Nonlinear Analysis: Modelling and Control, Vol. 30, No. 1, 40–57
https://doi.org/10.15388/namc.2025.30.38317

Absolute exponential stability of switching time-delay Lurie systems with the application to switching Hopfield neural networks

Wenxiu Zhao[a,b], **Yuangong Sun**[c]

[a]School of Artificial Intelligence, Shandong Women's University,
Jinan 250300, China
zhaowx@sdwu.edu.cn

[b]School of Mathematical Sciences, Qufu Normal University,
Qufu 273165, China

[c]School of Mathematical Sciences, University of Jinan,
Jinan 250022, China
sunyuangong@163.com

Received: November 16, 2023 / **Revised:** November 17, 2024 / **Published online:** January 2, 2025

Abstract. This paper investigates the problem of absolute exponential stability analysis for switching time-delay Lurie system (STDLS) with all modes unstable. By proposing a novel switching time-varying Lyapunov–Razumikhin function, a computable sufficient condition is formulated to guarantee absolute exponential stability of STDLS under mode-dependent range dwell-time (MDRDT) switching. Especially, theoretical results are applied to switching delay Hopfield neural network. Simulations are served to illustrate the developed theory.

Keywords: absolute exponential stability, switching Lurie system, time-varying delay, switching time-varying Lyapunov–Razumikhin function method.

1 Introduction

Switching systems usually appear in situations, where multiple dynamical models are needed to represent a hybrid system because of the structural changes of many actual systems during operation, or in the applications, where multiple controllers are switched between controllers for higher performance [18, 22, 31, 33, 34].

Stability of switching systems is a fundamental and significant research problem. Many interesting results for stability analysis of switching linear systems have been proposed by framing appropriate Lyapunov function with the family of subsystems for switching systems [10, 17, 26, 52]. However, for the switching nonlinear systems, the nonlinearity problem and the construction of Lyapunov function bring difficulties to the stability analysis [16, 25, 27, 42]. A special kind of switching nonlinear systems, switching Lurie systems, arises in sensor network problems [50], population model [3], model predictive control [30], and so on. Generally, we call stability of switching Lurie systems absolute stability. Based on common Lyapunov function technique, absolute stability criteria for switching Lurie system via arbitrary switching were formulated [2, 36]. Stability analysis for switching Lurie system was addressed by average dwell-time and multiple Lyapunov functions technique in [47, 48]. This technique has been further extended to more general switching Lurie system, and exponential stability criteria were derived [13]. Nevertheless, stability issue for switching Lurie system has not been completely solved so far, and how to construct appropriate Lyapunov functions numerically is still an interesting and open problem. These points motivate the work carried out in this paper.

In engineering practice, time delay occurs naturally and is often considered as an important factor affecting the stability of systems. Some discoveries related to stability of switching delay system have been proposed [11, 19, 20, 24, 29, 32, 37, 43]. There have also been some progress in the study of the stability of STDLS. In references [4, 36], stability analysis for STDLS was addressed by constructing a common Lyapunov–Krasovskii functional. Subsequently, average dwell-time and multiple Lyapunov–Krasovskii functionals technique was considered as a valid method resolving stability problems for STDLS [49]. This idea was further expanded to more general STDLS [23]. Notice that above research objects focus on switching time-delay systems including stable modes. However, for the switching system with all modes unstable, above methods are ineffective. Moreover, STDLS contains inherently many difficult research problems, many of which are still open. This adds to the challenges and complexity of stability analysis for STDLS with all modes unstable. It should be noted that switching behavior can be a favorable factor to stabilize the systems. Some related findings have been put forward [35, 38, 40, 41, 44–46, 51, 53]. However, to the best of our knowledge, nothing is known about absolute stability for STDLS with nonlinear delay when all modes are unstable. This inspires us to perform this work.

The main work in this paper is to investigate absolute exponential stability of STDLS composed fully of unstable modes. In the framework of MDRDT, a novel switching time-varying Lyapunov–Razumikhin function is proposed to derive computable stability criteria for STDLS. On this basis, stability region ensuring absolute exponential stability of STDLS is obtained. Moreover, above researches are applied to the stability analysis for switching delay Hopfield neural network. Finally, the conservativeness of the theoretical achievements is discussed.

The organization of the paper: Section 2 introduces some preparatory knowledge. Section 2.1 provides theoretical achievements. Main achievements are used to switching delay Hopfield neural network model in Section 3. Section 4 presents simulation examples to illustrate theoretical achievements. Section 5 summarizes our works.

Notations. $\mathbb{R}^+$, $\mathbb{N}$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ represent the set of nonnegative real numbers, natural numbers, n-tuples real vectors, and $n\times m$ real matrices, respectively. The symmetric matrix $\Lambda=\Lambda^{\mathrm{T}}>0$ ($\leqslant 0$) is positive definite (seminegative definite), where T represents transpose, and $*$ stands for symmetric elements. The minimum (maximum) eigenvalue of Λ is denoted by $\lambda_{\min}(\Lambda)$ ($\lambda_{\max}(\Lambda)$). $\vartheta(t_p^+)=\lim_{t\to t_p^+}\vartheta(t)$ ($\vartheta(t_p^-)=\lim_{t\to t_p^-}\vartheta(t)$) is right (left) limit of function $\vartheta(t)$ at a certain point. $\|\cdot\|$ is Euclidean norm. $\dot{\vartheta}(t)=\lim_{s\to 0^+}(\vartheta(t+s)-\vartheta(t))/s$ represents the right derivative of function $\vartheta(t)$.

2 Preliminaries

Consider the STDLS described by

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}x(t) + A_{d\sigma(t)}x\big(t-\epsilon(t)\big)\\ &\quad + B_{\sigma(t)}f\big(\varsigma(t)\big) + B_{d\sigma(t)}f\big(\varsigma\big(t-\epsilon(t)\big)\big),\\ \varsigma(t) &= C^{\mathrm{T}}x(t), \quad x(t)=\varpi(t), \quad t\in[-\hat{\epsilon},0],\end{aligned} \tag{1}$$

in which $x(t)\in\mathbb{R}^n$ denotes state variable, time delay $\epsilon(t)$ satisfies $0\leqslant\epsilon(t)\leqslant\hat{\epsilon}$, $\varpi(t):[-\hat{\epsilon},0]\to\mathbb{R}^n$, $\hat{\epsilon}\in\mathbb{R}^+$, is continuous initial function, switching law $\sigma(t)=i\in\mathbf{M}=\{1,2,\dots,M\}$ when $t\in R_p=[t_p,t_{p+1})$, $p\in\mathbb{N}$. Increasing sequence t_p, $p\in\mathbb{N}$, stands for switching moments, where $t_0=0$, and $t_p\to\infty$ for $p\to\infty$. $\mathcal{T}_p=t_{p+1}-t_p$ denotes dwell time. $A_i, A_{di}\in\mathbb{R}^{n\times n}$ and $B_i, B_{di}, C\in\mathbb{R}^{n\times m}$ are constant matrices, and $\varsigma\in\mathbb{R}^m$ denotes feedback variable. $f(\varsigma)=(f_1(\varsigma_1), f_2(\varsigma_2),\dots,f_m(\varsigma_m))^{\mathrm{T}}$ is continuous, and each component $f_j(\varsigma_j)$, $j=1,2,\dots,m$, belongs to the class $\mathcal{S}_{[0,k_j]}$, $k_j>0$, defined as follows:

$$\mathcal{S}_{[0,k_j]} := \big\{\phi\colon\ \phi(0)=0,\ 0\leqslant\varrho\phi(\varrho)\leqslant k_j\varrho^2,\ \varrho\neq0\big\}.$$

For brevity, we directly call $f(\varsigma)\in\mathcal{S}_{[0,K]}$.

Firstly, several important definitions are given.

Definition 1. (See [23].) The STDLS (1) achieves absolute exponential stability via switching law $\sigma(t)$ in the sector $[0,K]$ if there are positive scalars γ_0, α_0, for any $\varpi(t)\in C[-\hat{\epsilon},0]$, any $f(\varsigma)\in\mathcal{S}_{[0,K]}$, and $\sigma(t)\in\mathbf{M}$, satisfying $\|x(t)\|\leqslant\gamma_0\mathrm{e}^{-\alpha_0 t}\|\widehat{\varpi}\|_{\hat{\epsilon}}$, $t\in[0,\infty)$, $\|\widehat{\varpi}\|_{\hat{\epsilon}}=\sup_{-\hat{\epsilon}\leqslant t\leqslant0}\|\varpi(t)\|$.

Definition 2. (See [6].) Given a switching law $\sigma(t)$, if there are constants $\check{\mathcal{T}}_i=\inf_{p\in\mathbb{N}}\{\mathcal{T}_p\colon\ \sigma(t_p)=i,\ i\in\mathbf{M}\}$ and $\hat{\mathcal{T}}_i=\sup_{p\in\mathbb{N}}\{\mathcal{T}_p\colon\ \sigma(t_p)=i,\ i\in\mathbf{M}\}$, we call $\sigma(t)$ MDRDT switching law.

Note that when $\check{\mathcal{T}}_i\equiv\check{\mathcal{T}}$ and $\hat{\mathcal{T}}_i\equiv\hat{\mathcal{T}}$, MDRDT reduces to the range dwell time (RDT) in [7].?? bruksnelis?

Definition 3. (See [40].) Stability region is the set of dwell time that make the STDLS satisfies absolute exponential stability.

For the development of the work, we give two assumptions:

Assumption 1. $\hat{\epsilon}\leqslant\check{\mathcal{T}}$, where $\check{\mathcal{T}}=\min_{i\in\mathbf{M}}\{\check{\mathcal{T}}_i\}$.

Assumption 2. (See [14].) For switching law $\sigma(t)$, let $N_\sigma(0,t)$ represent discontinuities times for $\sigma(t)$ in time range $[0,t)$. Suppose that there are positive numbers N_0 and τ_0 satisfying

$$N_\sigma(0,t)\geqslant\frac{t}{\tau_0}-N_0.$$

This means that for a appropriate τ_0, it is switched at least once every interval of length τ_0.

To analyze the absolute exponential stability issue for system (1), a novel switching time-varying Lyapunov–Razumikhin function is proposed.

Before continuing, a time-varying matrix function $\varTheta_i(t)$ is introduced. The range R_p is first divided into G blocks of equal length. Every block is denoted by $R_{p,g} = [t_p + gd_p, t_p + (g+1)d_p)$ with $g \in \underline{\mathbf{G}}_0 = \{0, 1, \dots, G-1\}$ and $d_p = \mathcal{T}_p/G$, where $\bigcup_{g=0}^{G-1} R_{p,g} = R_p$ and $R_{p,u} \bigcap R_{p,s} = \emptyset$, $u \neq s$. For every fragment $R_{p,g}$, select matrices $\mathcal{Q}_{i,g} \in \mathbb{R}^{n\times n}$ with $\mathcal{Q}_{i,g} > 0$ for $i \in \mathbf{M}$ and $g \in \mathbf{G}_0 = \{0, 1, \dots, G\}$. Then $\varTheta_i(t)$ is depicted as

$$\varTheta_i(t) = \frac{t_p + (g+1)d_p - t}{d_p}\mathcal{Q}_{i,g} + \frac{t - t_p - gd_p}{d_p}\mathcal{Q}_{i,g+1}, \quad t \in R_{p,g}. \tag{2}$$

Now, for given invariable $\alpha < 0$, a switching time-varying Lyapunov–Razumikhin function is defined as

$$\mathcal{G}_{\sigma(t)}\big(t, x(t)\big) = \mathrm{e}^{\alpha t}\big(x^{\mathrm{T}}(t)\varTheta_{\sigma(t)}(t)x(t)\big). \tag{3}$$

Clearly, $\mathcal{G}_{\sigma(t)}(t, x(t)) \geqslant 0$ is right-continuous. For brevity, let

$$\mathcal{G}_{\sigma(t)}\big(t_p, x(t_p)\big) = \mathcal{G}_{\sigma(t)}\big(t_p^+, x(t_p^+)\big).$$

Remark 1. This idea for time-varying Lyapunov function has been worked for the research of switching linear system under minimum dwell-time framework, and it has had effective results [5, 39]. It is worth mentioning that the MDRDT approach has obvious advantages compared with minimum dwell-time approach. As we shall see, the absolute exponential stability of system (1) is not limited by fixed dwell-time switching (see [39, Cor. 2]). This implies that the results we obtained have a wider scope of application.

Remark 2. In the research of the switching linear system, based on minimal dwell-time method, [40] defined matrix function $\varTheta_i(t)$ as

$$\varTheta_i(t) = \begin{cases} \frac{t_p+(g+1)d-t}{d}\mathcal{Q}_{i,g} + \frac{t-t_p-gd}{d}\mathcal{Q}_{i,g+1}, & t \in \big[t_p + gd,\, t_p + (g+1)d\big), \\ \mathcal{Q}_{i,G}, \quad t \in [t_p + \check{\mathcal{T}},\, t_{p+1}), \end{cases} \tag{4}$$

where $\bigcup_{g=0}^{G-1}[t_p + gd,\, t_p + (g+1)d) = [t_p,\, t_p + \check{\mathcal{T}}) \subseteq R_p$ with $\check{\mathcal{T}} = \inf_{p\in\mathbb{N}}\{\mathcal{T}_p\}$ and $d = \check{\mathcal{T}}/G$. Compared with (4), (2) eliminates the constant matrix function defined on the interval $[t_p + \check{\mathcal{T}},\, t_{p+1})$ because the definition of the matrix function (4) on interval $[t_p + \check{\mathcal{T}},\, t_{p+1})$ is redundant under MDRDT switching. This means that the stability condition related to constant matrix function defined on range $[t_p + \check{\mathcal{T}},\, t_{p+1})$ in [40] can be removed. In the current context, a more relaxed stability condition will be proposed in the next section. Additionally, many previous research achievements have not considered the mode dependence property of switching law; see, e.g., those in [5,23,39,40,49]. Here MDRDT is used to research of STDLS, which will also improve the results.

2.1 Main results

Theorem 1. *Under Assumptions* 1 *and* 2*, if there are constants* $\ell_i > 0$*,* $\gamma > 0$*,* $\rho > 1/\mu > 1$*,* $\alpha < 0$ *and matrices* $\mathcal{Q}_{i,g} > 0$*,* $g \in \mathbf{G}_0$*,* $\bar{\mathcal{Q}} > 0$ *satisfying*

$$\Gamma_1 = \begin{bmatrix} \Phi_{i,g} & \mathcal{Q}_{i,g}A_{di} & \mathcal{Q}_{i,g}B_i + \gamma C & \mathcal{Q}_{i,g}B_{di} \\ * & -\ell_i\bar{\mathcal{Q}} & 0 & \gamma C \\ * & * & -2\gamma K^{-1} & 0 \\ * & * & * & -2\gamma K^{-1} \end{bmatrix} < 0, \tag{5}$$

$$\Gamma_2 = \begin{bmatrix} \Psi_{i,g} & \mathcal{Q}_{i,g+1}A_{di} & \mathcal{Q}_{i,g+1}B_i + \gamma C & \mathcal{Q}_{i,g+1}B_{di} \\ * & -\ell_i\bar{\mathcal{Q}} & 0 & \gamma C \\ * & * & -2\gamma K^{-1} & 0 \\ * & * & * & -2\gamma K^{-1} \end{bmatrix} < 0, \tag{6}$$

$$\bar{\mathcal{Q}} \leqslant \mathcal{Q}_{i,g}, \quad g \in \mathbf{G}_0, \tag{7}$$

$$\mathcal{Q}_{j,0} \leqslant \mu\mathcal{Q}_{i,G}, \quad i \neq j,\ j \in \mathcal{M}, \tag{8}$$

for any $i \in \mathbf{M}$*,* $g \in \underline{\mathbf{G}}_0$*, where*

$$\Phi_{i,g} = G\left(\frac{\mathcal{Q}_{i,g+1}}{\check{\mathcal{T}}_i} - \frac{\mathcal{Q}_{i,g}}{\hat{\mathcal{T}}_i}\right) + (\ell_i\rho + \alpha)\mathcal{Q}_{i,g} + A_i^{\mathrm{T}}\mathcal{Q}_{i,g} + \mathcal{Q}_{i,g}A_i,$$

$$\Psi_{i,g} = G\left(\frac{\mathcal{Q}_{i,g+1}}{\check{\mathcal{T}}_i} - \frac{\mathcal{Q}_{i,g}}{\hat{\mathcal{T}}_i}\right) + (\ell_i\rho + \alpha)\mathcal{Q}_{i,g+1} + A_i^{\mathrm{T}}\mathcal{Q}_{i,g+1} + \mathcal{Q}_{i,g+1}A_i,$$

and

$$K = \operatorname{diag}\{k_1, k_2, \dots, k_m\},$$

system (1) *maintains absolute exponential stability via MDRDT switching with* $\tau_0 < ln(\mu)/\alpha$*.*

Proof. For convenience, set

$$c_1 = \min_{i\in\mathbf{M},\, g\in\mathbf{G}_0}\bigl\{\lambda_{\min}(\mathcal{Q}_{i,g})\bigr\}, \qquad c_2 = \max_{i\in\mathbf{M},\, g\in\mathbf{G}_0}\bigl\{\lambda_{\max}(\mathcal{Q}_{i,g})\bigr\}$$

and

$$\mathcal{G}\bigl(t, x(t)\bigr) = \mathcal{G}_{\sigma(t)}\bigl(t, x(t)\bigr).$$

Next, the proof will be divided into two sections.

Part I. We will prove by induction that for $t \in R_p$ and $p \in \mathbb{N}$, $\mathcal{G}(t, x(t))$ satisfies

$$\mathcal{G}\bigl(t, x(t)\bigr) < \rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^p. \tag{9}$$

(i) Let us first demonstrate that (9) holds when $p = 0$. That is,

$$\mathcal{G}\bigl(t, x(t)\bigr) < \rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2, \quad t \in R_0.$$

Obviously, for $t = 0$, it is easy to obtain

$$\mathcal{G}\bigl(t, x(t)\bigr) \leqslant c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2 < \rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2.$$

Next, it is proved that (9) is valid when $t \in R_0$. Otherwise, for $t \in \underline{R}_0 = (0, t_1)$, it holds that

$$\mathcal{G}\big(t, x(t)\big) \geqslant \rho c_2 \|\widehat{\varpi}\|_{\hat{\epsilon}}^2.$$

Set

$$\underline{t} = \inf\big\{t \in \underline{R}_0\colon\ \mathcal{G}\big(t, x(t)\big) \geqslant \rho c_2 \|\widehat{\varpi}\|_{\hat{\epsilon}}^2\big\}$$

and

$$\bar{t} = \sup\big\{t \in [0, \underline{t})\colon\ \mathcal{G}\big(t, x(t)\big) \leqslant c_2 \|\widehat{\varpi}\|_{\hat{\epsilon}}^2\big\}.$$

Then, since $\mathcal{G}(t, x(t))$ remains continuous on R_0, we infer

$$\mathcal{G}\big(\bar{t}, x(\bar{t})\big) = c_2 \|\widehat{\varpi}\|_{\hat{\epsilon}}^2$$

and

$$\mathcal{G}\big(\underline{t}, x(\underline{t})\big) = \rho c_2 \|\widehat{\varpi}\|_{\hat{\epsilon}}^2.$$

Then one can obtain that for $t \in [\bar{t}, \underline{t}]$,

$$\mathcal{G}\big(t + \tilde{\tau}, x(t + \tilde{\tau})\big) \leqslant \rho \mathcal{G}\big(t, x(t)\big), \tag{10}$$

where $\tilde{\tau} \in [-\hat{\epsilon}, 0)$. Next, consider the derivation of $\mathcal{G}(t, x(t))$ in regard to system (1) on range $[\bar{t}, \underline{t}]$:

$$\begin{aligned}
\dot{\mathcal{G}}\big(t, x(t)\big) &= \mathrm{e}^{\alpha t}\big\{x^{\mathrm{T}}(t)\dot{\Theta}_i(t)x(t) + \alpha\big(x^{\mathrm{T}}(t)\Theta_i(t)x(t)\big) + 2x^{\mathrm{T}}(t)\Theta_i(t)\dot{x}(t)\big\} \\
&= \mathrm{e}^{\alpha t}\big\{x^{\mathrm{T}}(t)\big[\dot{\Theta}_i(t) + \alpha\Theta_i(t) + A_i^{\mathrm{T}}\Theta_i(t) + \Theta_i(t)A_i\big]x(t) \\
&\quad + 2x^{\mathrm{T}}(t)\Theta_i(t)\big[A_{di}x\big(t - \epsilon(t)\big) + B_i f\big(\varsigma(t)\big) + B_{di} f\big(\varsigma\big(t - \epsilon(t)\big)\big)\big]\big\}.
\end{aligned}$$

Using the S-process with $2\gamma\varsigma^{\mathrm{T}} f(\varsigma) (\gamma > 0)$ [21], the above equation can be rewritten as

$$\begin{aligned}
\dot{\mathcal{G}}\big(t, x(t)\big) &= \mathrm{e}^{\alpha t}\big\{x^{\mathrm{T}}(t)\big[\dot{\Theta}_i(t) + \alpha\Theta_i(t) + A_i^{\mathrm{T}}\Theta_i(t) + \Theta_i(t)A_i\big]x(t) \\
&\quad + 2x^{\mathrm{T}}(t)\Theta_i(t)\big[A_{di}x\big(t - \epsilon(t)\big) + B_i f\big(\varsigma(t)\big) + B_{di} f\big(\varsigma\big(t - \epsilon(t)\big)\big)\big] \\
&\quad + 2\gamma\varsigma^{\mathrm{T}}(t) f\big(\varsigma(t)\big) + 2\gamma\varsigma^{\mathrm{T}}\big(t - \epsilon(t)\big) f\big(\varsigma\big(t - \epsilon(t)\big)\big) \\
&\quad - 2\gamma\varsigma^{\mathrm{T}}(t) f\big(\varsigma(t)\big) - 2\gamma\varsigma^{\mathrm{T}}\big(t - \epsilon(t)\big) f\big(\varsigma\big(t - \epsilon(t)\big)\big)\big\}.
\end{aligned} \tag{11}$$

Clearly, $f(\varsigma) \in \mathcal{S}_{[0,K]}$ yields

$$f^{\mathrm{T}}(\varsigma)K^{-1} f(\varsigma) \leqslant \varsigma^{\mathrm{T}} f(\varsigma). \tag{12}$$

Then it renders from (2) to (10)–(12) that for any constant $\ell_i > 0$,

$$\begin{aligned}
\dot{\mathcal{G}}\big(t, x(t)\big) \leqslant{}& \mathrm{e}^{\alpha t}\big\{x^{\mathrm{T}}(t)\big[\dot{\Theta}_i(t) + \alpha\Theta_i(t) + A_i^{\mathrm{T}}\Theta_i(t) + \Theta_i(t)A_i\big]x(t) \\
& + 2x^{\mathrm{T}}(t)\Theta_i(t)\big[A_{di}x\big(t - \epsilon(t)\big) + B_i f\big(\varsigma(t)\big) + B_{di} f\big(\varsigma\big(t - \epsilon(t)\big)\big)\big] \\
& + 2\gamma\varsigma^{\mathrm{T}}(t) f\big(\varsigma(t)\big) + 2\gamma\varsigma^{\mathrm{T}}\big(t - \epsilon(t)\big) f\big(\varsigma\big(t - \epsilon(t)\big)\big) \\
& - 2\gamma\varsigma^{\mathrm{T}}(t) f\big(\varsigma(t)\big) - 2\gamma\varsigma^{\mathrm{T}}\big(t - \epsilon(t)\big) f\big(\varsigma\big(t - \epsilon(t)\big)\big) \\
& + \ell_i\big[\rho x^{\mathrm{T}}(t)\Theta_i(t)x(t) - x^{\mathrm{T}}\big(t - \epsilon(t)\big)\Theta_j\big(t - \epsilon(t)\big)x\big(t - \epsilon(t)\big)\big]\big\}
\end{aligned}$$

$$
\begin{aligned}
&\leqslant \mathrm{e}^{\alpha t}\Bigg\{x^{\mathrm{T}}(t)\bigg[G\bigg(\frac{\mathcal{Q}_{i,g+1}}{\check{\mathcal{T}}_i}-\frac{\mathcal{Q}_{i,g}}{\hat{\mathcal{T}}_i}\bigg)+(\ell_i\rho+\alpha)\Theta_i(t)\\
&\quad+A_i^{\mathrm{T}}\Theta_i(t)+\Theta_i(t)A_i\bigg]x(t)-\ell_i x^{\mathrm{T}}\big(t-\epsilon(t)\big)\bar{\mathcal{Q}}x\big(t-\epsilon(t)\big)\\
&\quad+2x^{\mathrm{T}}(t)\big(\Theta_i(t)B_i+\gamma C\big)f\big(\varsigma(t)\big)+2x^{\mathrm{T}}(t)\Theta_i(t)B_{di}f\big(\varsigma\big(t-\epsilon(t)\big)\big)\\
&\quad+2x^{\mathrm{T}}(t)\Theta_i(t)A_{di}x\big(t-\epsilon(t)\big)+2\gamma x^{\mathrm{T}}\big(t-\epsilon(t)\big)Cf\big(\varsigma\big(t-\epsilon(t)\big)\big)\\
&\quad-2\gamma f^{\mathrm{T}}\big(\varsigma(t)\big)K^{-1}f^{\mathrm{T}}\big(\varsigma(t)\big)-2\gamma f^{\mathrm{T}}\big(\varsigma\big(t-\epsilon(t)\big)\big)K^{-1}f\big(\varsigma\big(t-\epsilon(t)\big)\big)\Bigg\}\\
&=\frac{t_p+(g+1)d_p-t}{d_p}H_{i,g}+\frac{t-t_p-gd_p}{d_p}E_{i,g},
\end{aligned}
$$

where $H_{i,g}=\theta^{\mathrm{T}}\Gamma_1\theta$ and $E_{i,g}=\theta^{\mathrm{T}}\Gamma_2\theta$ with

$$\theta=\big[x^{\mathrm{T}}(t)\,x^{\mathrm{T}}\big(t-\epsilon(t)\big)\,f^{\mathrm{T}}\big(\varsigma(t)\big)\,f^{\mathrm{T}}\big(\varsigma\big(t-\epsilon(t)\big)\big)\big]^{\mathrm{T}}.$$

According to (5), $H_{i,g}<0$. Similarly, (6) yields $E_{i,g}<0$. Obviously, for $t\in[\bar{t},\underline{t}]$,

$$\dot{\mathcal{G}}\big(t,x(t)\big)<0,$$

which implies $\mathcal{G}(\bar{t},x(\bar{t}))\geqslant\mathcal{G}(\underline{t},x(\underline{t}))$, i.e., $c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\geqslant\rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2$. The contradiction arises. Therefore, we infer that (9) holds for $t\in R_0$.

(ii) Assume that (9) holds on R_{p-1} for $p=1,2,\dots,s$, $s\geqslant 1\in\mathbb{N}$, i.e.,

$$\mathcal{G}\big(t,x(t)\big)<\rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^{p-1},\quad t\in R_{p-1},\ p=1,2,\dots,s.$$

Next, let us display that (9) is also valid on R_s. For $t=t_s$, (8) implies

$$\mathcal{G}\big(t_s,x(t_s)\big)\leqslant\mu\mathcal{G}\big(t_s^-,x\big(t_s^-\big)\big)<\rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^s.$$

Based on this, we will prove

$$\mathcal{G}\big(t,x(t)\big)<\rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^s,\quad t\in R_s.$$

Otherwise, for $t\in\underline{R}_s$, it holds that

$$\mathcal{G}\big(t,x(t)\big)\geqslant\rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^s.$$

Set

$$\underline{t}=\inf\big\{t\in\underline{R}_s\colon\ \mathcal{G}\big(t,x(t)\big)\geqslant\rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^s\big\}$$

and

$$\bar{t}=\sup\big\{t\in[t_s,\underline{t})\colon\ \mathcal{G}\big(t,x(t)\big)\leqslant c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^{s-1}\big\}.$$

Note that, if $\{t\in[t_s,\underline{t})\colon\ \mathcal{G}(t,x(t))\leqslant c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^{s-1}\}=\emptyset$, set $\bar{t}=t_s$. Clearly,

$$c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^{s-1}<\rho c_2\|\widehat{\varpi}\|_{\hat{\epsilon}}^2\mu^s$$

results from $\rho > 1/\mu$. Thus, based on Assumption 1, (10) holds. Through an argument similar to the case R_0, we conclude that (9) is valid on R_s. This completes the induction, and we have (9) holds for $p = 1, 2, \dots$.

Part II. We shall show that system (1) satisfies absolute exponential stability. By (3), obviously,

$$\mathcal{G}\bigl(t, x(t)\bigr) \geqslant c_1 \bigl\|x(t)\bigr\|^2 \mathrm{e}^{\alpha t}.$$

Furthermore, associating with Assumption 2 and (9) yields

$$\begin{aligned} c_1 \bigl\|x(t)\bigr\|^2 \mathrm{e}^{\alpha t} &\leqslant \mathcal{G}\bigl(t, x(t)\bigr) \leqslant \rho c_2 \|\widehat{\varpi}\|_{\hat{\epsilon}}^2 \mu^p \leqslant \rho c_2 \|\widehat{\varpi}\|_{\hat{\epsilon}}^2 \mu^{N_\sigma(0,t)} \\ &\leqslant \rho c_2 \|\widehat{\varpi}\|_{\hat{\epsilon}}^2 \mathrm{e}^{\ln(\mu)t/\tau_0 - N_0 \ln \mu} \end{aligned}$$

for any $t \in [-\hat{\epsilon}, \infty)$, where $N_\sigma(0, t) = p$ for $t \in R_p$. Therefore, we get

$$\bigl\|x(t)\bigr\| \leqslant \gamma_0 \mathrm{e}^{-\alpha_0 t} \|\widehat{\varpi}\|_{\hat{\epsilon}},$$

where $\alpha_0 = (\alpha - \ln(\mu)/\tau_0)/2$ and $\gamma_0 = (\rho c_2/c_1)^{1/2} \mathrm{e}^{-N_0 \ln(\mu)/2}$. Consequently, system (1) satisfies absolute exponential stability. The proof is complete. □

Based on the linear matrix inequality, the admissible range of dwell time that guarantees absolute exponential stability of system (1) can be estimated by $\hat{\mathcal{T}}_i^* = \max_{\check{\mathcal{T}}_i < \hat{\mathcal{T}}_i} \{\hat{\mathcal{T}}_i\colon$ (5)–(8) hold$\}$; $\check{\mathcal{T}}_i^* = \min_{\check{\mathcal{T}}_i < \hat{\mathcal{T}}_i} \{\check{\mathcal{T}}_i\colon$ (5)–(8) hold$\}$. Then we can get the stability region ensuring the absolute exponential stability of STDLS under MDRDT switching.

Under RDT switching, a new criterion of absolute exponential stability of system (1) will be proposed.

Corollary 1. *Under Assumptions* 1 *and* 2, *if there are constants* $\ell_i > 0$, $\gamma > 0$, $\rho > 1/\mu > 1$, $\alpha < 0$ *and matrices* $\mathcal{Q}_{i,g} > 0$, $g \in \mathbf{G}_0$, $\bar{\mathcal{Q}} > 0$ *satisfying* (7), (8), *and*

$$\begin{bmatrix} \widetilde{\Phi}_{i,g} & \mathcal{Q}_{i,g} A_{di} & \mathcal{Q}_{i,g} B_i + \gamma C & \mathcal{Q}_{i,g} B_{di} \\ * & -\ell_i \bar{\mathcal{Q}} & 0 & \gamma C \\ * & * & -2\gamma K^{-1} & 0 \\ * & * & * & -2\gamma K^{-1} \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \widetilde{\Psi}_{i,g} & \mathcal{Q}_{i,g+1} A_{di} & \mathcal{Q}_{i,g+1} B_i + \gamma C & \mathcal{Q}_{i,g+1} B_{di} \\ * & -\ell_i \bar{\mathcal{Q}} & 0 & \gamma C \\ * & * & -2\gamma K^{-1} & 0 \\ * & * & * & -2\gamma K^{-1} \end{bmatrix} < 0 \tag{14}$$

for any $i \in \mathbf{M}$, $g \in \underline{\mathbf{G}}_0$, *where*

$$\begin{aligned} \widetilde{\Phi}_{i,g} &= G\biggl(\frac{\mathcal{Q}_{i,g+1}}{\check{\mathcal{T}}} - \frac{\mathcal{Q}_{i,g}}{\hat{\mathcal{T}}}\biggr) + (\ell_i \rho + \alpha)\mathcal{Q}_{i,g} + A_i^{\mathrm{T}} \mathcal{Q}_{i,g} + \mathcal{Q}_{i,g} A_i, \\ \widetilde{\Psi}_{i,g} &= G\biggl(\frac{\mathcal{Q}_{i,g+1}}{\check{\mathcal{T}}} - \frac{\mathcal{Q}_{i,g}}{\hat{\mathcal{T}}}\biggr) + (\ell_i \rho + \alpha)\mathcal{Q}_{i,g+1} + A_i^{\mathrm{T}} \mathcal{Q}_{i,g+1} + \mathcal{Q}_{i,g+1} A_i, \end{aligned}$$

and $K = \operatorname{diag}\{k_1, k_2, \dots, k_m\}$, *system* (1) *maintains absolute exponential stability via RDT switching with* $\tau_0 < \ln(\mu)/\alpha$.

Under RDT switching, the dwell time $\mathcal{T}_p$ satisfies $\check{\mathcal{T}} \leqslant \mathcal{T}_p \leqslant \hat{\mathcal{T}}$. Then the corollary could be derived from Theorem 1.

Remark 3. Some significant research achievements for stability of STDLS under arbitrary switching or average dwell-time switching have been put forward [4, 23, 36, 49]. However, these research are only aimed at STDLS with modes stable. For STDLS composed fully of unstable modes, above methods will be invalid. In this sense, Theorem 1 gives absolute exponential stability conditions of STDLS (1) with all modes unstable under MDRDT constraint.

Remark 4. This paper fully considers the stabilization characteristics of switching behaviors. In Theorem 1, even if all models are unstable, stability can be achieved through switching behaviors because of the decrease of the Lyapunov function value, i.e., $\mathcal{G}(t_p, x(t_p)) \leqslant \mu\mathcal{G}(t_p^-, x(t_p^-))$, $0 < \mu < 1$. Based on (3), the above inequality turns into $\mathcal{Q}_i(t_p) \leqslant \mu\mathcal{Q}_j(t_p^-)$. As we know, the technique of multiple Lyapunov functions based on average dwell time is an effective method to study the stability of the system, and some interesting results have also been put forward [8,9,12]. However, if we adopt the classical multiple Lyapunov functions technique by letting $\mathcal{Q}_{i,l} \equiv \mathcal{Q}_i$, the above condition will never hold. Thus, we construct a switching time-varying Lyapunov–Razumikhin function to effectively solve this question.

Remark 5. From Theorem 1 we can see that the value of G determines the complexity of calculation. Generally, we will give a smaller value of G in advance for the convenience of calculation. In fact, it also directly affects the conservativeness of the results. A less conservative result can be obtained with the increase of the value of G. The detailed analysis will be given in Section 5. In practice, we can choose the appropriate G according to the demand.

On this basis, Theorem 1 can be further extended to switching linear delay systems

$$\begin{aligned} &\dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x\bigl(t - \epsilon(t)\bigr), \\ &x(t) = \varpi(t), \quad t \in [-\hat{\epsilon}, 0]. \end{aligned} \tag{15}$$

Next, we shall provide a corresponding corollary.

Corollary 2. *Under Assumptions* 1 *and* 2*, if there are constants* $\ell_i > 0$*,* $\rho > 1/\mu > 1$*,* $\alpha < 0$ *and matrices* $\mathcal{Q}_{i,g} > 0$*,* $g \in \mathbf{G}_0$*,* $\bar{\mathcal{Q}} > 0$ *satisfying* (7)*,* (8)*, and*

$$\begin{bmatrix} \widehat{\Phi}_{i,g} & \mathcal{Q}_{i,g}A_{di} \\ * & -\ell_i\bar{\mathcal{Q}} \end{bmatrix} < 0, \qquad \begin{bmatrix} \widehat{\Psi}_{i,g} & \mathcal{Q}_{i,g+1}A_{di} \\ * & -\ell_i\bar{\mathcal{Q}} \end{bmatrix} < 0,$$

for $i \in \mathbf{M}$*,* $g \in \underline{\mathbf{G}}_0$*, where*

$$\begin{aligned} \widehat{\Phi}_{i,g} &= G\biggl(\frac{\mathcal{Q}_{i,g+1}}{\check{\mathcal{T}}_i} - \frac{\mathcal{Q}_{i,g}}{\hat{\mathcal{T}}_i}\biggr) + (\ell_i\rho + \alpha)\mathcal{Q}_{i,g} + A_i^{\mathrm{T}}\mathcal{Q}_{i,g} + \mathcal{Q}_{i,g}A_i, \\ \widehat{\Psi}_{i,g} &= G\biggl(\frac{\mathcal{Q}_{i,g+1}}{\check{\mathcal{T}}_i} - \frac{\mathcal{Q}_{i,g}}{\hat{\mathcal{T}}_i}\biggr) + (\ell_i\rho + \alpha)\mathcal{Q}_{i,g+1} + A_i^{\mathrm{T}}\mathcal{Q}_{i,g+1} + \mathcal{Q}_{i,g+1}A_i, \end{aligned}$$

system (15) *maintains exponential stability via MDRDT switching with* $\tau_0 < \ln(\mu)/\alpha$*.*

3 Application in switching Hopfield neural network

In this section, we shall use obtained achievements in absolute exponential stability analysis of switching delay Hopfield neural network. Consider the following switching delay Hopfield neural network:

$$\dot{\zeta}_i(t) = -a_i^{\sigma(t)}\zeta(t) + \sum_{j=1}^{n} d_{ij}^{\sigma(t)}\beta_j\big(\zeta_j\big(t-\epsilon(t)\big)\big) + r_i$$

or, equivalently,

$$\dot{\zeta}(t) = -A_{\sigma(t)}\zeta(t) + D_{\sigma(t)}\beta\big(\zeta\big(t-\epsilon(t)\big)\big) + r \tag{16}$$

in which $\zeta(t)$, $r \in \mathbb{R}^n$, denote neural state vector and constant external input, respectively, $\beta(\zeta(t-\epsilon(t))) \in \mathcal{S}_{[0,K]}$ represents neuron activation function vector, $A_i \in \mathbb{R}^{n\times n}$ are positive diagonal matrices, $D_i \in \mathbb{R}^{n\times n}$ are interconnection matrices.

Assume that ζ^* is the equilibrium point of (16). By a transformation $\overline{\nu} = \zeta(t) - \zeta^*$, system (16) is equivalent to

$$\dot{\overline{\nu}}(t) = -A_{\sigma(t)}\overline{\nu}(t) + D_{\sigma(t)}\psi\big(\overline{\nu}\big(t-\epsilon(t)\big)\big), \tag{17}$$

where $\psi(\overline{\nu}) = \beta(\overline{\nu}+\zeta^*) - \beta(\zeta^*)$ with $\psi_j(\overline{\nu}_j) \in \mathcal{S}_{[0,k_j]}$, $k_j > 0$, $j = 1, 2, \dots, n$. Based on supposition, the origin is the sole equilibrium in system (17).

By Theorem 1 we shall obtain directly stability criterion.

Theorem 2. *Under Assumptions* 1 *and* 2*, if there are constants* $\ell_i > 0$*,* $\gamma > 0$*,* $\rho > 1/\mu > 1$*,* $\alpha < 0$ *and matrices* $\mathcal{Q}_{i,g} > 0$*,* $g \in \mathbf{G}_0$*,* $\bar{\mathcal{Q}} > 0$ *satisfying* (7)*,* (8)*, and*

$$\begin{bmatrix} \Xi_{i,g} & 0 & \mathcal{Q}_{i,g}D_i \\ * & -\ell_i\bar{\mathcal{Q}} & \gamma I \\ * & * & -2\gamma K^{-1} \end{bmatrix} < 0, \tag{18}$$

$$\begin{bmatrix} \Pi_{i,g} & 0 & \mathcal{Q}_{i,g+1}D_i \\ * & -\ell_i\bar{\mathcal{Q}} & \gamma I \\ * & * & -2\gamma K^{-1} \end{bmatrix} < 0 \tag{19}$$

for $i \in \mathbf{M}$*,* $g \in \underline{\mathbf{G}}_0$*, where*

$$\Xi_{i,g} = G\left(\frac{\mathcal{Q}_{i,g+1}}{\check{\mathcal{T}}_i} - \frac{\mathcal{Q}_{i,g}}{\hat{\mathcal{T}}_i}\right) + (\ell_i\rho + \alpha)\mathcal{Q}_{i,g} - A_i^{\mathrm{T}}\mathcal{Q}_{i,g} - \mathcal{Q}_{i,g}A_i,$$

$$\Pi_{i,g} = G\left(\frac{\mathcal{Q}_{i,g+1}}{\check{\mathcal{T}}_i} - \frac{\mathcal{Q}_{i,g}}{\hat{\mathcal{T}}_i}\right) + (\ell_i\rho + \alpha)\mathcal{Q}_{i,g+1} - A_i^{\mathrm{T}}\mathcal{Q}_{i,g+1} - \mathcal{Q}_{i,g+1}A_i,$$

system (17) *maintains absolute exponential stability via MDRDT switching with* $\tau_0 < \ln(\mu)/\alpha$*.*

Remark 6. System (1) degenerates into system (17) when $A_{di} = 0$, $B_i = 0$ $B_{di} = D_i$, and $C = I$ in which I denotes unit matrix. Stability conditions of Theorem 2 are obtained by replacing θ with $[\overline{\nu}^{\mathrm{T}}(t)\,\overline{\nu}^{\mathrm{T}}(t-\epsilon(t))\psi^{\mathrm{T}}(\overline{\nu}(t-\epsilon(t)))]^{\mathrm{T}}$ in the proof of Theorem 1.

4 Numerical example

We now use simulation examples to elucidate the above achievements.

Example 1. Consider STDLS (1) with

$$A_1 = \begin{bmatrix} -1.8 & 0.5 \\ 0.5 & -0.1 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} 0.01 & 0.02 \\ 0.09 & 0.03 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.1 & -0.1 \\ 0.2 & 0.15 \end{bmatrix}, \qquad B_{d1} = \begin{bmatrix} -0.04 & -0.06 \\ 0.15 & 0.05 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.15 & -0.85 \\ 0.1 & -1.3 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 0.02 & 0.11 \\ 0.06 & 0.01 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.1 & -0.1 \\ 0.3 & 0.25 \end{bmatrix}, \qquad B_{d2} = \begin{bmatrix} -0.05 & -0.05 \\ 0.1 & 0.2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & -0.1 \end{bmatrix},$$

where $f_i(\varsigma_i) = 0.02\varsigma_i/(\varsigma_i^2 + 1)$, $i = 1, 2$, and $\epsilon(t) = 0.1 - 0.1 \sin t$. It follows that $k_i = 0.02$, $i = 1, 2$, and $\hat{\epsilon} = 0.2$. Each subsystem is unstable as shown in Fig. 1. This means that these researches in [4, 23, 36, 49], which are only aimed at STDLS with modes stable, cannot be applied to this situation. Next, we shall show that our conclusion is valid. Choose $G = 1$, $\ell_1 = \ell_2 = 1$, $\gamma = 0.2$, $\alpha = -0.01$, $\mu = 0.991$, and $\rho = 1.01$. Based on inequalities (7), (8), (13), (14) in Corollary 1 and given $\check{\mathcal{T}} = 0.248$, $\hat{\mathcal{T}} = 0.25$, we obtain the feasible solution

$$\mathcal{Q}_{1,0} = \begin{bmatrix} 4.6373 & 1.5794 \\ 1.5794 & 9.0033 \end{bmatrix}, \qquad \mathcal{Q}_{1,1} = \begin{bmatrix} 7.1815 & 0.2245 \\ 0.2245 & 6.6333 \end{bmatrix},$$

$$\mathcal{Q}_{2,0} = \begin{bmatrix} 7.1000 & 0.2237 \\ 0.2237 & 6.5647 \end{bmatrix}, \qquad \mathcal{Q}_{2,1} = \begin{bmatrix} 4.6915 & 1.5942 \\ 1.5942 & 9.0956 \end{bmatrix},$$

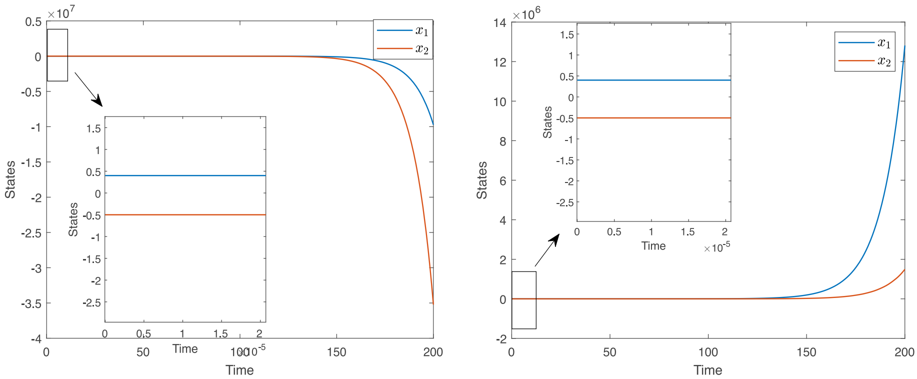
$$\bar{\mathcal{Q}} = \begin{bmatrix} 3.9881 & 1.0267 \\ 1.0267 & 4.9228 \end{bmatrix}.$$

On the other hand, using Theorem 1 and keeping $\check{\mathcal{T}}_2 = 0.248$, $\hat{\mathcal{T}}_1 = \hat{\mathcal{T}}_2 = 0.25$, we can find that Eqs. (5)–(8) hold for

$$\mathcal{Q}_{1,0} = \begin{bmatrix} 0.4777 & 0.1403 \\ 0.1403 & 0.8578 \end{bmatrix}, \qquad \mathcal{Q}_{1,1} = \begin{bmatrix} 0.7317 & 0.0040 \\ 0.0040 & 0.6291 \end{bmatrix},$$

$$\mathcal{Q}_{2,0} = \begin{bmatrix} 0.7249 & 0.0040 \\ 0.0040 & 0.6233 \end{bmatrix}, \qquad \mathcal{Q}_{2,1} = \begin{bmatrix} 0.4822 & 0.1415 \\ 0.1415 & 0.8658 \end{bmatrix},$$

$$\bar{\mathcal{Q}} = \begin{bmatrix} 0.4684 & 0.1144 \\ 0.1144 & 0.5567 \end{bmatrix},$$



(a) Subsystem 1

(b) Subsystem 2

Figure 1. State trajectories of subsystems without switching.

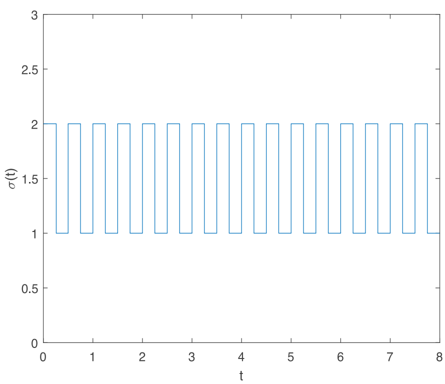


Figure 2. Switching law of system (1).

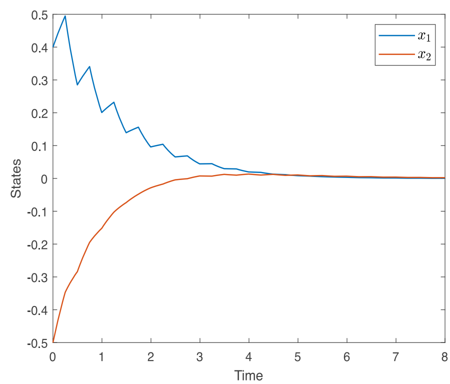


Figure 3. State trajectories of system (1).

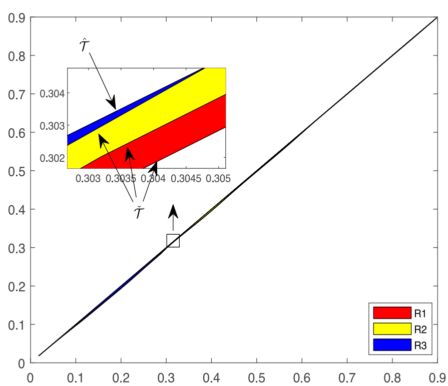


Figure 4. Stability region: R_3 is stability region for $G = 1$, $R_2 \cup R_3$ is stability region for $G = 2$, and $\bigcup_{i=1,2,3} R_i$ is stability region for $G = 8$, respectively.

and $\check{\mathcal{T}}_1 = 0.243$. Obviously, the allowable minimal dwell time $\check{\mathcal{T}}_1$ given in Theorem 1 is superior to that given in Corollary 1, which implies that the designed MDRDT switching signal is more general. Further, choosing $x(0) = [0.4; -0.5]$, Figs. 2 and 3 show switching law $\sigma(t)$ and trajectories of state, respectively.

Next, we will analyze the influence of G on stability region. The relationship between the stability region and G is shown in Fig. 4. As we can see, the stability region expands as the increase of G. Therefore, we can obtain the less conservative results by choosing a larger G.

Example 2. Consider switching delay Hopfield neural network (17) with:

$$A_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0.7 & 0.8 \\ 0.1 & -0.3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} -0.1 & 0.2 \\ 0.7 & 0.7 \end{bmatrix},$$

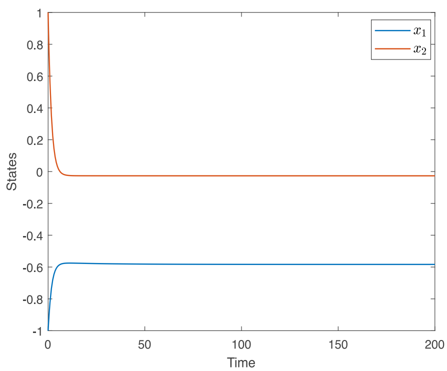
where $\psi_i(\overline{\nu}_i) = 0.3\tanh(\overline{\nu}_i)$, $i = 1, 2$ and $\epsilon(t) = 0.01 + 0.01\cos t$. It follows that $k_i = 0.3$, $i = 1, 2$ and $\hat{\epsilon} = 0.02$. Each subsystem is unstable as shown in Fig. 5. Thus, the methods in [1, 15, 28] are unavailable. Choose $G = 1$, $\ell_1 = \ell_2 = 0.46$, $\gamma = 0.01$, $\alpha = -0.75$, $\mu = 0.9$ and $\rho = 1.2$. Based on the inequalities (7), (8), (18), (19) in Theorem 2 and given $\check{\mathcal{T}}_1 = 0.192$, $\check{\mathcal{T}}_2 = 0.196$, $\hat{\mathcal{T}}_1 = 0.2$, $\hat{\mathcal{T}}_2 = 0.204$, we obtain

$$\mathcal{Q}_{1,0} = \begin{bmatrix} 0.0132 & 0.0015 \\ 0.0015 & 0.0079 \end{bmatrix}, \qquad \mathcal{Q}_{1,1} = \begin{bmatrix} 0.0134 & 0.0017 \\ 0.0017 & 0.0094 \end{bmatrix},$$

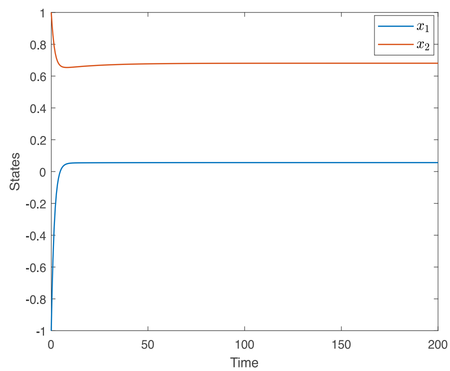
$$\mathcal{Q}_{2,0} = \begin{bmatrix} 0.0120 & 0.0015 \\ 0.0015 & 0.0085 \end{bmatrix}, \qquad \mathcal{Q}_{2,1} = \begin{bmatrix} 0.0147 & 0.0017 \\ 0.0017 & 0.0088 \end{bmatrix},$$

$$\bar{\mathcal{Q}} = \begin{bmatrix} 0.0102 & 0.0020 \\ 0.0020 & 0.0075 \end{bmatrix}.$$

Theorem 2 implies that designed MDRDT switching law with $\check{\mathcal{T}}_1 = 0.192$, $\check{\mathcal{T}}_2 = 0.196$, $\hat{\mathcal{T}}_1 = 0.2$, $\hat{\mathcal{T}}_2 = 0.204$ can stabilize system. Then, choosing $x(0) = [-1; 1]$, Figs. 6 and 7 show switching law $\sigma(t)$ and trajectory of state, respectively.



(a) Subsystem 1



(b) Subsystem 2

Figure 5. State trajectories of subsystems without switching.

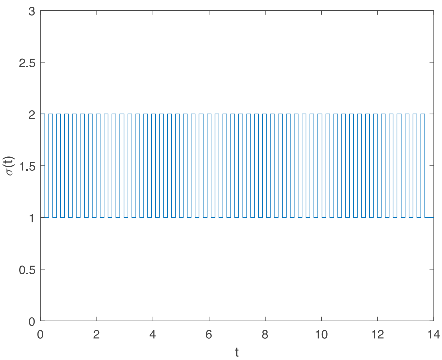


Figure 6. Switching law of system (17).

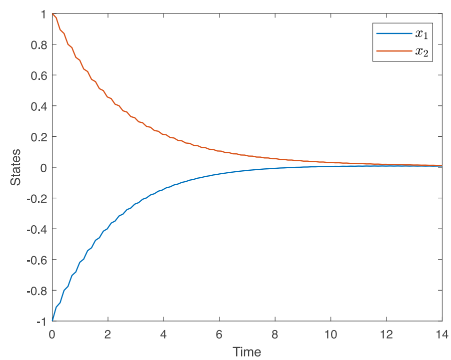


Figure 7. State trajectories of system (17).

5 Conclusion

The problem for absolute exponential stability of STDLS was investigated in this paper. Through switching time-varying Lyapunov–Razumikhin function technique and the stabilization performance of switching behaviors, computable sufficient conditions of the absolute exponential stability of STDLS were derived. It turns out that the absolute exponential stability of STDLS consisting entirely of unstable modes can be realized via MDRDT switching. Moreover, the obtained results were extended to switching delay Hopfield neural network model. Simulations have been carried out to elucidate the obtained conclusions.

In the control synthesis of real switching system, asynchronous behavior of the controller and the underlying plant is ubiquitous, which makes the research of stability issues more complicated. Thus, in the future, further research on stability of switching Lurie systems with asynchronous switching is of great significance.

References

1. C.K. Ahn, An H_∞ approach to stability analysis of switched Hopfield neural networks with time-delay, *Nonlinear Dyn.*, **60**(4):703–711, 2010, `https://doi.org/10.1007/s11071-009-9625-6`.
2. A. Aleksandrov, On the existence of a common Lyapunov function for a family of nonlinear positive systems, *Syst. Control Lett.*, **147**(2):104832, 2021, `https://doi.org/10.1016/j.sysconle.2020.104832`.
3. A.Yu. Aleksandrov, Y. Chen, A. Platonov, L. Zhang, Stability analysis for a class of switched nonlinear systems, *Automatica*, **47**(10):2286–2291, 2011, `https://doi.org/10.1016/j.automatica.2011.08.016`.
4. A.Yu. Aleksandrov, O. Mason, Absolute stability and Lyapunov–Krasovskii functionals for switched nonlinear systems with time-delay, *J. Franklin Inst.*, **351**(8):4381–4394, 2014, `https://doi.org/10.1016/j.jfranklin.2014.05.013`.
5. L.I. Allerhand, U. Shaked, Robust control of linear systems via switching, *IEEE Trans. Autom. Control*, **58**(2):506–512, 2013, `https://doi.org/10.1109/TAC.2012.2206715`.

6. C. Briat, Convex conditions for robust stabilization of uncertain switched systems with guaranteed minimum and mode-dependent dwell-time, *Syst. Control Lett.*, **78**:63–72, 2015, https://doi.org/10.1016/j.sysconle.2015.01.012.
7. C. Briat, $L_1 \times \ell_1$-to-$L_1 \times \ell_1$ analysis of linear positive impulsive systems with application to the $L_1 \times \ell_1$-to-$L_1 \times \ell_1$ interval observation of linear impulsive and switched systems, *Nonlinear Anal., Hybrid Syst.*, **34**:1–17, 2017, https://doi.org/10.1016/j.nahs.2019.03.010.
8. D. Cui, C.K. Ahn, Z. Xiang, Fault-tolerant fuzzy observer-based fixed-time tracking control for nonlinear switched systems, *IEEE Trans. Fuzzy Syst.*, **31**(12):4410–4420, 2023, https://doi.org/10.1109/TFUZZ.2023.3284917.
9. D. Cui, W. Zou, J. Guo, Z. Xiang, Neural network-based adaptive finite-time tracking control of switched nonlinear systems with time-varying delay, *Appl. Math. Comput.*, **428**:127216, 2022, https://doi.org/10.1016/j.amc.2022.127216.
10. X. Ding, X. Liu, On stabilizability of switched positive linear systems under state-dependent switching, *Appl. Math. Comput.*, **307**:92–101, 2017, https://doi.org/10.1016/j.amc.2017.03.007.
11. E. Fridman, *Introduction to Time-Delay Systems: Analysis and Control*, Birkhäuser, Basel, 2014.
12. Y. Fu, N. Cui, C. Zhang, T. Li, Finite-time stability of switched systems with delayed arguments and nonlinear perturbations, *Hacet. J. Math. Stat.*, **49**:162–169, 2020, https://doi.org/10.15672/HJMS.2019.669.
13. M. Hajiahmadi, B.D. Schutter, H. Hellendoorn, Stabilization and robust H_∞ control for sector-bounded switched nonlinear systems, *Automatica*, **50**(10):2726–2731, 2014, https://doi.org/10.1016/j.automatica.2014.08.015.
14. J.P. Hespanha, D. Liberzon, A.R. Teel, Lyapunov conditions for input-to-state stability of impulsive systems, *Automatica*, **44**(11):2735–2744, 2008, https://doi.org/10.1016/j.automatica.2008.03.021.
15. H. Huang, Y. Qu, H.X. Li, Robust stability analysis of switched Hopfield neural networks with time-varying delay under uncertainty, *Phys. Lett.*, **345**(4–6):345–354, 2005, https://doi.org/10.1016/j.physleta.2005.07.042.
16. T. Jiao, H. P. Ju, G. Zong, Y. Zhao, Q. Du, On stability analysis of random impulsive and switching neural networks, *Neurocomputing*, **350**:146–154, 2019, https://doi.org/10.1016/j.neucom.2019.03.039.
17. A. Kundu, D. Chatterjee, D. Liberzon, Generalized switching signals for input-to-state stability of switched systems, *Automatica*, **64**:270–277, 2016, https://doi.org/10.1016/j.automatica.2015.11.027.
18. S. Li, C. Ahn, Z. Xiang, Sampled-data adaptive output feedback fuzzy stabilization for switched nonlinear systems with asynchronous switching, *IEEE Trans. Fuzzy Syst.*, **27**(1):200–205, 2019, https://doi.org/10.1109/TFUZZ.2018.2881660.
19. Y. Li, Y. Sun, F. Meng, Exponential stabilization of switched time-varying systems with delays and disturbances, *Appl. Math. Comput.*, **324**:131–140, 2018, https://doi.org/10.1016/j.amc.2017.12.011.

20. J. Lian, K. Zhang, Z. Feng, Stability analysis for switched Hopfield neural networks with time delay, *Optim. Control Appl. Methods*, **33**(4):433–444, 2012, https://doi.org/10.1002/oca.1005.
21. X. Liao, L. Wang, P. Yu, *Stability of Dynamical Systems*, Elsevier, Amsterdam, 2007.
22. D. Liberzon, *Switching in Systems and Control*, Birkhauser, Boston, 2003.
23. J. Liu, Q. Huang, Absolute stability for a class of switched delay system with stable and unstable subsystems, in *The 31th Chinese Control and Decision Conference (CCDC), Nanchang, China, 3–5 June 2019*, IEEE, 2019, pp. 1816–1821, https://doi.org/10.1109/CCDC.2019.8833323.
24. X. Liu, Y. Yu, H. Chen, Stability of perturbed switched nonlinear systems with delays, *Nonlinear Anal., Hybrid Syst.*, **25**:114–125, 2017, https://doi.org/10.1016/j.nahs.2017.03.003.
25. X. Liu, S. Zhong, X. Ding, Robust exponential stability of nonlinear impulsive switched systems with time-varying delays, *Nonlinear Anal. Model. Control*, **17**(2):210–222, 2012, https://doi.org/10.15388/NA.17.2.14069.
26. Z. Liu, X. Zhang, X. Lu, Q. Liu, Stabilization of positive switched delay systems with all modes unstable, *Nonlinear Anal., Hybrid Syst.*, **29**:110–120, 2018, https://doi.org/10.1016/j.nahs.2018.01.004.
27. L. Long, Integral ISS for switched nonlinear time-varying systems using indefinite multiple Lyapunov functions, *IEEE Trans. Autom. Control*, **64**(1):404–411, 2019, https://doi.org/10.1109/TAC.2018.2833159.
28. X. Lou, B. Cui, Delay-dependent criteria for robust stability of uncertain switched Hopfield neural networks, *Int. J. Autom. Comput.*, **4**(3):304–314, 2007, https://doi.org/10.1007/s11633-007-0304-0.
29. J. Mao, C.K. Ahn, Z. Xiang, Global stabilization for a class of switched nonlinear time-delay systems via sampled-data output-feedback control, *IEEE Trans. Syst. Man Cybern., Syst.*, **52**(2):694–705, 2022, https://doi.org/10.1109/TSMC.2020.3048064.
30. J. Nygren, K. Pelckmans, A stability criterion for switching Lur'e systems with switching-path restrictions, *Automatica*, **96**:337–341, 2018, https://doi.org/10.1016/j.automatica.2018.06.038.
31. L. Pan, J. Cao, A. Alsaedi, Stability of reaction–diffusion systems with stochastic switching, *Nonlinear Anal. Model. Control*, **24**(3):315–331, 2019, https://doi.org/10.15388/NA.2019.3.1.
32. W. Qi, X. Yang, X. Gao, J. Cheng, Y. Wei, Stability for delayed switched systems with markov jump parameters and generally incomplete transition rates, *Appl. Math. Comput.*, **365**:124718, 2020, https://doi.org/10.1016/j.amc.2019.124718.
33. H. Ren, G. Zong, T. Li, Event-triggered finite-time control for networked switched linear systems with asynchronous switching, *IEEE Trans. Syst. Man Cybern.*, **48**(11):1874–1884, 2018, https://doi.org/10.1109/TSMC.2017.2789186.
34. L. Shi, R. Tang, X. Yang, J. Ding, Z. Xu, Y. Li, Event-triggered dynamic output quantized control for 2-D switched systems, *Nonlinear Anal. Model. Control*, **28**(1):34–55, 2022, https://doi.org/10.15388/namc.2023.28.29622.

35. Y. Sun, Stabilization of switched systems with nonlinear impulse effects and disturbances, *IEEE Trans. Autom. Control*, **56**(11):2739–2743, 2011, https://doi.org/10.1109/TAC.2011.2159889.
36. Y. Sun, L. Wang, On stability of a class of switched nonlinear systems, *Automatica*, **49**:305–307, 2013, https://doi.org/10.1016/j.automatica.2012.10.011.
37. Y. Tian, Y. Cai, Y. Sun, Stability of switched nonlinear time-delay systems with stable and unstable subsystems, *Nonlinear Anal., Hybrid Syst.*, **24**:58–68, 2017, https://doi.org/10.1016/j.nahs.2016.11.003.
38. Q. Wang, H. Sun, G. Zong, Stability analysis of switched delay systems with all subsystems unstable, *Int. J. Control Autom. Syst.*, **14**(5):1262–1269, 2016, https://doi.org/10.1007/s12555-015-0052-9.
39. W. Xiang, J. Lam, J. Shen, Stability analysis and L_1-gain characterization for switched positive systems under dwell-time constraint, *Automatica*, **85**:1–8, 2017, https://doi.org/10.1016/j.automatica.2017.07.016.
40. W. Xiang, J. Xiao, Stabilization of switched continuous-time systems with all modes unstable via dwell time switching, *Automatica*, **50**(3):940–945, 2014, https://doi.org/10.1016/j.automatica.2013.12.028.
41. H. Xiao, Q. Zhu, Stability analysis of switched stochastic delay system with unstable subsystems, *Nonlinear Anal., Hybrid Syst.*, **42**(98):101075, 2021, https://doi.org/10.1016/j.nahs.2021.101075.
42. D. Yang, X. Li, J. Qiu, Output tracking control of delayed switched systems via state-dependent switching and dynamic output feedback, *Nonlinear Anal., Hybrid Syst.*, **32**:294–305, 2019, https://doi.org/10.1016/j.nahs.2019.01.006.
43. D. Yang, X. Li, J. Shen, Z. Zhou, State-dependent switching control of delayed switched systems with stable and unstable modes, *Math. Methods Appl. Sci.*, **41**(16):6968–6983, 2018, https://doi.org/10.1002/mma.5209.
44. H. Yang, B. Jiang, V. Cocquempot, *Stabilization of Switched Nonlinear Systems with Unstable Modes*, Springer, Cham, 2014, https://doi.org/10.1007/978-3-319-07884-7.
45. X. Yang, X. Li, J. Lu, Z. Cheng, Synchronization of time-delayed complex networks with switching topology via hybrid actuator fault and impulsive effects control, *IEEE Trans. Cybern.*, **50**(9):4043–4052, 2020, https://doi.org/10.1109/TCYB.2019.2938217.
46. X. Yang, X. Wan, C. Zunshui, J. Cao, Y. Liu, L. Rutkowski, Synchronization of switched discrete-time neural networks via quantized output control with actuator fault, *IEEE Trans. Neural Netw. Learn. Syst.*, **32**(9):4191–4201, 2021, https://doi.org/10.1109/TNNLS.2020.3017171.
47. J. Zhang, Z. Han, F. Zhu, X. Zhao, Absolute exponential stability and stabilization of switched nonlinear systems, *Syst. Control Lett.*, **66**:51–57, 2014, https://doi.org/10.1016/j.sysconle.2013.12.021.
48. J. Zhang, T. Raïssi, Saturation control of switched nonlinear systems, *Nonlinear Anal., Hybrid Syst.*, **32**:320–336, 2019, https://doi.org/10.1016/j.nahs.2019.01.005.
49. J. Zhang, X. Zhao, X. Cai, Absolute exponential L_1-gain analysis and synthesis of switched nonlinear positive systems with time-varying delay, *Appl. Math. Comput.*, **284**(5):24–36, 2016, https://doi.org/10.1016/j.amc.2016.02.050.

50. Y. Zhang, Q. Yang, P. Li, W. Yan, Synchronization criterion and control scheme for Lur'e type complex dynamical networks with switching topology and coupling time-varying delay, *Asian J. Control*, **17**(5):1696–1706, 2015, `https://doi.org/10.1002/asjc.980`.

51. W. Zhao, Y. Sun, Absolute exponential stability of switching Lurie systems with time-varying delay via dwell time switching, *J. Franklin Inst.*, **50**:11871–11891, 2023, `https://doi.org/10.1016/j.jfranklin.2023.09.023`.

52. X. Zhao, S. Yin, H. Li, B. Niu, Switching stabilization for a class of slowly switched systems, *IEEE Trans. Autom. Control*, **60**(1):221–226, 2015, `https://doi.org/10.1109/TAC.2014.2322961`.

53. X. Zhao, Y. Yin, B. Niu, X. Zheng, Stabilization for a class of switched nonlinear systems with novel average dwell time switching by T–S fuzzy modeling, *IEEE Trans. Cybern.*, **46**(8):1952–1957, 2016, `https://doi.org/10.1109/TCYB.2015.2458896`.