Nonlinear Analysis: Modelling and Control, Vol. 30, No. 2, 157–175
https://doi.org/10.15388/namc.2025.30.38319

Analysis of exponential stability and L_1-gain performance of positive switched impulsive systems with all unstable subsystems*

Xiukun Zhang, Yuangong Sun[1], Xingao Zhu

School of Mathematical Sciences, University of Jinan,
Jinan 250022, Shandong, China

zhangxiukun1007@163.com; sunyuangong@163.com
zhuxingao2021@163.com

Received: April 29, 2024 / **Revised:** November 21, 2024 / **Published online:** January 2, 2025

Abstract. This article investigates the exponential stability and L_1-gain performance of time-varying positive switched impulsive systems even when all modes are unstable. By employing the discretized switched copositive Lyapunov function approach and the analytical method developed for positive systems, we derive a sufficient condition to ensure the exponential stability for such systems. An algorithm for computing the stability region of admissible dwell time is also introduced. Furthermore, building on this stability result, the L_1-gain performance of positive switched impulsive systems is further studied. Finally, numerical examples are presented to demonstrate the validity of our results.

Keywords: positive system, switched system, exponential stability, L_1-gain, mode-dependent interval dwell time.

1 Introduction

Switched systems, a specialized category of dynamical system, consist of a collection of continuous or discrete dynamic subsystems that switch among themselves based on a switching signal [12]. A positive switched system is a subset of switched systems characterized by nonnegative state variables and outputs, provided that both the initial conditions and inputs are nonnegative [4]. In recent years, scholarly interest in positive switched systems has surged, driven by their wide application in areas such as chemical reaction networks [1], power electronics [10], congestion control [22], formation flying [39], and so on. The theoretical analysis of positive switched system, especially, the stability of positive switched system, has been developed rapidly [14, 21, 23, 36].

*This research was supported by the Taishan Scholar Foundation of Shandong Province (ts20190938), Natural Science Foundation of Shandong Province (ZR2023MF057), and National Natural Science Foundation of China (61873110).

[1]Corresponding author.

Impulsive systems, characterized by instantaneous discrete-time state jumps, have experienced significant advancements over the past few decades, particularly, in the area of stability problems [6,11,15]. It is worth noting that the impulsive phenomenon does not always adversely impact system stability, rather, it can sometimes be exploited to achieve stability. The impulsive system has also garnered considerable research interest due to its wide applications in various fields, including biological systems [25], network control systems [7], robotics [3], and multiagent systems [30].

The stability of switched systems, a fundamental property of dynamic systems, has garnered significant attention. To analyze the stability of switched systems with given switching signals, the dwell-time method and the multiple Lyapunov functions approach are commonly used. Numerous classical works have focused on the stability of switched systems when all modes are stable [19, 27, 32, 38]. Additionally, there has been a wealth of results dedicated to the stability analysis of systems that incorporate both stable and unstable subsystems [5, 18, 26, 31, 33]. The primary concept is that the decay of states within stable subsystems counteracts the growth of the Lyapunov function in unstable subsystems, thus maintaining overall system stability. Consequently, in most existing works, it is often assumed that at least one subsystem within the switched system is asymptotically stable. Designing appropriate switching signals to guarantee stability becomes more challenging when all modes are unstable. The application of a discretized Lyapunov function in [8, 16, 28, 29, 34] provides stability conditions for switched systems with all unstable subsystems. Among these works, sufficient criteria were introduced in [28] to guarantee the asymptotic stability of the positive switched systems even when all subsystems are unstable. Furthermore, by constructing a new discretized copositive Lyapunov–Krasovskii function, stability conditions for positive switched linear delay systems were provided in [34]. With the assistance of a discretized Lyapunov function and analytical methods in positive systems, exponential stability criteria were derived for the positive switched impulsive systems in [8].

It is noteworthy that L_1-gain [28,37], L_2-gain [13,35], and H_∞ performance [20] are typically used to analyze input-output performance. Some common performance indicators, such as the H_∞ norm, which relies on the L_2 signal space [9], fail to naturally capture specific characteristics of actual physical systems. Moreover, the 1-norm represents the sum of the values of the components [2]. Consequently, the L_1-gain has been introduced and subsequently employed for input-output performance analysis in the study of positive systems, which has been proven to provide a more effective description. For instance, when measuring the population of species, we often use the 1-norm to determine the total number. Therefore, it is necessary to use L_1-gain to evaluate input-output performance of positive switched systems, and numerous significant results have been developed in existing research [17, 24]. Given that prior works have not explored the L_1-gain performance analysis of positive switched impulsive systems, further investigation is warranted. Our research encounters greater challenges due to the mixed characteristics exhibited by the positive switched impulsive systems.

Compared to previous studies [8,28], this article offers three contributions. Firstly, we introduce a novel analytical approach to establish less restrictive sufficient conditions for the exponential stability of time-varying positive switched impulsive system. Secondly, based on stability theory, we propose an algorithm to compute the stability region of admissible dwell time. For specific details, please refer to Algorithm 1. Finally, we utilize a different analytical approach than that used in [28] to address the L_1-gain performance for positive switched impulsive system.

The structure of this article is as follows. The problem formulation is introduced in Section 1. In Section 3, the new criterion for the exponential stability of positive switched impulsive system is presented. Besides, an algorithm for determining the stability region is proposed. The L_1-gain performance for the positive switched impulsive system is further analyzed in Section 4. In Section 5, numerical examples demonstrate the effectiveness of our main results. Section 6 summarizes this article.

2 Problem formulation

Notations. In the following discussion, $\mathbb{N}$ represents the set of natural numbers. $\mathbb{R}^n$ ($\mathbb{R}^n_+$) denotes the set of n-dimensional (positive) real vectors, and $\mathbb{R}^{n\times n}$ is the space of $n \times n$ real matrices. For a vector $x \in \mathbb{R}^n$, the notation x_l refers to the lth component of x, the notation $x \prec 0$ means $x_l < 0$ for $1 \leqslant l \leqslant n$, and its 1-norm is defined as $\|x\| = \Sigma_{k=1}^n |x_k|$. Consider a piecewise continuous function $\rho(t)$ defined on $[0, \infty)$. $\rho(t^+)$, $\rho(t^-)$ are the right and left limits of $\rho(t)$, respectively, and $\rho(t) = \rho(t^+)$ at the discontinuous point. Matrix A is called Metzler, whose off-diagonal entries are nonnegative. Say a matrix $B \in \mathbb{R}^{n\times n}$ is nonnegative if all its elements are nonnegative.

Let us consider time-varying switched impulsive system as follows:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}(t)x(t) + B_{\sigma(t)}(t)\omega(t), \quad t \geqslant 0,\ t \neq t_k, \\ x(t^+) &= F_{\sigma(t^+)}x(t^-), \quad t = t_k, \\ y(t) &= C_{\sigma(t)}(t)x(t) + D_{\sigma(t)}(t)\omega(t), \end{aligned} \tag{1}$$

where $x(t)$, $\omega(t)$, and $y(t) \in \mathbb{R}^n$ represent the state vector, the disturbance input, and the system output, respectively. $\bar{\mathcal{M}} = \{1, 2, \dots, m\}$ denotes the set of subsystem indices with m being the total number of subsystems. The switching signal $\sigma(t) : [0, \infty) \to \bar{\mathcal{M}}$ is a right-continuous function over time. The matrices $A_i(t), B_i(t), C_i(t), D_i(t) \in \mathbb{R}^{n\times n}$, $i \in \bar{\mathcal{M}}$, represent the time-varying system matrices of the ith subsystem. Additionally, $F_i \in \mathbb{R}^{n\times n}$, $i \in \bar{\mathcal{M}}$, is a impulse matrix. For a switching sequence $0 < t_1 < t_2 < \cdots < t_k < \cdots$ with $k \in \mathbb{N}$, t_k is the switching instant with the dwell time $\tau_k = t_{k+1} - t_k$, $x(t_k) = x(t_k^+)$, and $\lim_{k\to\infty} t_k = +\infty$.

It is known that if $A_i(t)$ is a Metzler matrix and $B_i(t), C_i(t), D_i(t), F_i \in \mathbb{R}^{n\times n}$ are nonnegative and $\omega(t) \succcurlyeq 0$ for each $i \in \bar{\mathcal{M}}$, $t \geqslant 0$, then system (1) is positive.

Definition 1. The zero solution of system (1) is globally uniformly exponentially stable (GUES) if there exist two positive constants b and c such that $\|x(t)\| \leqslant b\|x(0)\|\mathrm{e}^{-ct}$, $t \geqslant 0$, for all initial conditions, where c is referred to as the exponential convergence rate.

Definition 2. A system (1) is said to be GUES with an L_1-gain of ε for a given switching signal and a constant $\varepsilon > 0$ if it satisfies the following conditions:

(i) System (1) exhibits GUES in the absence of the input variable $\omega(t)$;

(ii) When the initial conditions are set to zero, the following inequality holds for any nonzero $\omega(t) \in L_1[0,\infty)$:

$$\int_{t_0}^{\infty} \|y(t)\| \,\mathrm{d}t \leqslant \varepsilon \int_{t_0}^{\infty} \|\omega(t)\| \,\mathrm{d}t.$$

To further investigate the exponential stability of the time-varying switched impulsive system under the switching signal $\sigma(t)$, we proceed under the following assumptions.

Assumption 1. $A_i(t) \in \mathbb{R}^{n\times n}$ is Metzler, and there exist a class of constant matrices $\bar{A}_i \in \mathbb{R}^{n\times n}$ such that $A_i(t) \preccurlyeq \bar{A}_i$, $i \in \bar{\mathcal{M}}$, $t \geqslant 0$.

Assumption 2. $B_i(t), C_i(t), D_i(t), F_i \in \mathbb{R}^{n\times n}$ are nonnegative, and there exist a series of constant matrices $\bar{B}_i, \bar{C}_i, \bar{D}_i \in \mathbb{R}^{n\times n}$ such that $B_i(t) \preccurlyeq \bar{B}_i$, $C_i(t) \preccurlyeq \bar{C}_i$, $D_i(t) \preccurlyeq \bar{D}_i$, $i \in \bar{\mathcal{M}}$, $t \geqslant 0$.

Assumption 3. $\omega(t) \succcurlyeq 0$ for $t \geqslant 0$.

Under Assumptions 1–3, it is evident that the time-varying switched impulsive system (1) is positive.

Assumption 4. There exist positive constants $\tau_1 = \inf_{k\in\mathbb{N}} \tau_k$ and $\tau_2 = \sup_{k\in\mathbb{N}} \tau_k$. The switching signal that meets such conditions is termed interval dwell time (IDT).

Assumption 5. There exist a series of positive constants $\tau_{i1} = \inf_{k\in\mathbb{N}}\{\tau_k\colon \sigma(t_k) = i, i \in \bar{\mathcal{M}}\}$ and $\tau_{i2} = \sup_{k\in\mathbb{N}}\{\tau_k\colon \sigma(t_k) = i, i \in \bar{\mathcal{M}}\}$. The switching signal that meets such conditions is termed mode-dependent interval dwell time (MDIDT).

When $t = t_k$, the $\sigma(t_k)$th subsystem is active. Therefore, in this article, the time interval during which the $\sigma(t_k)$th subsystem operates is designated as $R_k = [t_k, t_{k+1})$. Subsequently, we divide the interval R_k into L segments averagely with length $d_k = (t_{k+1} - t_k)/L$, and each segment is marked as $R_{kq} = [t_k + \theta_{kq},\, t_k + \theta_{kq+1})$, where $\theta_{kq} = qd_k = q(t_{k+1} - t_k)/L$, $q = 0, 1, \dots, L-1$. It is obvious that $\bigcup_{n=0}^{L-1} R_{kn} = R_k$ and $R_{kn} \cap R_{km} = \emptyset$ when $n \neq m$.

For the switching moment t_k, several piecewise linear functions are introduced as follows:

$$\kappa(t) = \frac{1}{t_{k+1} - t_k}, \quad t \in R_k,\ k \in \mathbb{N}, \tag{2}$$

$$\overline{\kappa}(t) = \frac{t - t_k - \theta_{kq}}{d_k}, \quad \widetilde{\kappa}(t) = \frac{t_k + \theta_{kq+1} - t}{d_k}, \quad t \in R_{kq},\ k \in \mathbb{N}. \tag{3}$$

For (2), we employ the convex combination technique. For ith subsystem, it is established that there exists a function $\overline{\alpha}(t) \in [0,1]$, $t \in R_k$, $k \in \mathbb{N}$, such that

$$\kappa(t) = \frac{\widetilde{\alpha}(t)}{\tau_{i1}} + \frac{\overline{\alpha}(t)}{\tau_{i2}},$$

where $\widetilde{\alpha}(t) = 1 - \overline{\alpha}(t)$.

For convenience, we set $\sigma(t) = i \in \bar{\mathcal{M}}$ for $t \in R_k$, $k \in \mathbb{N}$, in the following analysis. For each switching interval R_{kq}, a set of positive constant vectors $\varsigma_{iq+1} \in \mathbb{R}^n_+$ and $\varsigma_{iq} \in \mathbb{R}^n_+$ for $i \in \bar{\mathcal{M}}$, $q = 0, 1, \dots, L$, are selected. With the aid of (3), we construct the following continuous vector function, which depends on the switching signal $\sigma(t)$:

$$\varsigma_{\sigma(t)}(t) = \overline{\kappa}(t)\varsigma_{iq+1} + \widetilde{\kappa}(t)\varsigma_{iq}. \tag{4}$$

Taking the derivative of (4) with respect to t, we obtain

$$\dot{\varsigma}_i(t) = \frac{\varsigma_{iq+1} - \varsigma_{iq}}{d_k} = L\biggl(\frac{\varsigma_{iq+1} - \varsigma_{iq}}{t_{k+1} - t_k}\biggr), \quad t \in R_k,\ k \in \mathbb{N}. \tag{5}$$

3 Stability analysis

In this section, a discretized switched time-varying copositive Lyapunov function is defined as

$$V_{\sigma(t)}(t, x) = \varsigma^{\mathrm{T}}_{\sigma(t)}(t)x(t), \quad t \geqslant 0,\ x \in \mathbb{R}^n,\ x \succcurlyeq 0.$$

Consider system (1) with $\omega(t) = 0$, $t \geqslant 0$, as follows:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}(t)x(t), \quad t \geqslant 0,\ t \neq t_k, \\ x(t^+) &= F_{\sigma(t^+)}x(t^-), \quad t = t_k. \end{aligned} \tag{6}$$

Next, we present new criterion on exponential stability for the positive switched impulsive system (6).

Theorem 1. *Suppose that Assumptions* 1 *and* 2 *hold true. Given a positive integer* L *and constants* $\eta > 0$, $0 < \rho < 1$, *if there exist a group of constant vectors* $\varsigma_{iq} \in \mathbb{R}^n_+$, $i \in \bar{\mathcal{M}}$, $q = 0, 1, \dots, L$, *such that for all* $q = 0, 1, \dots, L-1$, *we have*

$$\varsigma^{\mathrm{T}}_{iq}\bar{A}_i + \frac{L}{\tau_{i2}}\bigl(\varsigma^{\mathrm{T}}_{iq+1} - \varsigma^{\mathrm{T}}_{iq}\bigr) - \eta\varsigma^{\mathrm{T}}_{iq} \prec 0, \tag{7}$$

$$\varsigma^{\mathrm{T}}_{iq}\bar{A}_i + \frac{L}{\tau_{i1}}\bigl(\varsigma^{\mathrm{T}}_{iq+1} - \varsigma^{\mathrm{T}}_{iq}\bigr) - \eta\varsigma^{\mathrm{T}}_{iq} \prec 0, \tag{8}$$

$$\varsigma^{\mathrm{T}}_{iq+1}\bar{A}_i + \frac{L}{\tau_{i2}}\bigl(\varsigma^{\mathrm{T}}_{iq+1} - \varsigma^{\mathrm{T}}_{iq}\bigr) - \eta\varsigma^{\mathrm{T}}_{iq+1} \prec 0, \tag{9}$$

$$\varsigma^{\mathrm{T}}_{iq+1}\bar{A}_i + \frac{L}{\tau_{i1}}\bigl(\varsigma^{\mathrm{T}}_{iq+1} - \varsigma^{\mathrm{T}}_{iq}\bigr) - \eta\varsigma^{\mathrm{T}}_{iq+1} \prec 0, \tag{10}$$

$$\varsigma^{\mathrm{T}}_{i0}F_i - \rho\varsigma^{\mathrm{T}}_{jL} \preccurlyeq 0, \quad i, j \in \bar{\mathcal{M}},\ i \neq j, \tag{11}$$

$$\tau_2\eta + \ln\rho < 0, \tag{12}$$

where $\tau_2 = \max_{i\in\bar{\mathcal{M}}} \tau_{i2}$, *then system* (6) *is GUES under any MDIDT switching signal satisfying Assumption* 5.

Proof. Since Assumptions 1 and 2 hold, system (6) is positive. Given $x(0) \succcurlyeq 0$, the trajectory of system (6) will remain nonnegative for any $t \geqslant 0$ under arbitrary switching. Letting $\sigma(t) = i$ for $t \in R_k$, $k \in \mathbb{N}$, we obtain

$$\begin{aligned}
&\dot{V}_i(t, x(t)) - \eta V_i\bigl(t, x(t)\bigr) \\
&\quad = \dot{\varsigma}_i^{\mathrm{T}}(t)x(t) + \varsigma_i^{\mathrm{T}}(t)\dot{x}(t) - \eta\varsigma_i^{\mathrm{T}}(t)x(t) \\
&\quad = \dot{\varsigma}_i^{\mathrm{T}}(t)x(t) + \varsigma_i^{\mathrm{T}}(t)A_i(t)x(t) - \eta\varsigma_i^{\mathrm{T}}(t)x(t) \\
&\quad \leqslant \dot{\varsigma}_i^{\mathrm{T}}(t)x(t) + \varsigma_i^{\mathrm{T}}(t)\bar{A}_i x(t) - \eta\varsigma_i^{\mathrm{T}}(t)x(t) \\
&\quad = \bigl(\dot{\varsigma}_i^{\mathrm{T}}(t) + \varsigma_i^{\mathrm{T}}(t)\bar{A}_i - \eta\varsigma_i^{\mathrm{T}}(t)\bigr)x(t) \\
&\quad = \left(\frac{\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}}{d_k} + \bigl(\overline{\kappa}(t)\varsigma_{iq+1}^{\mathrm{T}} + \widetilde{\kappa}(t)\varsigma_{iq}^{\mathrm{T}}\bigr)\bar{A}_i - \eta\bigl(\overline{\kappa}(t)\varsigma_{iq+1}^{\mathrm{T}} + \widetilde{\kappa}(t)\varsigma_{iq}^{\mathrm{T}}\bigr)\right)x(t) \\
&\quad = \left(\overline{\kappa}(t)\overline{\alpha}(t)\left(\varsigma_{iq+1}^{\mathrm{T}}\bar{A}_i + \frac{L}{\tau_{i2}}\bigl(\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}\bigr) - \eta\varsigma_{iq+1}^{\mathrm{T}}\right)\right. \\
&\qquad + \overline{\kappa}(t)\widetilde{\alpha}(t)\left(\varsigma_{iq+1}^{\mathrm{T}}\bar{A}_i + \frac{L}{\tau_{i1}}\bigl(\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}\bigr) - \eta\varsigma_{iq+1}^{\mathrm{T}}\right) \\
&\qquad + \widetilde{\kappa}(t)\overline{\alpha}(t)\left(\varsigma_{iq}^{\mathrm{T}}\bar{A}_i + \frac{L}{\tau_{i2}}\bigl(\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}\bigr) - \eta\varsigma_{iq}^{\mathrm{T}}\right) \\
&\qquad \left.+ \widetilde{\kappa}(t)\widetilde{\alpha}(t)\left(\varsigma_{iq}^{\mathrm{T}}\bar{A}_i + \frac{L}{\tau_{i1}}\bigl(\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}\bigr) - \eta\varsigma_{iq}^{\mathrm{T}}\right)\right)x(t).
\end{aligned} \tag{13}$$

By (7)–(10) and the fact $\bigcup_{n=0}^{L-1} R_{kn} = R_k$ and $R_{kn} \cap R_{km} = \emptyset$, we have

$$\dot{V}_i\bigl(t, x(t)\bigr) \leqslant \eta V_i\bigl(t, x(t)\bigr), \quad t \in R_k,\ k \in \mathbb{N}. \tag{14}$$

Let $\sigma(t_k^-) = j$. Due to (11), it is concluded that

$$\begin{aligned}
&V_i\bigl(t_k, x(t_k)\bigr) - \rho V_j\bigl(t_k^-, x(t_k^-)\bigr) \\
&\quad = \varsigma_i^{\mathrm{T}}(t_k)x(t_k) - \rho\varsigma_j^{\mathrm{T}}(t_k^-)x(t_k^-) = \bigl[\varsigma_i^{\mathrm{T}}(t_k)F_i - \rho\varsigma_j^{\mathrm{T}}(t_k^-)\bigr]x(t_k^-) \\
&\quad = \bigl(\varsigma_{i0}^{\mathrm{T}}F_i - \rho\varsigma_{jL}^{\mathrm{T}}\bigr)x(t_k^-) \leqslant 0.
\end{aligned} \tag{15}$$

Inequalities (14) and (15) lead to the fact that

$$\begin{aligned}
V_i\bigl(t, x(t)\bigr) &\leqslant \mathrm{e}^{\eta(t-t_k)}V_i\bigl(t_k, x(t_k)\bigr) \leqslant \rho\mathrm{e}^{\eta(t-t_k)}V_j\bigl(t_k^-, x(t_k^-)\bigr) \\
&\leqslant \rho\mathrm{e}^{\eta(t-t_{k-1})}V_j\bigl(t_{k-1}, x(t_{k-1})\bigr) \\
&\leqslant \rho^k\mathrm{e}^{\eta t}V_{\sigma(0)}\bigl(0, x(0)\bigr) \leqslant \rho^{t/\tau_2}\mathrm{e}^{\eta t}V_{\sigma(0)}\bigl(0, x(0)\bigr) \\
&= \mathrm{e}^{(\ln(\rho)/\tau_2+\eta)t}V_{\sigma(0)}\bigl(0, x(0)\bigr), \quad t \in R_k,\ k \in \mathbb{N},
\end{aligned} \tag{16}$$

where $\tau_2 = \max_{i\in\bar{\mathcal{M}}}\tau_{i2}$. We can get from (12) that

$$\frac{\ln\rho}{\tau_2} + \eta < 0.$$

On the basis of (5) and (6), we can conclude that there exists an n-dimensional vector $\lambda > 0$ ensuring $V_{\sigma(t)}(t,x) \geqslant \lambda^{\mathrm{T}} x(t)$ for $t \geqslant 0$. Therefore, inequality (16) implies that system (1) is GUES under the MDIDT switching signal satisfying Assumption 5. The proof of Theorem 1 is completed. $\square$

Remark 1. In Theorem 1, conditions (7)–(10) assess the evolution of subsystems between every two switching behaviors. Inequality (11) means that changes in energy of the switching behaviors with impulse are evaluated, indicating the impulse should not be too large.

Remark 2. In certain applications, the subsystem does not switch arbitrarily; it can only switch to specific subsystems. For this scenario, we define the allowable switching set $\bar{\mathcal{Q}}_i = \{j\colon \sigma(t^+) = j,\, \sigma(t^-) = i\}$ for $t > 0$, where $i \neq j$ and $i \in \bar{\mathcal{M}}$, which represents the possible destinations for the ith subsystem. Consequently, the switching condition (11) can be relaxed to

$$\varsigma_{i0}^{\mathrm{T}} F_i - \rho \varsigma_{jL}^{\mathrm{T}} \preccurlyeq 0, \quad j \in \bar{\mathcal{Q}}_i.$$

Remark 3. The definition of $\varsigma(t)$ by [28] is in the form of

$$\varsigma_i(t) = \begin{cases} \frac{t-t_k-\theta_q}{d}\varsigma_{iq+1} + \frac{t_k+\theta_{q+1}-t}{d}\varsigma_{iq}, & t \in [t_k+\theta_q,\, t_k+\theta_{q+1}),\ q = 0,1,\dots,L-1, \\ \varsigma_{iL+1}, & t \in [t_k+\tau_{\min},\, t_{k+1}), \end{cases}$$

where $\theta_q = qd$, $d = \tau_{\min}/L$, $\sigma(t_k) = i$, and $\cup_{q=0}^{L-1}[t_k+\theta_q,\, t_k+\theta_{q+1}) = [t_k,\, t_k+\tau_{\min}) \subseteq [t_k,\, t_{k+1})$. This paper adopts a different segmentation method from [28] for the interval $R_k = [t_k, t_{k+1})$ and considers the time-varying positive switched system under the influence of pulse factors.

When $A_i(t) \equiv \bar{A}_i$, system (6) reduces to a time-invariant positive switched impulsive system, and Theorem 1 still holds in this case. The stability criterion of the time-invariant positive switched impulsive system in [8] is as follows.

Proposition 1. *Given a positive integer L, if there exist a group of positive vectors $\varsigma_{iq} \in \mathbb{R}_+^n$, $i \in \bar{\mathcal{M}}$, $q = 0,1,\dots,L$, and two constants $0 < \rho < 1$, $\eta > 0$ such that for all $q = 0,1,\dots,L-1$, we have*

$$\varsigma_{iq}^{\mathrm{T}} \bar{A}_i + L\left(\frac{\varsigma_{iq+1}^{\mathrm{T}}}{\tau_{i1}} - \frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i2}}\right) - \eta \varsigma_{iq}^{\mathrm{T}} \prec 0, \tag{17}$$

$$\varsigma_{iq+1}^{\mathrm{T}} \bar{A}_i + L\left(\frac{\varsigma_{iq+1}^{\mathrm{T}}}{\tau_{i1}} - \frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i2}}\right) - \eta \varsigma_{iq+1}^{\mathrm{T}} \prec 0, \tag{18}$$

$$\varsigma_{i0}^{\mathrm{T}} F_i - \rho \varsigma_{jL}^{\mathrm{T}} \preccurlyeq 0, \quad i,j \in \bar{\mathcal{M}},\ i \neq j,$$

$$\tau_2 \eta + \ln \rho < 0,$$

where $\tau_2 = \max_{i\in\bar{\mathcal{M}}} \tau_{i2}$, then system (6) *is GUES under MDIDT switching signal.*

Next, this paper compares the conservativeness of Proposition 1 and Theorem 1 through formula derivation.

In (7), we have

$$\frac{L}{\tau_{i2}}\left(\varsigma_{iq+1}^{\mathrm{T}}-\varsigma_{iq}^{\mathrm{T}}\right)=L\left(\frac{\varsigma_{iq+1}^{\mathrm{T}}}{\tau_{i2}}-\frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i2}}\right).$$

The fact that $\tau_{i2}>\tau_{i1}$ leads to the conclusion that

$$\frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i2}}\prec\frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i1}},$$

which further implies that

$$L\left(\frac{\varsigma_{iq+1}^{\mathrm{T}}}{\tau_{i2}}-\frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i2}}\right)\prec L\left(\frac{\varsigma_{iq+1}^{\mathrm{T}}}{\tau_{i1}}-\frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i2}}\right).$$

Similarly, in (8), we obtain

$$\frac{L}{\tau_{i1}}\left(\varsigma_{iq+1}^{\mathrm{T}}-\varsigma_{iq}^{\mathrm{T}}\right)=L\left(\frac{\varsigma_{iq+1}^{\mathrm{T}}}{\tau_{i1}}-\frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i1}}\right),$$

we can also deduce that

$$L\left(\frac{\varsigma_{iq+1}^{\mathrm{T}}}{\tau_{i1}}-\frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i1}}\right)\prec L\left(\frac{\varsigma_{iq+1}^{\mathrm{T}}}{\tau_{i1}}-\frac{\varsigma_{iq}^{\mathrm{T}}}{\tau_{i2}}\right).$$

Based on these findings, it is evident that when inequality (17) holds, inequalities (7) and (8) also hold. Similarly, when (18) holds, the inequalities (9) and (10) are also satisfied. Therefore, the restrictions on the stability criterion obtained in this paper are less conservative compared to those in [8].

We also consider the stability of system (6) under an IDT switching signal that satisfies Assumption 4.

Corollary 1. *Suppose that Assumptions* 1 *and* 2 *hold true. Given a positive integer* L *and constants* $\eta>0$, $0<\rho<1$, *if there exist a group of positive constant vectors* $\varsigma_{iq}\in\mathbb{R}_+^n$, $i\in\bar{\mathcal{M}}$, $q=0,1,\dots,L$, *such that for all* $q=0,1,\dots,L-1$, *we have*

$$\varsigma_{iq}^{\mathrm{T}}\bar{A}_i+\frac{L}{\tau_2}\left(\varsigma_{iq+1}^{\mathrm{T}}-\varsigma_{iq}^{\mathrm{T}}\right)-\eta\varsigma_{iq}^{\mathrm{T}}\prec 0, \tag{19}$$

$$\varsigma_{iq}^{\mathrm{T}}\bar{A}_i+\frac{L}{\tau_1}\left(\varsigma_{iq+1}^{\mathrm{T}}-\varsigma_{iq}^{\mathrm{T}}\right)-\eta\varsigma_{iq}^{\mathrm{T}}\prec 0, \tag{20}$$

$$\varsigma_{iq+1}^{\mathrm{T}}\bar{A}_i+\frac{L}{\tau_2}\left(\varsigma_{iq+1}^{\mathrm{T}}-\varsigma_{iq}^{\mathrm{T}}\right)-\eta\varsigma_{iq+1}^{\mathrm{T}}\prec 0, \tag{21}$$

$$\varsigma_{iq+1}^{\mathrm{T}}\bar{A}_i+\frac{L}{\tau_1}\left(\varsigma_{iq+1}^{\mathrm{T}}-\varsigma_{iq}^{\mathrm{T}}\right)-\eta\varsigma_{iq+1}^{\mathrm{T}}\prec 0, \tag{22}$$

$$\varsigma_{i0}^{\mathrm{T}}F_i-\rho\varsigma_{jL}^{\mathrm{T}}\preccurlyeq 0,\quad i,j\in\bar{\mathcal{M}},\ i\neq j, \tag{23}$$

$$\tau_2\eta+\ln\rho<0, \tag{24}$$

then system (1) *is GUES under any IDT switching signal satisfying Assumption* 4.

We now present an algorithm designed to compute the maximum and minimum dwell times, which can further delve into the stability region.

Algorithm 1. IDT maximum-minimum dwell-times algorithm

Steps:

1. Initialization:

Initial guess τ_2, ς_{iq}, $i \in \bar{\mathcal{M}}$, $q = 0, 1, \ldots, L$;

Configure the $\mathrm{fmincon}$ function with the SQP and define the objective function $f = -\tau_2$ along with the constraint functions (19)–(24).

2. Optimization:

Utilize the $\mathrm{fmincon}$ function to find the optimal solution of τ_2, ς_{iq} and their corresponding objective function value f;

Obtain the optimization success flag ($\mathrm{exitflag}$) and output information.

3. Results:

If $\mathrm{exitflag} > 0$, print "Optimal solution found!" and display τ_2, ς_{iq}, and the objective function value f;

Otherwise, print "No solution found".

4. Save:

Save results for further analysis.

Considering Corollary 1, our goal is to compute the maximum dwell time τ_2 and the minimum dwell time τ_1 under the IDT switching signal and to seek $m(L+1)$ positive vectors $\varsigma_{iq} \in \mathbb{R}^n$ for $i \in \bar{\mathcal{M}}$ and $q = 0, 1, \ldots, L$. By treating τ_1, τ_2, and ς_{iq} as variables, the problem transforms into a nonlinear optimization problem subject to constraints (19)–(24). To tackle this optimization problem, we employ the sequential quadratic programming (SQP) algorithm, which is not constrained by step size limitations and can provide solutions with high precision.

Given the mutual constraints between τ_1 and τ_2 as defined by conditions (19)–(24), it is not feasible to search for τ_1 or τ_2 independently. Therefore, we employ the SQP algorithm to treat either τ_1 or τ_2 as an independent variable and the other as a dependent variable. This method enables us to construct a relationship graph that visually represents the variations between the two variables and subsequently identifies the stability region. Assuming that τ_1 is the independent variable and assigning an initial assumption to τ_1, when conditions (19)–(24) are satisfied, we can compute the corresponding maximum dwell time. For details, we refer to Algorithm 1.

4 L_1-gain analysis

In this section, based on Theorem 1, the L_1-gain performance of the positive switched impulsive system (1) is further analyzed.

Theorem 2. *Consider system* (1) *under Assumptions* 1–3. *Given a positive integer L and a constant* $0 < \rho < 1$, *if there exist a group of constant vectors* $\varsigma_{iq} \in \mathbb{R}^n_+$, $i \in \bar{\mathcal{M}}$, $q = 0, 1, \ldots, L$, *such that for all* $q = 0, 1, \ldots, L-1$, *we have*

$$\begin{bmatrix} \varsigma_{iq}^{\mathrm{T}} \bar{A}_i + \frac{L}{\tau_{i2}} (\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}) + \mathbf{1}^{\mathrm{T}} \bar{C}_i \\ \varsigma_{iq}^{\mathrm{T}} \bar{B}_i + \mathbf{1}^{\mathrm{T}} \bar{D}_i - \varepsilon \mathbf{1}^{\mathrm{T}} \end{bmatrix} \prec 0, \tag{25}$$

$$\begin{bmatrix} \varsigma_{iq}^{\mathrm{T}} \bar{A}_i + \frac{L}{\tau_{i1}} (\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}) + \mathbf{1}^{\mathrm{T}} \bar{C}_i \\ \varsigma_{iq}^{\mathrm{T}} \bar{B}_i + \mathbf{1}^{\mathrm{T}} \bar{D}_i - \varepsilon \mathbf{1}^{\mathrm{T}} \end{bmatrix} \prec 0, \tag{26}$$

$$\begin{bmatrix} \varsigma_{iq+1}^{\mathrm{T}} \bar{A}_i + \frac{L}{\tau_{i2}} (\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}) + \mathbf{1}^{\mathrm{T}} \bar{C}_i \\ \varsigma_{iq+1}^{\mathrm{T}} \bar{B}_i + \mathbf{1}^{\mathrm{T}} \bar{D}_i - \varepsilon \mathbf{1}^{\mathrm{T}} \end{bmatrix} \prec 0, \tag{27}$$

$$\begin{bmatrix} \varsigma_{iq+1}^{\mathrm{T}} \bar{A}_i + \frac{L}{\tau_{i1}} (\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}) + \mathbf{1}^{\mathrm{T}} \bar{C}_i \\ \varsigma_{iq+1}^{\mathrm{T}} \bar{B}_i + \mathbf{1}^{\mathrm{T}} \bar{D}_i - \varepsilon \mathbf{1}^{\mathrm{T}} \end{bmatrix} \prec 0, \tag{28}$$

$$\varsigma_{i0}^{\mathrm{T}} F_i - \rho \varsigma_{jL}^{\mathrm{T}} \preccurlyeq 0, \quad i, j \in \bar{\mathcal{M}},\ i \neq j, \tag{29}$$

where $\tau_2 = \max_{i \in \bar{\mathcal{M}}} \tau_{i2}$*, then system* (1) *is GUES with an* L_1*-gain* ε *under MDIDT switching signal satisfying Assumption* 5.

Proof. Since Assumptions 1–3 hold, system (1) is positive. Given $x(0) \succcurlyeq 0$, the trajectory of system (1) will remain nonnegative under arbitrary switching. Based on Theorem 1, we can easily arrive at system (1) without $\omega(t)$ is GUES. Subsequently, we proceed to investigate the L_1-gain performance. Let

$$\mathfrak{K}(t) = \|y(t)\| - \varepsilon \|\omega(t)\| = \mathbf{1}^{\mathrm{T}} y(t) - \varepsilon \mathbf{1}^{\mathrm{T}} \omega(t), \quad t \in R_k,\ k \in \mathbb{N}. \tag{30}$$

Moreover, for $\sigma(t) = i$, we have that

$$\begin{aligned} \dot{V}_i(t) + \mathfrak{K}(t) &= \dot{\varsigma}_i^{\mathrm{T}}(t) x(t) + \varsigma_i^{\mathrm{T}}(t) \dot{x}(t) + \mathbf{1}^{\mathrm{T}} y(t) - \varepsilon \mathbf{1}^{\mathrm{T}} \omega(t) \\ &= \big(\dot{\varsigma}_i^{\mathrm{T}}(t) + \varsigma_i^{\mathrm{T}}(t) A_i(t)\big) x(t) + \varsigma_i^{\mathrm{T}}(t) B_i(t) \omega(t) \\ &\quad + \mathbf{1}^{\mathrm{T}} C_i(t) x(t) + \mathbf{1}^{\mathrm{T}} D_i(t) \omega(t) - \varepsilon \mathbf{1}^{\mathrm{T}} \omega(t) \\ &\leqslant \big(\dot{\varsigma}_i^{\mathrm{T}}(t) + \varsigma_i^{\mathrm{T}}(t) \bar{A}_i\big) x(t) + \varsigma_i^{\mathrm{T}}(t) \bar{B}_i \omega(t) \\ &\quad + \mathbf{1}^{\mathrm{T}} \bar{C}_i x(t) + \mathbf{1}^{\mathrm{T}} \bar{D}_i \omega(t) - \varepsilon \mathbf{1}^{\mathrm{T}} \omega(t) \\ &= \begin{bmatrix} \varsigma_i^{\mathrm{T}}(t) \bar{A}_i + \dot{\varsigma}_i^{\mathrm{T}}(t) + \mathbf{1}^{\mathrm{T}} \bar{C}_i \\ \varsigma_i^{\mathrm{T}}(t) \bar{B}_i + \mathbf{1}^{\mathrm{T}} \bar{D}_i - \varepsilon \mathbf{1}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}, \end{aligned}$$

where $\varsigma_i(t)$ is defined by (5). Thus, similar to the proof of inequality (13), inequalities (25)–(28) imply that

$$\dot{V}_i(t) + \mathfrak{K}(t) < 0, \quad t \in R_k,\ k \in \mathbb{N}. \tag{31}$$

Taking the integral of (31) with respect to t, $t \in R_k$, $k \in \mathbb{N}$, we have

$$V_i(t) < V_i(t_k) - \int_{t_k}^{t} \mathfrak{K}(s)\, \mathrm{d}s. \tag{32}$$

Set $\sigma(t_k^-) = j$. Based on (29), one has

$$\begin{aligned} & V_i\big(t_k, x(t_k)\big) - V_j\big(t_k^-, x(t_k^-)\big) \\ &\quad \leqslant V_i\big(t_k, x(t_k)\big) - \rho V_j\big(t_k^-, x(t_k^-)\big) = \varsigma_i^{\mathrm{T}}(t_k) x(t_k) - \rho \varsigma_j^{\mathrm{T}}(t_k^-) x(t_k^-) \\ &\quad = \big[\varsigma_i^{\mathrm{T}}(t_k) F_i - \rho \varsigma_j^{\mathrm{T}}(t_k^-)\big] x(t_k^-) = \big(\varsigma_{i0}^{\mathrm{T}} F_i - \rho \varsigma_{jL}^{\mathrm{T}}\big) x(t_k^-) \leqslant 0, \quad k \in \mathbb{N}. \end{aligned} \tag{33}$$

It further follows from (32) and (33) that

$$V_i(t) < V_j(t_k^-) - \int\limits_{t_k}^{t} \mathfrak{K}(s)\,\mathrm{d}s. \tag{34}$$

Moreover,

$$V_j(t_k^-) < V_j(t_{k-1}) - \int\limits_{t_{k-1}}^{t_k} \mathfrak{K}(s)\,\mathrm{d}s. \tag{35}$$

Substitute (35) into (34)

$$V_i(t) < V_j\big(t_{k-1}\big) - \int\limits_{t_{k-1}}^{t_k} \mathfrak{K}(s)\,\mathrm{d}s - \int\limits_{t_k}^{t} \mathfrak{K}(s)\,\mathrm{d}s.$$

Replicating the above protocol, we can obtain

$$\begin{aligned} V_i(t) &< V_{\sigma(0)}(t_0) - \int\limits_{t_0}^{t_1} \mathfrak{K}(s)\,\mathrm{d}s - \int\limits_{t_1}^{t_2} \mathfrak{K}(s)\,\mathrm{d}s \\ &\quad - \int\limits_{t_2}^{t_3} \mathfrak{K}(s)\,\mathrm{d}s - \cdots - \int\limits_{t_k}^{t} \mathfrak{K}(s)\,\mathrm{d}s \\ &= V_{\sigma(0)}(t_0) - \int\limits_{t_0}^{t} \mathfrak{K}(s)\,\mathrm{d}s,\ t \geqslant t_0. \end{aligned}$$

On the basis of initial condition and definition of $V_i(t)$, it is already known that $V_{\sigma(0)}(t_0) = 0$ and $V_i(t) > 0$. Obviously, $\int_{t_0}^{t} \mathfrak{K}(s)\,\mathrm{d}s < 0$, $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. Then, $\int_{t_0}^{\infty} \mathfrak{K}(s)\,\mathrm{d}s < 0$ holds, which can derive $\int_{t_0}^{\infty} \|y(t)\|\,\mathrm{d}t < \varepsilon \int_{t_0}^{\infty} \|\omega(t)\|\,\mathrm{d}t$. The proof is now complete. □

We also evaluate the L_1-gain performance of system (1) under IDT switching signals, leading to the following conclusions.

Corollary 2. *Suppose that Assumptions* 1–3 *hold true. Given a positive integer L and constants $\eta > 0$, $0 < \rho < 1$, if there exist a group of positive constant vectors $\varsigma_{iq} \in \mathbb{R}_+^n$, $i \in \bar{\mathcal{M}}$, $q = 0, 1, \dots, L$, such that for all $q = 0, 1, \dots, L-1$, we have*

$$\begin{bmatrix} \varsigma_{iq}^{\mathrm{T}} \bar{A}_i + \frac{L}{\tau_2}(\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}) + \mathbf{1}^{\mathrm{T}} \bar{C}_i \\ \varsigma_{iq}^{\mathrm{T}} \bar{B}_i + \mathbf{1}^{\mathrm{T}} \bar{D}_i - \varepsilon \mathbf{1}^{\mathrm{T}} \end{bmatrix} \prec 0,$$

$$\begin{bmatrix} \varsigma_{iq}^{\mathrm{T}} \bar{A}_i + \frac{L}{\tau_1}(\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}) + \mathbf{1}^{\mathrm{T}} \bar{C}_i \\ \varsigma_{iq}^{\mathrm{T}} \bar{B}_i + \mathbf{1}^{\mathrm{T}} \bar{D}_i - \varepsilon \mathbf{1}^{\mathrm{T}} \end{bmatrix} \prec 0,$$

$$\begin{bmatrix} \varsigma_{iq+1}^{\mathrm{T}}\bar{A}_i + \frac{L}{\tau_2}(\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}) + \mathbf{1}^{\mathrm{T}}\bar{C}_i \\ \varsigma_{iq+1}^{\mathrm{T}}\bar{B}_i + \mathbf{1}^{\mathrm{T}}\bar{D}_i - \varepsilon\mathbf{1}^{\mathrm{T}} \end{bmatrix} \prec 0,$$

$$\begin{bmatrix} \varsigma_{iq+1}^{\mathrm{T}}\bar{A}_i + \frac{L}{\tau_1}(\varsigma_{iq+1}^{\mathrm{T}} - \varsigma_{iq}^{\mathrm{T}}) + \mathbf{1}^{\mathrm{T}}\bar{C}_i \\ \varsigma_{iq+1}^{\mathrm{T}}\bar{B}_i + \mathbf{1}^{\mathrm{T}}\bar{D}_i - \varepsilon\mathbf{1}^{\mathrm{T}} \end{bmatrix} \prec 0,$$

$$\varsigma_{i0}^{\mathrm{T}}F_i - \rho\varsigma_{jL}^{\mathrm{T}} \preccurlyeq 0,\ i,j \in \bar{\mathcal{M}},\ i \neq j,$$

then system (1) *is GUES with an L_1-gain ε under IDT switching signal satisfying Assumption* 5.

5 Numerical examples

This section presents two numerical examples to show the effectiveness of our main results. The first example involves a time-invariant positive switched impulsive system. By employing Algorithm 1, this example aims to illustrate the reduced conservatism of our findings in comparison to prior works [8,28]. The second numerical example concerns a time-varying positive switched impulsive system, which is introduced to demonstrate the validity of our results.

Example 1. When $A_i(t) \equiv \bar{A}_i$. Consider system (6) consisting of two subsystems as

$$\bar{A}_1 = \begin{bmatrix} 0.15 & 0.2 \\ 0 & -1 \end{bmatrix}, \qquad \bar{A}_2 = \begin{bmatrix} -1.5 & 0.2 \\ 0 & 0.37 \end{bmatrix},$$

impulse matrices as follows:

$$F_1 = \begin{bmatrix} 1.5 & 1 \\ 0 & 0.5 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.6 \end{bmatrix}.$$

Both subsystems are unstable due to the system matrices A_1 and A_2 not being Hurwitz. Let $x(0) = [1, 3]^{\mathrm{T}}$. Figs. 1 and 2 show that the subsystems 1 and 2 are unstable, respectively.

We specify a set of fixed constants satisfying $L = 1$, $\rho = 0.6$, and $\eta = 0.1$. When a guess at $\tau_1 = 0.84$ is provided, we are able to find the values of ς_{10}, ς_{11}, ς_{20}, ς_{21} and the maximum dwell time τ_2 that satisfy the stability criterion of Corollary 1

$$\varsigma_{10} = \begin{bmatrix} 0.0112 \\ 13.0795 \end{bmatrix}, \qquad \varsigma_{11} = \begin{bmatrix} 0.0100 \\ 25.0000 \end{bmatrix}, \qquad \varsigma_{20} = \begin{bmatrix} 0.0120 \\ 25.0000 \end{bmatrix}, \qquad \varsigma_{21} = \begin{bmatrix} 0.0280 \\ 10.9183 \end{bmatrix},$$

and $\tau_2 = 2.08$.

The switching signal $\sigma(t)$ designed based on IDT with $\tau_1 = 0.84$ and $\tau_2 = 2.08$ is shown in Fig. 3. The trajectory of the positive switched impulsive system (6) is depicted in Fig. 4, demonstrating that the positive switched impulsive system is exponentially stable under the IDT switching signal.

By utilizing the linear programming (LP) tool in MATLAB, we are unable to find a suitable vector $\varsigma_{iL} \in \mathbb{R}^n$ with $\varsigma_{iL} \succ 0$ that satisfies $A_i^{\mathrm{T}}\varsigma_{iL} \prec 0$, indicating that the

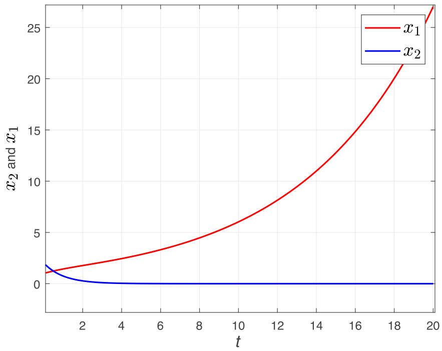

Figure 1. The state trajectory of subsystem 1.

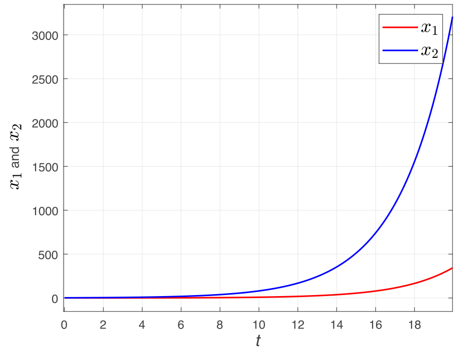

Figure 2. The state trajectory of subsystem 2.

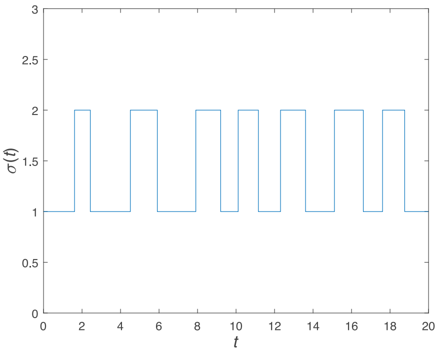

Figure 3. The given switching signal for Example 1.

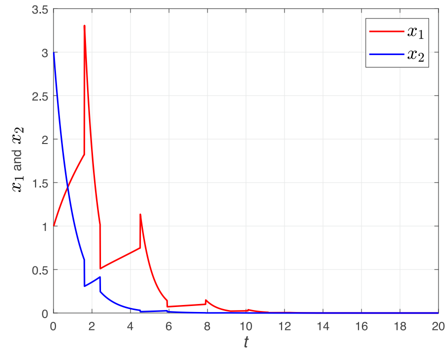

Figure 4. The state trajectory of Example 1.

result from [28] is not applicable to this specific scenario. Employing Algorithm 1, we illustrate the stability region that satisfies Corollary 1 and Proposition 1 for this example in Fig. 5. The dwell time within this stability region guarantees the exponential stability as demonstrated by the above simulation result with the switching signals $\tau_1 = 0.84$ and $\tau_2 = 2.08$. As shown in Fig. 5, region R_2 represents the stability region of Proposition 1, while region $R_1 \cup R_2$ represents the stability region of Corollary 1. It can be observed that R_2 is entirely contained within $R_1 \cup R_2$, indicating that the conservativeness of our result is lower than that of Proposition 1 from [8].

Finally, the L_1-gain performance of this positive switched impulsive system is considered. We consider system (1) with

$$\begin{bmatrix} \bar{A}_1 & \bar{B}_1 \\ \bar{C}_1 & \bar{D}_1 \end{bmatrix} = \begin{bmatrix} 0.15 & 0.2 & 0.5 \\ 0 & -1 & 1 \\ 1 & 2 & 0.5 \end{bmatrix}, \qquad \begin{bmatrix} \bar{A}_2 & \bar{B}_2 \\ \bar{C}_2 & \bar{D}_2 \end{bmatrix} = \begin{bmatrix} -1.5 & 0.2 & 2 \\ 0 & 0.37 & 1 \\ 1 & 0.5 & 0.5 \end{bmatrix},$$

the system matrices $\bar{A}_1$ and $\bar{A}_2$, as well as the impulse matrices F_1 and F_2, are consistent with the above. Given the input function $\omega(t) = 2t^3$, under the switching signal shown in Fig. 3, we can calculate the L_1-gain as $\varepsilon = 19.8200$ using MATLAB.

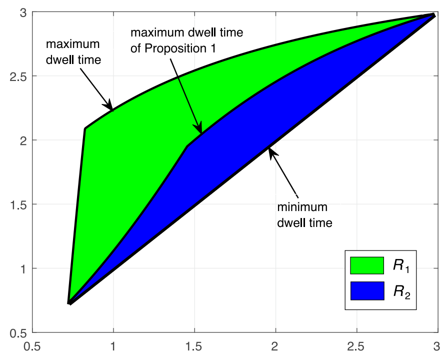

Figure 5. Stability region for admissible dwell time: R_2 is the stability region of Proposition 1, and $R_1 \cup R_2$ is the stability region of Corollary 1, respectively.

Example 2. Consider the time-varying system (6), which consists of two subsystems with the parameters setting in the form of

$$A_1(t) = \begin{bmatrix} -1.5 & 0 \\ 0.2|\cos(t)| & 0.1|\sin(t)| + 0.01 \end{bmatrix}, \qquad A_2(t) = \begin{bmatrix} 0.1 - 2^{-t} & 0.3 \\ 0.04|\sin(t)| & -1.1 \end{bmatrix},$$

which implies

$$\bar{A}_1 = \begin{bmatrix} -1.5 & 0 \\ 0.2 & 0.11 \end{bmatrix}, \qquad \bar{A}_2 = \begin{bmatrix} 0.1 & 0.3 \\ 0.04 & -1.1 \end{bmatrix},$$

impulse matrices as follows:

$$F_1 = \begin{bmatrix} 1.05 & 0.2 \\ 0 & 0.6 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 1 & 0.4 \\ 0 & 0.75 \end{bmatrix}.$$

Each subsystem is unstable, as evidenced by the fact that $A_1(t)$ and $A_2(t)$ are not Hurwitz matrices. We choose the initial state as $x(0) = [2, 2]^{\mathrm{T}}$. The state trajectories of subsystems 1 and 2 are shown in Figs. 6 and 7, respectively.

In [8, 28], the stability criterion for the time-varying positive switched impulsive system is not provided, thus they cannot address this case. Given $L = 1$, $\rho = 0.95$, and $\eta = 0.01$, an initial guess for τ_1 is set to 0.91. By utilizing Algorithm 1, we can determine that

$$\varsigma_{10} = \begin{bmatrix} 0.4476 \\ 1.1334 \end{bmatrix}, \qquad \varsigma_{11} = \begin{bmatrix} 0.8565 \\ 0.7480 \end{bmatrix}, \qquad \varsigma_{20} = \begin{bmatrix} 0.8136 \\ 0.5135 \end{bmatrix}, \ \varsigma_{21} = \begin{bmatrix} 0.4948 \\ 0.8101 \end{bmatrix},$$

and $\tau_2 = 3.40$.

The switching signal $\sigma(t)$ is designed with $\tau_1 = 0.91$ and $\tau_2 = 3.40$. This design is shown in Fig. 8. The trajectory of the time-varying positive switched impulsive system is shown in Fig. 9, clearly demonstrating its exponential stability.

With the aid of Algorithm 1, the stability region of this example is illustrated in Fig. 10. The dwell time within the stability region can ensure the exponential stability,

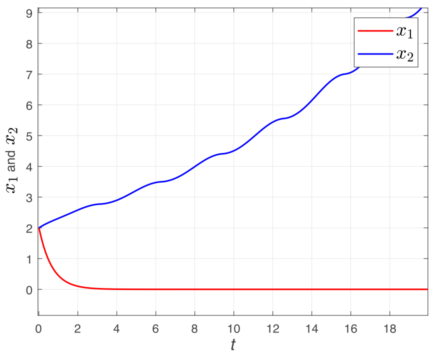

Figure 6. The state trajectory of subsystem 1.

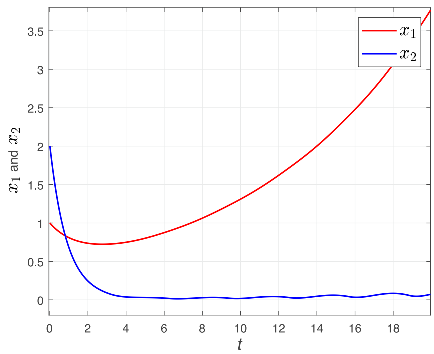

Figure 7. The state trajectory of subsystem 2.

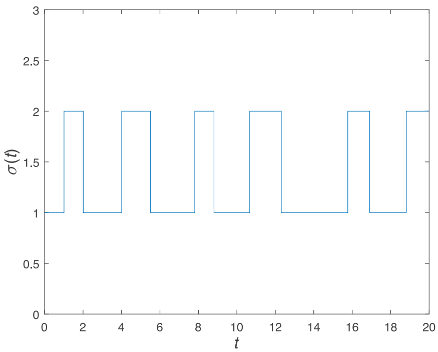

Figure 8. The given switching signal for Example 2.

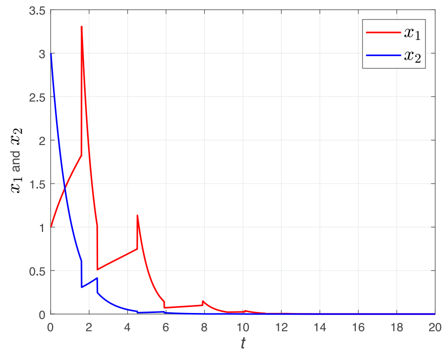

Figure 9. The state trajectory of Example 2.

as demonstrated by the above simulation result with the switching signals $\tau_1 = 0.91$ and $\tau_2 = 3.40$.

Lastly, the L_1-gain performance of the positive switched impulsive system is considered. We consider system (1) with

$$\begin{bmatrix} A_1(t) & B_1(t) \\ C_1(t) & D_1(t) \end{bmatrix} = \begin{bmatrix} -1.5 & 0 & 0.5|\sin(t)| \\ 0.2|\cos(t)| & 0.1|\sin(t)| + 0.01 & 0.1 \\ 0.9 + 0.1|\cos(t)| & 1 & 1.5 + 0.5|\sin(t)| \end{bmatrix},$$

$$\begin{bmatrix} A_2(t) & B_2(t) \\ C_2(t) & D_2(t) \end{bmatrix} = \begin{bmatrix} 0.1 - 2^{-t} & 0.3 & 0.2 \\ 0.04|\sin(t)| & -1.1 & 0.2 + 0.1|\sin(t)| \\ 0.5 & 0.5|\cos(t)| & 1.5 + 0.5|\sin(t)| \end{bmatrix},$$

which implies

$$\begin{bmatrix} \bar{A}_1 & \bar{B}_1 \\ \bar{C}_1 & \bar{D}_1 \end{bmatrix} = \begin{bmatrix} -1.5 & 0 & 0.5 \\ 0.2 & 0.11 & 0.1 \\ 1 & 1 & 2 \end{bmatrix}, \qquad \begin{bmatrix} \bar{A}_2 & \bar{B}_2 \\ \bar{C}_2 & \bar{D}_2 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.3 & 0.2 \\ 0.04 & -1.1 & 0.3 \\ 0.5 & 0.5 & 2 \end{bmatrix},$$

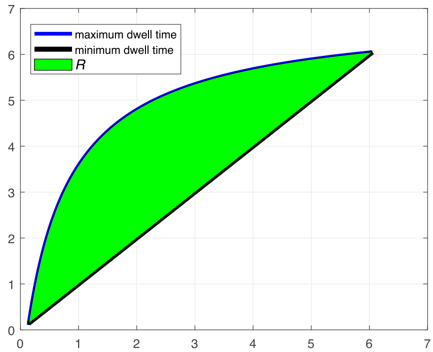

Figure 10. Stability region for admissible dwell time: R is the stability region.

the system matrices $A_1(t)$ and $A_2(t)$, as well as the impulse matrices F_1 and F_2, are consistent with the preceding descriptions. Assuming an input function $\omega(t) = 4\mathrm{e}^{-0.1t}$, the L_1-gain is computed as $\varepsilon = 6.3118$ using MATLAB under the switching signal depicted in Fig. 8.

6 Conclusions

In this paper, we have delved into the exponential stability and L_1-gain performance of time-varying positive switched impulsive systems when all subsystems are unstable under the MDIDT switching. By constructing a switched copositive Lyapunov function, we have derived a novel exponential stability criterion for such systems, which offers less conservative approach than prior results. Furthermore, an algorithm for calculating the maximum and minimum dwell times has been introduced, which is implemented based on SQP in MATLAB. Subsequently, based on the established stability result, we have explored the unweighted L_1-gain performance. Finally, the validity of our main results is demonstrated by numerical examples and simulations.

References

1. D. Angeli, P. De Leenheer, E.D. Sontag, Chemical networks with inflows and outflows: A positive linear differential inclusions approach, *Biotechnol. Progr.*, **25**(3):632–642, 2009, `https://doi.org/10.1002/btpr.162`.

2. X. Chen, J. Lam, P. Li, Z. Shu, L_1-induced norm and controller synthesis of positive systems, *Automatica*, **49**(5):1377–1385, 2013, `https://doi.org/10.1016/j.automatica.2013.02.023`.

3. Y. Farid, F. Ruggiero, Finite-time extended state observer and fractional-order sliding mode controller for impulsive hybrid port-Hamiltonian systems with input delay and actuators saturation: Application to ball-juggler robots, *Mech. Mach. Theory*, **167**:104577, 2022, `https://doi.org/10.1016/j.mechmachtheory.2021.104577`.

4. L. Farina, S. Rinaldi, *Positive Linear Systems: Theory and Applications*, John Wiley & Sons, New York, 2000, `https://doi.org/10.1002/9781118033029`.
5. E. Fornasini, M.E. Valcher, Stability and stabilizability criteria for discrete-time positive switched systems, *IEEE Trans. Autom. Control*, **57**(5):1208–1221, 2012, `https://doi.org/10.1109/TAC.2011.2173416`.
6. J. Hu, G. Sui, X. Lu, X. Li, Fixed-time control of delayed neural networks with impulsive perturbations, *Nonlinear Anal. Model. Control*, **23**(6):904–920, 2018, `https://doi.org/10.15388/NA.2018.6.6`.
7. Z. Hu, X. Mu, Event-triggered impulsive control for stochastic networked control systems under cyber attacks, *IEEE Trans. Syst. Man Cybern., Syst.*, **52**(9):5636–5645, 2022, `https://doi.org/10.1109/TSMC.2021.3130614`.
8. Y. Ju, F. Meng, Y. Sun, New criteria for exponential stability of positive switched linear impulsive systems with all modes unstable and its application, *Math. Methods Appl. Sci.*, **43**:1254–1277, 2021, `https://doi.org/10.1002/mma.7886`.
9. H.R. Karimi, N. Luo, Robust mixed H_2/H_∞ control of uncertain neutral systems with time-varying delays, *IFAC Proceedings Volumes*, **41**(2):4899–4904, 2008, `https://doi.org/10.3182/20080706-5-KR-1001.00823`.
10. J. Li, K. Pan, Q. Su, Sensor fault detection and estimation for switched power electronics systems based on sliding mode observer, *Appl. Math. Comput.*, **353**:282–294, 2019, `https://doi.org/10.1016/j.amc.2019.02.010`.
11. X. Li, D. Peng, J. Cao, Lyapunov stability for impulsive systems via event-triggered impulsive control, *IEEE Trans. Autom. Control*, **65**(11):4908–4913, 2020, `https://doi.org/10.1109/TAC.2020.2964558`.
12. D. Liberzon, *Switching in Systems and Control*, Birkhäuser Boston, MA, 2003, `https://doi.org/10.1007/978-1-4612-0017-8`.
13. X. Lin, H. Du, S. Li, Finite-time boundedness and L_2-gain analysis for switched delay systems with norm-bounded disturbance, *Appl. Math. Comput.*, **217**(12):5982–5993, 2011, `https://doi.org/10.1016/j.amc.2010.12.032`.
14. X. Liu, C. Dang, Stability analysis of positive switched linear systems with delays, *IEEE Trans. Autom. Control*, **56**(7):1684–1690, 2011, `https://doi.org/10.1109/TAC.2011.2122710`.
15. X. Liu, S. Zhong, X. Ding, Robust exponential stability of nonlinear impulsive switched systems with time-varying delays, *Nonlinear Anal. Model. Control*, **17**(2):210–222, 2012, `https://doi.org/10.15388/NA.17.2.14069`.
16. Z. Liu, X. Zhang, X. Lu, Q. Liu, Stabilization of positive switched delay systems with all modes unstable, *Nonlinear Anal., Hybrid Syst.*, **29**:110–120, 2018, `https://doi.org/10.1016/j.nahs.2018.01.004`.
17. M.A. Nojoumian, M.R. Zakerzadeh, M. Ayati, Stabilization of delayed switched positive nonlinear systems under mode dependent average dwell time: A bumpless control scheme, *Nonlinear Anal., Hybrid Syst.*, **47**:101300, 2023, `https://doi.org/10.1016/j.nahs.2022.101300`.
18. L. Pan, J. Cao, A. Alsaedi, Stability of reaction-diffusion systems with stochastic switching, *Nonlinear Anal. Model. Control*, **24**(3):315–331, 2019, `https://doi.org/10.15388/NA.2019.3.1`.

19. A. Sakly, M. Kermani, Stability and stabilization studies for a class of switched nonlinear systems via vector norms approach, *ISA Trans.*, **57**:144–161, 2015, https://doi.org/10.1016/j.isatra.2015.01.013.
20. H. Shen, X. Hu, J. Wang, J. Cao, W. Qian, Non-fragile H_∞ synchronization for Markov jump singularly perturbed coupled neural networks subject to double-layer switching regulation, *IEEE Trans. Neural Networks Learn. Syst.*, **34**(5):2682–2692, 2023, https://doi.org/10.1109/TNNLS.2021.3107607.
21. H. Shen, Y. Men, Z.-G. Wu, J. Cao, G. Lu, Network-based quantized control for fuzzy singularly perturbed semi-Markov jump systems and its application, *IEEE Trans. Circuits Syst. I Regul. Pap.*, **66**(3):1130–1140, 2019, https://doi.org/10.1109/TCSI.2018.2876937.
22. R. Shorten, F. Wirth, D. Leith, A positive systems model of TCP-like congestion control: asymptotic results, *IEEE/ACM Trans. Networking*, **14**(3):616–629, 2006, https://doi.org/10.1109/TNET.2006.876178.
23. Y. Sun, Y. Tian, X.-J. Xie, Stabilization of positive switched linear systems and its application in consensus of multiagent systems, *IEEE Trans. Autom. Control*, **62**(12):6608–6613, 2017, https://doi.org/10.1109/TAC.2017.2713951.
24. P. Wang, D. Yang, Stability and L_1-gain analysis for switched positive fuzzy systems with time-delay: A state-dependent switching policy, *Fuzzy Sets Syst.*, **464**:108440, 2023, https://doi.org/10.1016/j.fss.2022.11.011.
25. S. Wang, Q. Huang, Bifurcation of nontrivial periodic solutions for a Beddington–DeAngelis interference model with impulsive biological control, *Appl. Math. Modelling*, **39**(5–6):1470–1479, 2015, https://doi.org/10.1016/j.apm.2014.09.011.
26. Y.-E. Wang, H.R. Karimi, D. Wu, Conditions for the stability of switched systems containing unstable subsystems, *IEEE Trans. Circuits Syst. II Express Briefs*, **66**(4):617–621, 2019, https://doi.org/10.1109/TCSII.2018.2852766.
27. X. Wu, Y. Tang, J. Cao, X. Mao, Stability analysis for continuous-time switched systems with stochastic switching signals, *IEEE Trans. Autom. Control*, **63**(9):3083–3090, 2018, https://doi.org/10.1109/TAC.2017.2779882.
28. W. Xiang, J. Lam, J. Shen, Stability analysis and L_1-gain characterization for switched positive systems under dwell-time constraint, *Automatica*, **85**:1–8, 2017, https://doi.org/10.1016/j.automatica.2017.07.016.
29. W. Xiang, J. Xiao, Stabilization of switched continuous-time systems with all modes unstable via dwell time switching, *Automatica*, **50**(3):940–945, 2014, https://doi.org/10.1016/j.automatica.2013.12.028.
30. Y.-Q. Xue, P. Zhao, Input-to-state consensus of nonlinear positive multi-agent systems under state feedback and impulsive control, *Int. J. Control Autom. Syst.*, **21**(7):2099–2111, 2023, https://doi.org/10.1007/s12555-021-1069-x.
31. Q. Yu, X. Yuan, Stability analysis for positive switched systems having stable and unstable subsystems based on a weighted average dwell time scheme, *ISA Trans.*, **136**:275–283, 2023, https://doi.org/10.1016/j.isatra.2022.10.019.
32. W. Yu, J. Cao, W. Lu, Synchronization control of switched linearly coupled neural networks with delay, *Neurocomputing*, **73**(4):858–866, 2010, https://doi.org/10.1016/j.neucom.2009.10.009.

33. N. Zhang, Y. Kang, P. Yu, Stability analysis of discrete-time switched positive nonlinear systems with unstable subsystems under different switching strategies, *IEEE Trans. Circuits Syst. II Express Briefs*, **68**(6):1957–1961, 2021, https://doi.org/10.1109/TCSII.2020.3029911.
34. N. Zhang, Y. Sun, Stabilization of switched positive linear delay system with all subsystems unstable, *Int. J. Robust Nonlinear Control*, **34**(4):2441–2456, 2024, https://doi.org/10.1002/rnc.7091.
35. X. Zhang, Z. Lu, X. Yuan, Y. Wang, X. Shen, L_2-gain adaptive robust control for hybrid energy storage system in electric vehicles, *IEEE Trans. Power Electron.*, **36**(6):7319–7332, 2021, https://doi.org/10.1109/TPEL.2020.3041653.
36. X. Zhao, L. Zhang, P. Shi, M. Liu, Stability of switched positive linear systems with average dwell time switching, *Automatica*, **48**(6):1132–1137, 2012, https://doi.org/10.1016/j.automatica.2012.03.008.
37. S. Zhu, Q.-L. Han, C. Zhang, L_1-gain performance analysis and positive filter design for positive discrete-time Markov jump linear systems: A linear programming approach, *Automatica*, **50**(8):2098–2107, 2014, https://doi.org/10.1016/j.automatica.2014.05.022.
38. X. Zhu, S. Liu, On reachable set bounding for discrete-time switched nonlinear positive systems with mixed time-varying delays and disturbance, *Nonlinear Anal., Hybrid Syst.*, **49**:101373, 2023, https://doi.org/10.1016/j.nahs.2023.101373.
39. G. Zong, H. Ren, L. Hou, Finite-time stability of interconnected impulsive switched systems, *IET Control Theory Appl.*, **10**(6):648–654, 2016, https://doi.org/10.1049/iet-cta.2015.0617.