t_l}^{t} \dot{\mathfrak{p}}^*(s)P_4\dot{\mathfrak{p}}(s)\,\mathrm{d}s + h_l(t_{l+1} - t)\dot{\mathfrak{p}}^*(t)P_4\dot{\mathfrak{p}}(t) - h_l\int\limits_{t}^{t_{l+1}} \dot{\mathfrak{p}}^*(s)P_5\dot{\mathfrak{p}}(s)\,\mathrm{d}s \\
& + h_l(t - t_l)\dot{\mathfrak{p}}^*(t)P_5\dot{\mathfrak{p}}(t) + \dot{\mathfrak{p}}^*(t)M\big[\mathfrak{p}(t_{l+1}) - \mathfrak{p}(t)\big] - \big[\mathfrak{p}(t) - \mathfrak{p}(t_l)\big]^* M\dot{\mathfrak{p}}(t) \\
& - \dot{\mathfrak{p}}^*(t)M^*\big[\mathfrak{p}(t) - \mathfrak{p}(t_l)\big] + \big[\mathfrak{p}(t_{l+1}) - \mathfrak{p}(t)\big]^* M^*\dot{\mathfrak{p}}(t) + \varpi_1^*\mathfrak{X}\dot{\varpi}_3(t) \\
& + \dot{\varpi}_3^*(t)\mathfrak{X}^*\varpi_1 + \dot{\varpi}_2^*(t)\mathfrak{L}\varpi_3(t) + \dot{\varpi}_3^*(t)\mathfrak{L}^*\varpi_2(t) + \varpi_2^*(t)\mathfrak{L}\dot{\varpi}_3(t) \\
& + \varpi_3^*(t)\mathfrak{L}^*\dot{\varpi}_2(t) + (t_{l+1} - t)\varpi_1^*\mathfrak{Z}\varpi_1 - (t - t_l)\varpi_1^*\mathfrak{Z}\varpi_1 \\
\leqslant {}& -h_l\int\limits_{t_l}^{t} \dot{\mathfrak{p}}^*(s)P_4\dot{\mathfrak{p}}(s)\,\mathrm{d}s + h_M(t_{l+1} - t)\dot{\mathfrak{p}}^*(t)P_4\dot{\mathfrak{p}}(t) - h_l\int\limits_{t}^{t_{l+1}} \dot{\mathfrak{p}}^*(s)P_5\dot{\mathfrak{p}}(s)\,\mathrm{d}s \\
& + h_M(t - t_l)\dot{\mathfrak{p}}^*(t)P_5\dot{\mathfrak{p}}(t) + \dot{\mathfrak{p}}^*(t)M\big[\mathfrak{p}(t_{l+1}) - \mathfrak{p}(t)\big] - \big[\mathfrak{p}(t) - \mathfrak{p}(t_l)\big]^* M\dot{\mathfrak{p}}(t) \\
& - \dot{\mathfrak{p}}^*(t)M^*\big[\mathfrak{p}(t) - \mathfrak{p}(t_l)\big] + \big[\mathfrak{p}(t_{l+1}) - \mathfrak{p}(t)\big]^* M^*\dot{\mathfrak{p}}(t) + \varpi_1^*\mathfrak{X}\dot{\varpi}_3(t) \\
& + \dot{\varpi}_3^*(t)\mathfrak{X}^*\varpi_1 + \dot{\varpi}_2^*(t)\mathfrak{L}\varpi_3(t) + \dot{\varpi}_3^*(t)\mathfrak{L}^*\varpi_2(t) + \varpi_2^*(t)\mathfrak{L}\dot{\varpi}_3(t) \\
& + \varpi_3^*(t)\mathfrak{L}^*\dot{\varpi}_2(t) + (t_{l+1} - t)\varpi_1^*\mathfrak{Z}\varpi_1 - (t - t_l)\varpi_1^*\mathfrak{Z}\varpi_1.
\end{aligned} \tag{11}
$$

Using condition (6), Lemmas 1 and 2 result in

$$-h_l \int_{t_l}^{t} \dot{\mathfrak{p}}^*(s) P_4 \dot{\mathfrak{p}}(s)\,\mathrm{d}s - h_l \int_{t}^{t_{l+1}} \dot{\mathfrak{p}}^*(s) P_5 \dot{\mathfrak{p}}(s)\,\mathrm{d}s$$
$$\begin{aligned}
&\leqslant -\frac{h_l}{t-t_l}\big(\mathfrak{p}(t)-\mathfrak{p}(t_l)\big)^* P_4\big(\mathfrak{p}(t)-\mathfrak{p}(t_l)\big) \\
&\quad - \frac{h_l}{t_{l+1}-t}\big(\mathfrak{p}(t_{l+1})-\mathfrak{p}(t)\big)^* P_5\big(\mathfrak{p}(t_{l+1})-\mathfrak{p}(t)\big) \\
&\leqslant -\begin{bmatrix} \mathfrak{p}(t)-\mathfrak{p}(t_l) \\ \mathfrak{p}(t_{l+1})-\mathfrak{p}(t) \end{bmatrix}^* \begin{bmatrix} P_4 & S \\ S^* & P_5 \end{bmatrix} \begin{bmatrix} \mathfrak{p}(t)-\mathfrak{p}(t_l) \\ \mathfrak{p}(t_{l+1})-\mathfrak{p}(t) \end{bmatrix} \\
&= \mathfrak{p}^*(t)(-P_4+S+S^*-P_5)\mathfrak{p}(t) + \mathfrak{p}^*(t)(P_4-S^*)\mathfrak{p}(t_l) + \mathfrak{p}^*(t)(-S+P_5)\mathfrak{p}(t_{l+1}) \\
&\quad + \mathfrak{p}^*(t_l)(P_4-S)\mathfrak{p}(t) - \mathfrak{p}^*(t_l)P_4\mathfrak{p}(t_l) + \mathfrak{p}^*(t_l)Sp(t_{l+1}) \\
&\quad + \mathfrak{p}^*(t_{l+1})(-S^*+P_5)\mathfrak{p}(t) + \mathfrak{p}^*(t_{l+1})S^*\mathfrak{p}(t_l) - \mathfrak{p}^*(t_{l+1})P_5\mathfrak{p}(t_{l+1}).
\end{aligned} \tag{12}$$

It is easy to compute that

$$\begin{aligned}
&\varpi_1^*\mathfrak{X}\dot{\varpi}_3(t) + \dot{\varpi}_3^*(t)\mathfrak{X}^*\varpi_1 \\
&\quad = -\mathfrak{p}^*(t)(\mathfrak{X}_{11}^*+\mathfrak{X}_{12}^*)\mathfrak{p}(t_l) - \mathfrak{p}^*(t)(\mathfrak{X}_{21}^*+\mathfrak{X}_{22}^*)\mathfrak{p}(t_{l+1}) \\
&\qquad - \mathfrak{p}^*(t_l)(\mathfrak{X}_{11}+\mathfrak{X}_{12})\mathfrak{p}(t) + \mathfrak{p}^*(t_l)(\mathfrak{X}_{11}+\mathfrak{X}_{11}^*)\mathfrak{p}(t_l) + \mathfrak{p}^*(t_l)(\mathfrak{X}_{12}+\mathfrak{X}_{21}^*)\mathfrak{p}(t_{l+1}) \\
&\qquad - \mathfrak{p}^*(t_{l+1})(\mathfrak{X}_{21}+\mathfrak{X}_{22})\mathfrak{p}(t) + \mathfrak{p}^*(t_{l+1})(\mathfrak{X}_{21}+\mathfrak{X}_{12}^*)\mathfrak{p}(t_l) \\
&\qquad + \mathfrak{p}^*(t_{l+1})(\mathfrak{X}_{22}+\mathfrak{X}_{22}^*)\mathfrak{p}(t_{l+1}) \\
&\qquad + (t_{l+1}-t)\big[\mathfrak{p}^*(t_l)\mathfrak{X}_{11}\dot{\mathfrak{p}}(t) \\
&\qquad + \mathfrak{p}^*(t_{l+1})\mathfrak{X}_{21}\dot{\mathfrak{p}}(t) + \dot{\mathfrak{p}}^*(t)\mathfrak{X}_{11}^*\mathfrak{p}(t_l) + \dot{\mathfrak{p}}^*(t)\mathfrak{X}_{21}^*\mathfrak{p}(t_{l+1})\big] \\
&\qquad - (t-t_l)\big[\mathfrak{p}^*(t_l)\mathfrak{X}_{12}\dot{\mathfrak{p}}(t) + \dot{\mathfrak{p}}^*(t)\mathfrak{X}_{12}^*\mathfrak{p}(t_l) + \mathfrak{p}^*(t_{l+1})\mathfrak{X}_{22}\dot{\mathfrak{p}}(t) \\
&\qquad + \dot{\mathfrak{p}}^*(t)\mathfrak{X}_{22}^*\mathfrak{p}(t_{l+1})\big],
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\dot{\varpi}_2^*(t)\mathfrak{L}\varpi_3(t) + \varpi_3^*(t)\mathfrak{L}^*\dot{\varpi}_2(t) \\
&\quad = (t_{l+1}-t)\big[\dot{\mathfrak{p}}^*(t)(\mathfrak{L}_{11}-\mathfrak{L}_{21})\mathfrak{p}(t) + \dot{\mathfrak{p}}^*(t)(-\mathfrak{L}_{11}+\mathfrak{L}_{21})\mathfrak{p}(t_l) \\
&\qquad + \mathfrak{p}^*(t)(\mathfrak{L}_{11}^*-\mathfrak{L}_{21}^*)\dot{\mathfrak{p}}(t) + \mathfrak{p}^*(t_l)(-\mathfrak{L}_{11}^*+\mathfrak{L}_{21}^*)\dot{\mathfrak{p}}(t)\big] \\
&\qquad + (t-t_l)\big[\dot{\mathfrak{p}}^*(t)(\mathfrak{L}_{12}-\mathfrak{L}_{22})\mathfrak{p}(t_{l+1}) + \dot{\mathfrak{p}}^*(t)(-\mathfrak{L}_{12}+\mathfrak{L}_{22})\mathfrak{p}(t) \\
&\qquad + \mathfrak{p}^*(t_{l+1})(\mathfrak{L}_{12}^*-\mathfrak{L}_{22}^*)\dot{\mathfrak{p}}(t) + \mathfrak{p}^*(t)(-\mathfrak{L}_{12}^*+\mathfrak{L}_{22}^*)\dot{\mathfrak{p}}(t)\big],
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\varpi_2^*(t)\mathfrak{L}\dot{\varpi}_3(t) + \dot{\varpi}_3^*(t)\mathfrak{L}^*\varpi_2(t) \\
&\quad = \mathfrak{p}^*(t)(-\mathfrak{L}_{11}+\mathfrak{L}_{21}-\mathfrak{L}_{12}+\mathfrak{L}_{22}-\mathfrak{L}_{11}^*+\mathfrak{L}_{21}^*-\mathfrak{L}_{12}^*+\mathfrak{L}_{22}^*)\mathfrak{p}(t) \\
&\qquad + \mathfrak{p}^*(t)(\mathfrak{L}_{11}-\mathfrak{L}_{21}+\mathfrak{L}_{11}^*+\mathfrak{L}_{12}^*)\mathfrak{p}(t_l) + \mathfrak{p}^*(t)(\mathfrak{L}_{12}-\mathfrak{L}_{22}-\mathfrak{L}_{21}^*-\mathfrak{L}_{22}^*)\mathfrak{p}(t_{l+1}) \\
&\qquad + \mathfrak{p}^*(t_l)(\mathfrak{L}_{11}+\mathfrak{L}_{12}+\mathfrak{L}_{11}^*-\mathfrak{L}_{21}^*)\mathfrak{p}^*(t) + \mathfrak{p}^*(t_l)(-\mathfrak{L}_{11}-\mathfrak{L}_{11}^*)\mathfrak{p}(t_l) \\
&\qquad + \mathfrak{p}^*(t_l)(-\mathfrak{L}_{12}+\mathfrak{L}_{21}^*)\mathfrak{p}(t_{l+1}) + \mathfrak{p}^*(t_{l+1})(-\mathfrak{L}_{21}-\mathfrak{L}_{22}+\mathfrak{L}_{12}^*-\mathfrak{L}_{22}^*)\mathfrak{p}(t) \\
&\qquad + \mathfrak{p}^*(t_{l+1})(\mathfrak{L}_{21}-\mathfrak{L}_{12}^*)\mathfrak{p}(t_l) + \mathfrak{p}^*(t_{l+1})(\mathfrak{L}_{22}+\mathfrak{L}_{22}^*)\mathfrak{p}(t_{l+1})
\end{aligned}$$

$$
\begin{aligned}
&+ (t_{l+1}-t)\big[\mathfrak{p}^*(t)(\mathfrak{L}_{11}-\mathfrak{L}_{21})\dot{\mathfrak{p}}(t) - \mathfrak{p}^*(t_l)\mathfrak{L}_{11}\dot{\mathfrak{p}}(t) + \mathfrak{p}^*(t_{l+1})\mathfrak{L}_{21}\dot{\mathfrak{p}}(t) \\
&+ \dot{\mathfrak{p}}^*(t)(\mathfrak{L}_{11}^*-\mathfrak{L}_{21}^*)\mathfrak{p}(t) - \dot{\mathfrak{p}}^*(t)\mathfrak{L}_{11}^*\mathfrak{p}(t_l) + \dot{\mathfrak{p}}^*(t)\mathfrak{L}_{21}^*\mathfrak{p}(t_{l+1})\big] \\
&+ (t-t_l)\big[\mathfrak{p}^*(t)(-\mathfrak{L}_{12}+\mathfrak{L}_{22})\dot{\mathfrak{p}}(t) + \mathfrak{p}^*(t_l)\mathfrak{L}_{12}\dot{\mathfrak{p}}(t) - \mathfrak{p}^*(t_{l+1})\mathfrak{L}_{22}\dot{\mathfrak{p}}(t) \\
&+ \dot{\mathfrak{p}}^*(t)(-\mathfrak{L}_{12}^*+\mathfrak{L}_{22}^*)\mathfrak{p}(t) + \dot{\mathfrak{p}}^*(t)\mathfrak{L}_{12}^*\mathfrak{p}(t_l) - \dot{\mathfrak{p}}^*(t)\mathfrak{L}_{22}^*\mathfrak{p}(t_{l+1})\big], \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
&(t_{l+1}-t)\varpi_1^*\mathfrak{Z}\varpi_1 - (t-t_l)\varpi_1^*\mathfrak{Z}\varpi_1 \\
&\quad = (t_{l+1}-t)\big[\mathfrak{p}^*(t_l)\mathfrak{Z}_{11}\mathfrak{p}(t_l) + \mathfrak{p}^*(t_l)\mathfrak{Z}_{12}\mathfrak{p}(t_{l+1}) \\
&\qquad + \mathfrak{p}^*(t_{l+1})\mathfrak{Z}_{12}^*\mathfrak{p}(t_l) + \mathfrak{p}^*(t_{l+1})\mathfrak{Z}_{22}\mathfrak{p}(t_{l+1})\big] \\
&\qquad - (t-t_l)\big[\mathfrak{p}^*(t_l)\mathfrak{Z}_{11}\mathfrak{p}(t_l) + \mathfrak{p}^*(t_l)\mathfrak{Z}_{12}\mathfrak{p}(t_{l+1}) \\
&\qquad + \mathfrak{p}^*(t_{l+1})\mathfrak{Z}_{12}^*\mathfrak{p}(t_l) + \mathfrak{p}^*(t_{l+1})\mathfrak{Z}_{22}\mathfrak{p}(t_{l+1})\big]. \qquad (16)
\end{aligned}
$$

Employing assumption (H1) results in

$$0 \leqslant \mathfrak{p}^*(t)\mathfrak{F}R_1\mathfrak{F}\mathfrak{p}(t) - f^*\big(\mathfrak{p}(t)\big)R_1 f\big(\mathfrak{p}(t)\big), \qquad (17)$$

$$0 \leqslant \mathfrak{p}^*(t-\varsigma)\mathcal{G}R_2\mathcal{G}\mathfrak{p}(t-\varsigma) - g^*\big(\mathfrak{p}(t-\varsigma)\big)R_2 g\big(\mathfrak{p}(t-\varsigma)\big). \qquad (18)$$

Moreover, (4) indicates

$$
\begin{aligned}
0 &= \big[\dot{\mathfrak{p}}(t)+\mathfrak{p}(t_l)\big]^* Q\big[-\dot{\mathfrak{p}}(t) - D\mathfrak{p}(t) + Af\big(\mathfrak{p}(t)\big) + Bg\big(\mathfrak{p}(t-\varsigma)\big) \\
&\quad + C\dot{\mathfrak{p}}(t-\sigma) + K\mathfrak{p}(t_l)\big] \\
&\quad + \big[-\dot{\mathfrak{p}}(t) - D\mathfrak{p}(t) + Af\big(\mathfrak{p}(t)\big) + Bg\big(\mathfrak{p}(t-\varsigma)\big) + C\dot{\mathfrak{p}}(t-\sigma) + K\mathfrak{p}(t_l)\big]^* \\
&\quad \times Q^*\big[\dot{\mathfrak{p}}(t)+\mathfrak{p}(t_l)\big]. \qquad (19)
\end{aligned}
$$

Let

$$\vartheta(t) = \big[\mathfrak{p}^*(t),\ f^*\big(\mathfrak{p}(t)\big),\ g^*\big(\mathfrak{p}(t-\varsigma)\big),\ \dot{\mathfrak{p}}^*(t),\ \mathfrak{p}^*(t-\varsigma),\ \dot{\mathfrak{p}}^*(t-\sigma),\ \mathfrak{p}^*(t_l), \mathfrak{p}^*(t_{l+1})\big]^*.$$

From (10)–(19) and noting (8), we can get that

$$
\begin{aligned}
\dot{W}(t) &\leqslant \vartheta^*(t)\big[\varOmega + (t_{l+1}-t)\varGamma + (t-t_l)\varPi\big]\vartheta(t) \\
&= \vartheta^*(t)\bigg[\frac{t_{l+1}-t}{h_l}(\varOmega + h_l\varGamma) + \frac{t-t_l}{h_l}(\varOmega + h_l\varPi)\bigg]\vartheta(t).
\end{aligned}
$$

Let $\rho \in [0,1]$, then $h_l \in [h_m, h_M]$ can be rewritten as $h_l = \rho h_m + (1-\rho)h_M$. Thus

$$
\begin{aligned}
\dot{W}(t) \leqslant \vartheta^*(t)\bigg\{&\frac{t_{l+1}-t}{h_l}\big[\rho(\varOmega + h_m\varGamma) + (1-\rho)(\varOmega + h_M\varGamma)\big] \\
&+ \frac{t-t_l}{h_l}\big[\rho(\varOmega + h_m\varPi) + (1-\rho)(\varOmega + h_M\varPi)\big]\bigg\}\vartheta(t). \qquad (20)
\end{aligned}
$$

By using condition (7), we have from (20) that

$$\dot{W}(t) \leqslant 0, \quad t \geqslant 0,$$

which implies that model (4) is globally stable, and the gain matrix of (3) is

$$K = Q^{-1}N.$$

The proof is finished. □

Remark 1. When neutral delay is not considered, model (1) degenerates into the following model:

$$\dot{\mathfrak{p}}(t) = -D\mathfrak{p}(t) + Af\big(\mathfrak{p}(t)\big) + Bg\big(\mathfrak{p}(t-\varsigma)\big) + u(t), \quad t \geqslant 0. \tag{21}$$

At this time, the controller is

$$u(t) = K\mathfrak{p}(t_l), \quad t_l \leqslant t < t_{l+1},\ w \in \mathbb{N}^+. \tag{22}$$

From Theorem 1 we have the following result by taking $P_3 = 0$ in (9) and $C = 0$ in (19).

Corollary 1. *Suppose there exist four positive matrices P_1, P_2, P_4, P_5, two diagonal positive matrices R_1, R_2, and fifteen matrices $M, S, Q, \mathfrak{X}_{ij}, \mathfrak{L}_{ij}, \mathfrak{Z}_{ij}, N \in \mathbb{Q}^{n\times n}$ $(i, j = 1, 2, \mathfrak{Z}_{21} = \mathfrak{Z}_{12}^*)$ such that*

$$\begin{bmatrix} P_4 & S \\ S^* & P_5 \end{bmatrix} > 0,$$

$$\varOmega + h_m\varGamma < 0, \qquad \varOmega + h_m\varPi < 0,$$
$$\varOmega + h_M\varGamma < 0, \qquad \varOmega + h_M\varPi < 0,$$

where

$$\varOmega = (\varOmega_{ij})_{7\times 7}, \qquad \varGamma = (\varGamma_{ij})_{7\times 7}, \qquad \varPi = (\varPi_{ij})_{7\times 7}$$

in which

$$\begin{aligned}
\varOmega_{11} &= P_2 - P_4 + S + S^* - P_5 - \mathfrak{L}_{11} - \mathfrak{L}_{11}^* - \mathfrak{L}_{12} - \mathfrak{L}_{12}^* + \mathfrak{L}_{21} \\
&\quad + \mathfrak{L}_{21}^* + \mathfrak{L}_{22} + \mathfrak{L}_{22}^* + \mathcal{F}R_1\mathcal{F}, \\
\varOmega_{14} &= P_1 - M - M^* - DQ^*, \\
\varOmega_{17} &= P_4 - S^* - \mathfrak{X}_{11}^* - \mathfrak{X}_{12}^* + \mathfrak{L}_{11} + \mathfrak{L}_{11}^* + \mathfrak{L}_{12}^* - \mathfrak{L}_{21} - DQ^*, \\
\varOmega_{18} &= -S + P_5 - \mathfrak{X}_{21}^* - \mathfrak{X}_{22}^* + \mathfrak{L}_{12} - \mathfrak{L}_{21}^* - \mathfrak{L}_{22} - \mathfrak{L}_{22}^*, \\
\varOmega_{22} &= -R_1, \qquad \varOmega_{24} = A^*Q^*, \qquad \varOmega_{27} = A^*Q^*, \qquad \varOmega_{33} = -R_2, \\
\varOmega_{34} &= B^*Q^*, \qquad \varOmega_{37} = B^*Q^*, \qquad \varOmega_{44} = -Q - Q^*, \\
\varOmega_{47} &= M^* + N - Q^*, \qquad \varOmega_{48} = M, \qquad \varOmega_{55} = -P_2 + \mathcal{G}R_2\mathcal{G}, \\
\varOmega_{77} &= -P_4 + \mathfrak{X}_{11} + \mathfrak{X}_{11}^* - \mathfrak{L}_{11} - \mathfrak{L}_{11}^* + N + N^*, \\
\varOmega_{78} &= S + \mathfrak{X}_{12} + \mathfrak{X}_{21}^* - \mathfrak{L}_{12} + \mathfrak{L}_{21}^*, \\
\varOmega_{88} &= -P_5 + \mathfrak{X}_{22} + \mathfrak{X}_{22}^* + \mathfrak{L}_{22} + \mathfrak{L}_{22}^*, \\
\varGamma_{14} &= \mathfrak{L}_{11} + \mathfrak{L}_{11}^* - \mathfrak{L}_{21} - \mathfrak{L}_{21}^*, \qquad \varGamma_{44} = h_M P_4,
\end{aligned}$$

$$\begin{aligned}
&\Gamma_{47} = \mathfrak{X}_{11}^{*} - \mathfrak{L}_{11} - \mathfrak{L}_{11}^{*} + \mathfrak{L}_{21}, \qquad \Gamma_{48} = \mathfrak{X}_{21}^{*} + \mathfrak{L}_{21}^{*},\\
&\Gamma_{77} = \mathfrak{Z}_{11}, \qquad \Gamma_{78} = \mathfrak{Z}_{12}, \qquad \Gamma_{88} = \mathfrak{Z}_{22};\\
&\Pi_{14} = -\mathfrak{L}_{12} - \mathfrak{L}_{12}^{*} + \mathfrak{L}_{22} + \mathfrak{L}_{22}^{*}, \qquad \Pi_{44} = h_M P_5,\\
&\Pi_{47} = -\mathfrak{X}_{12}^{*} + \mathfrak{L}_{12}^{*}, \qquad \Pi_{48} = -\mathfrak{X}_{22}^{*} + \mathfrak{L}_{12} - \mathfrak{L}_{22} - \mathfrak{L}_{22}^{*},\\
&\Pi_{77} = -\mathfrak{Z}_{11}, \qquad \Pi_{78} = -\mathfrak{Z}_{12}, \qquad \Pi_{88} = -\mathfrak{Z}_{22}.
\end{aligned}$$

Then model (21) *is globally stable under assumptions* (H1) *and* (H2)*, and the gain matrix of* (22) *is*

$$K = Q^{-1}N.$$

Remark 2. The two-sided looped-functional method is first employed in this paper to investigate the stability of QVNNs with sampled-data control. It should be pointed out that the positivity of the constructed energy function $W(t) = V(t) + v(t)$ is not required in this paper. The positivity of Lyapunov functional $V(t)$ and the conditions $v(t_l) = v(t_{l+1}) = 0$ are only required.

Remark 3. The obtained criteria in this paper are in the form of LMIs, and the YALMIP toolbox in Matlab can be applied to calculate it. The numbers of decision variables in Theorem 1 and Corollary 1 are $17.5n^2 + 4.5n$ and $17n^2 + 4n$, respectively.

Remark 4. It needs to be emphasized the acquired criteria holds for both real-valued and complex-valued NNs.

4 Example

For model (21), we take into account the following parameters:

$$\begin{aligned}
&D = \begin{bmatrix} 0.035 & 0 \\ 0 & 0.042 \end{bmatrix},\\
&A = \begin{bmatrix} 0.295+0.0141i+0.055j+0.0098k & 0.0072+0.001i-0.0095j+0.0048k \\ 0.0086+0.001i+0.005j+0.0039k & 0.395+0.0097i+0.017j+0.026k \end{bmatrix},\\
&B = \begin{bmatrix} -0.808+0.0058i+0.0012j+0.0021k & 0.026+0.003i+0.0054j-0.0038k \\ -0.037+0.022i+0.0035j+0.0189k & -0.817+0.016i-0.0146j+0.0084k \end{bmatrix},\\
&\varsigma = 2, \qquad \begin{aligned} &f_1(\mathfrak{p}(t)) = f_2(\mathfrak{p}(t)) = 0.2\tanh(\mathfrak{p}(t)),\\ &g_1(\mathfrak{p}(t)) = g_2(\mathfrak{p}(t)) = 0.6\tanh(\mathfrak{p}(t)). \end{aligned}
\end{aligned}$$

When $u(t) = 0$, the state trajectories of model (21) with initial conditions

$$\mathfrak{p}_1(s) = 1.24 + 0.26i - 0.35j - 0.4k$$

and

$$\mathfrak{p}_2(s) = -2.37 - 0.24i + 0.88j + 0.39k,$$

$s \in [-2, 0]$, are shown in Fig. 1, which are stable.

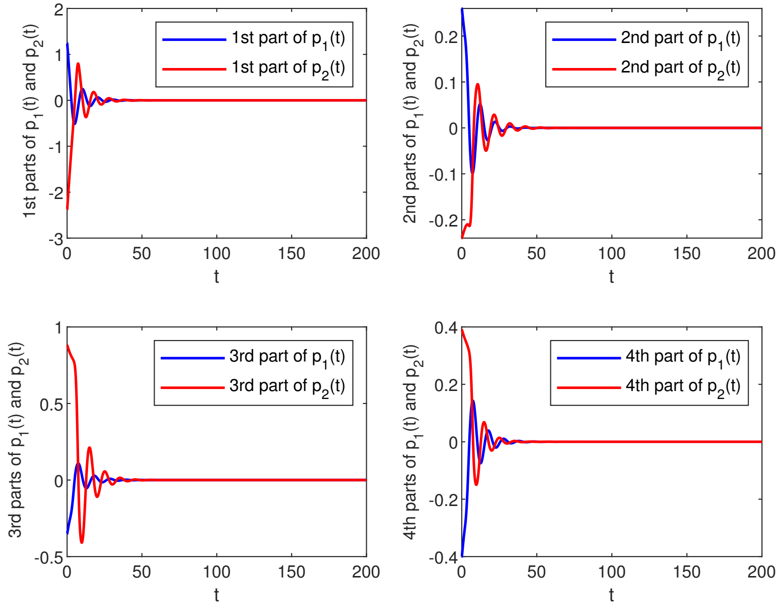

Figure 1. The state trajectories of model (21) when $u(t) = 0$.

For model (1), based on the parameters of model (21), we add the following parameters:

$$\sigma = 2,$$

$$C = \begin{bmatrix} -0.2055+0.006i+0.0032j+0.001k & 0.0079-0.0014i+0.0031j+0.001k \\ 0.0096+0.0005i+0.0093j+0.001k & -0.2598+0.005i+0.0076j+0.001k \end{bmatrix}.$$

When $u(t) = 0$, the state trajectories of model (1) with initial conditions

$$\mathfrak{p}_1(s) = 1.24 + 0.26i - 0.35j - 0.4k$$

and

$$\mathfrak{p}_2(s) = -2.37 - 0.24i + 0.88j + 0.39k,$$

$s \in [-2, 0]$, are shown in Fig. 2, which are chaotic. This means that neutral delay has a significant impact on the stability of quaternion-valued neural networks.

We consider the sampling instants as

$$t_l = \frac{l^2}{1+l}, \quad l \in \mathbb{N}^+,$$

and the controller is

$$u(t) = K\mathfrak{p}(t_l), \quad t_l \leqslant t < t_{l+1},\ l \in \mathbb{N}^+.$$

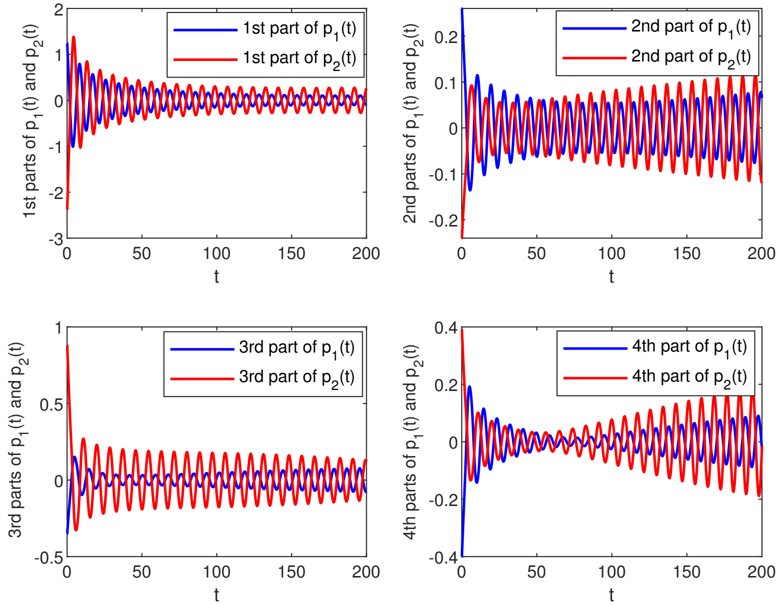

Figure 2. The state trajectories of model (1) when $u(t) = 0$.

Obviously, assumptions (H1) and (H2) are satisfied with

$$\mathcal{F} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad \mathcal{G} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}$$

and $h_m = 0.5$, $h_M = 1$. The feasible solutions of (6)–(7) are shown as follows:

$$P_1 = \begin{bmatrix} 178.3 & 1.163-2.722i+1.561j-1.719k \\ 1.163+2.722i-1.561j+1.719k & 124.3 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 145.4 & 0.8455-0.6711i-0.0044j-0.0192k \\ 0.8455+0.6711i+0.0044j+0.0192k & 107.4 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 55.58 & -1.26-0.9598i+1.732j-1.078k \\ -1.26+0.9598i-1.732j+1.078k & 47.43 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 183.7 & 5.635-5.36i+0.9132j-1.33k \\ 5.635+5.36i-0.9132j+1.33k & 81.11 \end{bmatrix},$$

$$P_5 = \begin{bmatrix} 106.3 & 5.972-5.056i+0.3751j-0.4483k \\ 5.972+5.056i-0.3751j+0.4483k & 20.21 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 343.6748 & 0 \\ 0 & 386.1514 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} 368.8447 & 0 \\ 0 & 294.7742 \end{bmatrix},$$

$$M = \begin{bmatrix} 93.86-0.0427i+0.2664j-0.1082k & 1.63-1.907i+0.8891j-0.9145k \\ 1.313+2.209i-0.8698j+0.2246k & 55.24+0.056i+0.1063j+0.0305k \end{bmatrix},$$

$$S = \begin{bmatrix} 25.61+0.0276i+0.0881j-0.089k & 3.889-3.823i+0.9876j-0.1352k \\ 4.054+3.758i-0.5478j-0.001k & -28.95-0.0151i+0.0363j+0.0052k \end{bmatrix},$$

$$Q = \begin{bmatrix} 299.8 & -0.7233-2.442i+2.495j-4.814k \\ -0.7233+2.442i-2.495j+4.814k & 239.4 \end{bmatrix},$$

$$\mathfrak{X}_{11} = \begin{bmatrix} 110.9+0.0229i-0.2893j+0.1034k & 3.605-3.216i-0.0875j-1.359k \\ 3.523+3.414i-0.1112j+2.334k & 38.57-0.0271i-0.1129j+0.0096k \end{bmatrix},$$

$$\mathfrak{X}_{12} = \begin{bmatrix} 41.64-0.0447i-0.0351j+0.0966k & -3.318+3.014i-0.4697j-0.4561k \\ -3.507-2.321i-0.0048j+0.2166k & 81.69+0.0459i+0.0092j+0.0037k \end{bmatrix},$$

$$\mathfrak{X}_{21} = \begin{bmatrix} 45.87+0.0092i+0.251j-0.0582k & -2.598+2.144i-0.2568j-3.93k \\ -2.217-2.574i+0.5551j+2.741k & 77.5-0.0089i+0.1319j+0.011k \end{bmatrix},$$

$$\mathfrak{X}_{22} = \begin{bmatrix} -70.81-0.0274i-0.1262j+0.0166k & 2.958-2.17i+0.4945j+3.873k \\ 2.667+1.862i-0.4787j-3.444k & -98.93+0.0095i-0.049j-0.0002k \end{bmatrix},$$

$$\mathfrak{L}_{11} = \begin{bmatrix} -2.813+0.002i-0.1908j-0.0026k & -0.7884+0.0047i-0.1033j-0.0007k \\ -0.7806+0.013i-0.2684j+0.0008k & -4.364+0.0023i+0.527j+0.0029k \end{bmatrix},$$

$$\mathfrak{L}_{12} = \begin{bmatrix} 13.26+0.0087i-0.0741j-0.0378k & -0.4019+0.9477i-0.7015j-0.7193k \\ -0.1955-0.4023i+0.608j+1.543k & 25.56-0.0068i-0.0612j+0.0231k \end{bmatrix},$$

$$\mathfrak{L}_{21} = \begin{bmatrix} 15.41-0.0026i-0.1336j+0.1255k & -1.521+0.7683i-0.1487j-1.361k \\ -1.825-0.4614i+0.0001j+1.143k & 20.62+0.0108i+0.0058j+0.0136k \end{bmatrix},$$

$$\mathfrak{L}_{22} = \begin{bmatrix} 1.279-0.0041i-0.6159j+0.0087k & 0.544+0.0027i-0.7223j+0.0042k \\ 0.8657+0.0008i+0.3925j+0.0004k & 4.727+0.0003i-0.8124j+0.0009k \end{bmatrix},$$

$$\mathfrak{Z}_{11} = \begin{bmatrix} 120.6 & 0.595-0.6618i-0.06153j-1.983k \\ 0.595+0.6618i+0.06153j+1.983k & 91.91 \end{bmatrix},$$

$$\mathfrak{Z}_{12} = \begin{bmatrix} -44.64+0.0292i+0.0503j-0.0035k & -0.1612+0.1288i-0.0167j+0.4577k \\ 0.0083-0.3308i+0.1581j-0.6561k & -35.59-0.0238i+0.0194j-0.015k \end{bmatrix},$$

$$\mathfrak{Z}_{22} = \begin{bmatrix} 8.218 & 0.5521-0.41i+0.00589j-0.00493k \\ 0.5521+0.41i-0.00589j+0.00493k & 1.053 \end{bmatrix},$$

$$N = \begin{bmatrix} -233.6-0.0174i+0.1317j-0.1239k & -0.0163+1.054i-0.5862j+4.923k \\ -0.0707-1.038i+0.6608j-4.669k & -187.6+0.0033i-0.0163j-0.0247k \end{bmatrix}.$$

According to Theorem 1, this model is globally stable, and the gain matrix is

$$K = \begin{bmatrix} -0.7791+0.0001i+0.0005j-0.0005k & -0.0019-0.0029i+0.0046j+0.0038k \\ -0.0027+0.0036i-0.0054j-0.0038k & -0.7836+0.0002i+0.00002j-0.0001k \end{bmatrix}.$$

The state trajectories of model (1) with the initial conditions

$$\mathfrak{p}_1(s) = 1.24 + 0.26i - 0.35j - 0.4k$$

and

$$\mathfrak{p}_2(s) = -2.37 - 0.24i + 0.88j + 0.39k,$$

$s \in [-2, 0]$, are shown in Fig. 3, which is stable.

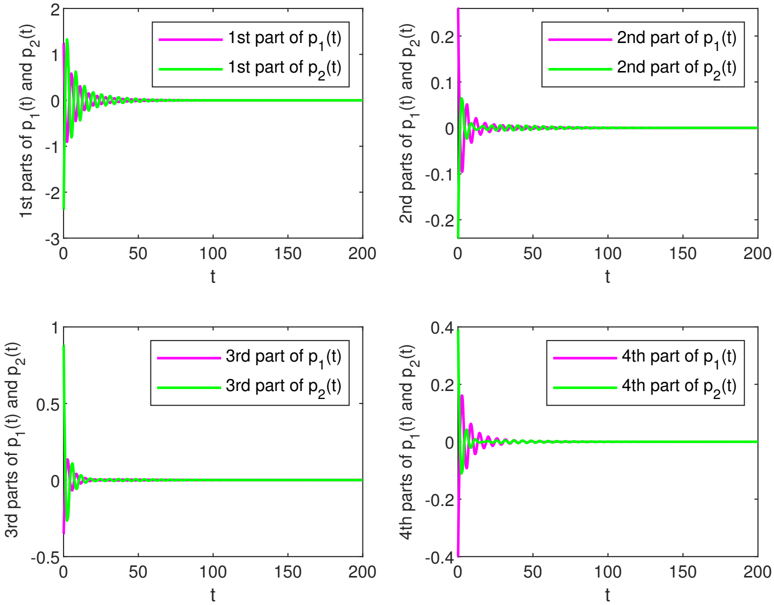

Figure 3. The state trajectories of model (1) when $u(t) = K\mathfrak{p}(t_l)$.

5 Conclusions

The stability of QVNTNNs has been investigated by designing sampled-data controller. Based on the two-sided functional method, a main stability criterion of the considered NNs has been derived in the form of LMI. A numerical example was provided to demonstrate the effectiveness of the obtained result. It should be noted that the obtained results in this paper are valid for real-valued NNs and complex-valued NNs.

We would like to point out that it is possible to generalize our main results to more complex systems, such as quaternion-valued neural networks with time-varying delays [10], stochastic quaternion-valued neural networks [27], and coupled quaternion-valued neural networks [20]. The corresponding results will be carried out in the near future.

Author contributions. All Authors (Q.S., Q.W., and Y.L.) have contributed as follows: methodology, Q.S.; formal analysis, Q.S, and Y.L.; software, Q.W.; validation, Q.S.; writing – original draft preparation, Q.S. and Q.W.; writing – review and editing, Q.S., Q.W., and Y.L. All authors have read and approved the published version of the manuscript.

Conflicts of interest. The authors declare no conflicts of interest.

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