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Message spreading modeling from the perspective of social psychology, differential equation dynamics, and deep learning technique

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Abstract. To gain a deeper understanding of the characteristics of message spreading, it is crucial to explore various methods of modeling this phenomenon. Given that message spreading is significantly influenced by social media, we propose a modified spreading model informed by social psychology analysis. This approach also incorporates differential equation dynamics and deep learning technique. The proposed model accounts for a cross-transmission mechanism between individuals and social media platforms, as well as a nonlinear spreading rate, to effectively characterize the saturation effect of messages. Utilizing Lyapunov functionals, we carry out a dynamical analysis of the message spreading model. Furthermore, we develop physics-informed neural networks based on deep learning technique that merges the efficiency inherent in data-driven modeling with the precision offered by mathematical modeling. Numerical simulations demonstrate that our prediction method can accurately capture real-time changes in data while correcting deviations observed in data-driven predictions, which highlights the robust potential for multidisciplinary integration among social psychology, differential equation dynamics, and deep learning technique.

Keywords: message spreading, dynamical modeling, deep learning technique, social psychology, cross-transmission mechanism.

1 Introduction

People can understand and transform the world by obtaining and identifying different information about nature and society. With the development of information globalization

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and network technology, information dissemination through online social networks has become the norm [6, 13]. As a kind of information, rumor harms our daily life, which exists in nearly every major social event [17]. The effect of rumors is always negative, and they can hurt the individual, society, and even the country [12, 18]. In numerous cases, gossip usually becomes dishonest people's means and tools of political struggle; it can make the original relatively stable interpersonal relationship become mutual suspicion, conflict, and tension; it can paralyze people's ideological vigilance, weaken people's defensive psychology so that they unknowingly become the captive of rumors; it can destroy people's unity, weaken mutual trust, create internal friction, collapse the effectiveness of the other side. Due to the nasty effects of rumors, powerful means should be put forward to prevent and control them.

On the one hand, the openness and equality of media networks provide a relaxed environment for people to obtain information and express their opinions. On the other hand, its immediacy also facilitates the breeding and spreading of rumors. Pelletier et al. [10] point out that the spread of true and false information promoted by social media can be likened to the transmission of infectious diseases. To accurately simulate the propagation dynamics of rumors in real social networks and effectively alleviate the harm of rumors to society, message spreading modeling by differential equations is a great way. Yu et al. [17] divide the total population into four groups: ignorants, discussants, spreaders, and removers, and reveal the dynamic behavior and mechanism of rumor propagation among these groups by dynamic analysis. Besides, Zhou et al. [21] explain the influence of the information intervention mechanism on the dynamics of rumor propagation by group division. To help design effective rumor control strategies, Yin et al. [16] propose the two-stage rumor propagation dynamics model in which the susceptible/educated–infected–recovered (S/EIR) dynamics model is used to characterize the first stage and the susceptible/educated–infected–denied–recovered (S/EIDR) dynamics model is to describe the second stage. S/EIR or S/EIDR is another group division and similar to the warehouse model widely used in disease transmission modeling.

The mentioned literature mainly focuses on the spread of rumors among people. The spreaders publish false and malicious rumors on public social networks media, and individuals visit these networks and share them with their friends through the friendship network, which provides a convenient hotbed for the indiscriminate spread of rumors. Hence, social media is also the virtual environment in which rumors spread, which is similar to friendship networks. Besides, Zhang et al. [19] utilize a classic susceptible–infected–recovered (SIR) disease transmission model to explore the influence of official refutation. After releasing official rumor refutation, some rumor spreaders are persuaded to alter their views on the rumors due to the elevated credibility and professionalism of official information. As well as changing the opinions of rumor-mongers and reducing their numbers, social media can begin to wash up the mess. To better depict these characteristics, a more scientific model should be established for rumor control, taking into account the impact of official refutations. The model based on partial differential equations (PDEs) may ameliorate this problem [4, 9, 20]. For instance, Ke et al. [5] have established a reaction–diffusion rumor propagation model to reflect the government and media refutation to rumor propagation. But we find that the theoretical analysis of

PDEs is more complex, and the interpretability of the reaction–diffusion term is more complicated.

In addition to using the reaction–diffusion model based on PDEs, media networks and friend networks can be both considered in the modeling process to describe the spatial characteristics of rumor propagation. Xia et al. [15] propose a different message spreading model based on ordinary differential equations (ODEs), combining with the incubation mechanism, cross-transmission mechanism, and general nonlinear spreading rate. By involving the media and people's environment together, message spreading's spatial propagation characteristics is depicted by ODEs. Similarly, Cheng et al. [1, 2] also explore the interaction between media websites and friendship networks.

To establish a more realistic rumor model, it is also necessary to fully investigate the literature related to social psychology. A social psychology study from the University of Kent claimed that while there was no evidence that people exposed to the Internet were more likely to believe conspiracy theories, the Internet made the conspiracy theorists more convinced [8]. Coincidentally, Gustave Le Bon described in his work *The Crowd-A Study of the Popular Mind*: "A lie repeated a thousand times does not become truth, but it will convince the crowd that it is a scientific truth" [7]. Wang et al. [14] study the relationship between rumor spread and emotional polarity, then found that negative sentiment and rumor spread are causally interrelated. When negative emotions accumulate in large numbers, people need to find a way to release them, and spreading rumors can vent negative sentiments without cost. Thus, social psychological factors should be thoroughly considered in modeling the rumor propagation process.

In this paper, we consider a modified message spreading model, which simultaneously considers the general nonlinear spreading rate and cross-transmission mechanism. From the perspective of social psychology, the modeling process is in accord with the realistic phenomenon; by means of differential equation dynamics, we carry out the spreading dynamical behaviors by credible mathematical proof; through deep learning technique, the modeling error of differential equation model is compensated by the data law of message propagation. Hence, the spreading mechanism will be illustrated more clearly.

The organizational structure of this paper is as follows: In Section 2, through social psychology analysis, we propose a modified message spreading model for media-friendship networks; In Section 3, the dynamical analysis of message spreading model is carried out by relevant theories of differential equation dynamics; In Section 4, we carry out rumor prediction analysis powered by artificial intelligence; In Section 5, a message spreading prediction case is given to illustrate the powerful ability of multidisciplinary coupling of social psychology, differential equation dynamics, and deep learning technique. Finally, our conclusions are summarized in Section 6.

2 Social psychology analysis and mathematical modeling

In this section, we introduce modeling ideas of the message spreading. First, we divide media-friendship networks into five groups: people who do not have access to the message $i(t)$, message spreader $s(t)$, people who are not interested in the message $m(t)$, mass

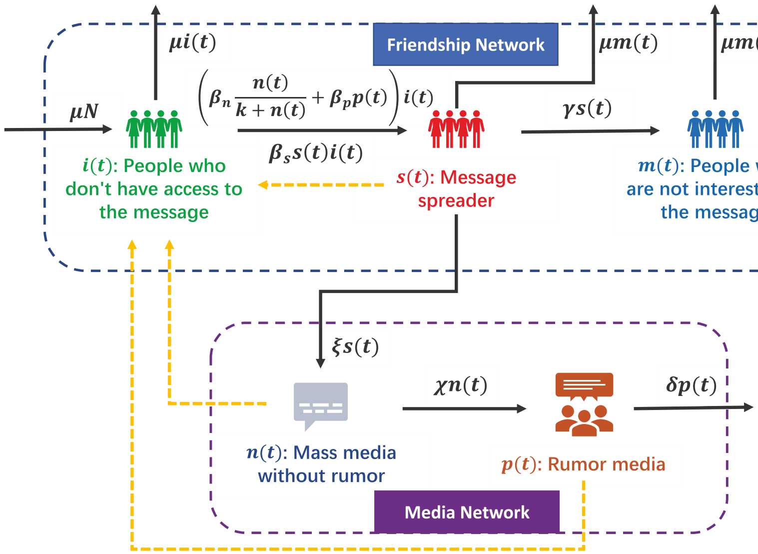

Figure 1. Schematic diagram of message spreading.

media without rumor $n(t)$, and rumor media $p(t)$. Given the ferment of any social event, there are a lot of messages and rumors that are part of them. Throughout the paper, “message” is relatively neutral against “rumor” but includes rumor. By simulating the occurrence of a significant social event, we study the dynamics of message transmission in situations where people spread “messages” on social media, and then people with ulterior motives walk unfounded “rumors”.

In media networks, due to the openness of websites, any user can freely visit these websites, comment on, or spread any information they are interested in. Thus, media websites provide a convenient platform for spreading rumors across geographic space. When malicious people release rumors on media websites, the original rumor-free websites will become sites that carry rumors. Due to that, the disseminator uploads the message to the mass media, such as TikTok or Sina Weibo, $n(t)$ increases at rate $\xi s(t)$. When the spreaders visit the mass media, they post or leave unfounded even harmful comments about the message, making it a rumor media. Thus, $n(t)$ decreases at rate χ, changing the mass media into rumor media. For $p(t)$, it will disappear at rate δ as an official refutation happens, or the truth of the matter comes to light.

As for friendship networks, the message spreaders $s(t)$, after visiting the mass media, spread the message through posts or comments, making part of the mass media evolve into the rumor media. Vice versa, when the $i(t)$ visit $p(t)$, the rumor will have a specific impact on them, which will prompt the $i(t)$ to become spreaders $s(t)$ with a certain even strong probability. Definitions of used variables in the message spreading model are summarized in Table 1. The diagram of message propagation in media-friendship networks is shown in Fig. 1.

Table 1. Definitions of used variables in message spreading model.

No.	Implication
N	The constant of total population in the friendship network
μ	The probability that people move out the group
β_s	Message spreading rate by message spreader
β_n	Message spreading rate by mass media without rumor
β_p	Message spreading rate by rumor media
k	Message concentration in media that yields 50% chance of become spreader
γ	Ratio of $s(t)$ that become not interested in the message
ξ	Ratio of $s(t)$ that spread the message in mass media
χ	Change ratio from message to rumor
δ	Scatter rate of rumor media

The process of message spreading involves extensive expertise in social psychology. Here are some possible points:

(a) *Social influence and norms*. Rumors are a kind of social influence whose spread is influenced by social norms and other people's attitudes or behaviors. In the modeling, γ is involved in describing the ratio of $s(t)$ that become not interested in the rumor, which is influenced by social influence and norms.

(b) *Source trust*. The source credibility of a message has much to do with its possibility of spreading. People are likely to believe information from well-known individuals, authorities, or reliable media. Therefore, the research on the process of rumor propagation needs to consider the source of trust, credibility, influence, and other factors. In the modeling, the value of δ could measure the degree of source trust such as official refutation.

(c) *Group psychology*. The spread of rumors also involves numerous concepts in group psychology such as group belonging, standardization, conformity, alienation, and resistance. When studying the process of rumor propagation, it is necessary to consider how these factors affect people's acceptance and propagation behavior of rumors. In the modeling, the message spreading rate by mass media β_n is set to depict the conformity psychology, and k means the saturation effect as a result of message flooding.

(d) *Psychological defense mechanisms*. When people feel upset, anxious, or unable to cope, they may use a variety of psychological defense mechanisms to alleviate the situation. These conditions may lead people to spread or believe rumors more easily. In the modeling, the value of β_p frequently becomes larger owing to the psychological defense mechanisms.

(e) *Overly aggressive and emotional*. Research shows that it is easy to become overly aggressive and passionate when encountering something, which can lead to rumors spreading more quickly. Some individuals exaggerate or partially distort the event itself to vent their emotions. Thus, the rumor media emerge from the mass media at rate χ naturally.

(f) *Media psychology*. Social media platforms play an essential role in the spread of rumors. Media psychology studies the interaction, influence, and reaction of media and people. When looking at the message spreading process, it is necessary to consider the role of media and social media platforms in influencing rumors and how people use them to spread rumors. Due to network media's virtuality, numerous individuals share their messages in the mass media that increase at rate ξ of spreaders.

(g) *Social identity theory*. People tend to conform to the opinions and behaviors of the majority, which is known as the "conformity effect". If most people believe a rumor, individuals may be influenced to believe the rumor as well. Thus, we introduce β_s to measure the message spreading rate by message spreader.

Based on the above social psychological analysis and the mathematical basis of differential equation dynamic modeling, we give the following modified model from the XY-ISR message spreading network [1]:

$$\begin{aligned}
\frac{\mathrm{d}i(t)}{\mathrm{d}t} &= \mu N - \left(\beta_n \frac{n(t)}{k+n(t)} + \beta_p p(t) + \beta_s s(t)\right) i(t) \\
&\quad - \mu i(t), \\
\frac{\mathrm{d}s(t)}{\mathrm{d}t} &= \left(\beta_n \frac{n(t)}{k+n(t)} + \beta_p p(t) + \beta_s s(t)\right) i(t) \\
&\quad - (\gamma+\mu) s(t), \\
\frac{\mathrm{d}m(t)}{\mathrm{d}t} &= \gamma s(t) - \mu m(t), \qquad \frac{\mathrm{d}n(t)}{\mathrm{d}t} = \xi s(t) - \chi n(t), \\
\frac{\mathrm{d}p(t)}{\mathrm{d}t} &= \chi n(t) - \delta p(t),
\end{aligned} \tag{1}$$

The initial condition for system (1) takes the form

$$i(0) = i^0, \qquad s(0) = s^0, \qquad m(0) = m^0, \qquad n(0) = n^0, \qquad p(0) = p^0, \tag{2}$$

where constants i^0, s^0, m^0, n^0, p^0 are defined on $[0, +\infty)$. It can be proved by the fundamental theory of functional differential equations [3] that system (1) has a unique solution $(i(t), s(t), m(t), n(t), p(t))$ satisfying the initial condition (2). It is easy to show that all solutions of system (1) with initial condition (2) are defined on $[0, +\infty)$ and remain positive for all $t \geqslant 0$, while Ω is a positively invariant set for system (1), where

$$\Omega = \left\{(i, s, m, n, p);\ i, s, m < N,\ n < \frac{\xi N}{\chi},\ p < \frac{\xi N}{\delta}\right\},$$

which implies that $i(t)$, $s(t)$, $m(t)$, $n(t)$, $p(t)$ are bounded in the invariant set Ω.

3 Dynamical analysis of message spreading model

In this section, the dynamical analysis of message spreading model (1) is carried out by relevant theories of differential equation dynamics.

System (1) always has a message-free equilibrium

$$E_0(i_0, 0, 0, 0, 0) = (N, 0, 0, 0, 0).$$

If system (1) has the message-prevailing equilibrium E^*, the following holds:

$$\begin{aligned}
&\mu N - \left(\beta_n \frac{n^*}{k+n^*} + \beta_p p^* + \beta_s s^*\right) i^* - \mu i^* = 0,\\
&\left(\beta_n \frac{n^*}{k+n^*} + \beta_p p^* + \beta_s s^*\right) i^* - (\gamma+\mu) s^* = 0,\\
&\gamma s^* - \mu m^* = 0, \qquad \xi s^* - \chi n^* = 0, \qquad \chi n^* - \delta p^* = 0.
\end{aligned} \tag{3}$$

From Eq.3 we have

$$i^* = N - \frac{\gamma+\mu}{\mu} s^*, \qquad m^* = \frac{\gamma s^*}{\mu}, \qquad n^* = \frac{\xi s^*}{\chi}, \quad p^* = \frac{\xi s^*}{\delta},$$

where s^* is the root of the following quadratic equation:

$$\begin{aligned}
&\frac{\xi}{\chi}\frac{\gamma+\mu}{\mu}\left(\frac{\beta_p \xi}{\delta} + \beta_s\right) s^{*2}\\
&\quad + \left[\frac{\beta_n \xi}{\chi}\frac{\gamma+\mu}{\mu} + k\frac{\gamma+\mu}{\mu}\left(\frac{\beta_p \xi}{\delta} + \beta_s\right) + \frac{\xi}{\chi}(\gamma+\mu) - N\frac{\xi}{\chi}\left(\frac{\beta_p \xi}{\delta} + \beta_s\right)\right] s^*\\
&\quad + k(\gamma+\mu)\left[1 - \left(\frac{\beta_n \xi}{k\chi} + \frac{\beta_p \xi}{\delta} + \beta_s\right)\frac{N}{\gamma+\mu}\right] = 0.
\end{aligned}$$

Define the basic reproduction number of system (1) as

$$R_0 = \left(\frac{\beta_n \xi}{k\chi} + \frac{\beta_p \xi}{\delta} + \beta_s\right)\frac{N}{\gamma+\mu},$$

which represents the average number of new message spreaders generated by a single newly spreader during the full message spread period.

Next, we study the global stability of each of the equilibria to system (1). The approach of proofs is to use suitable Lyapunov functionals and LaSalle's invariance principle. Since the variable $m(t)$ does not appear explicitly in the first two and last two equations in system (1), we do not need to consider it in later analysis.

Theorem 1. *If $\mathcal{R}_0 < 1$, the message-free equilibrium E_0 of system* (1) *is globally asymptotically stable.*

Proof. Let $(i(t), s(t), n(t), p(t))$ be any positive solution of system (1) with initial condition (2). Define

$$V_1(t) = i_0\left(\frac{i(t)}{i_0} - 1 - \ln\frac{i(t)}{i_0}\right) + s(t) + c_1 n(t) + c_2 p(t),$$

where positive constants c_1 and c_2 will be determined later. Calculating the derivative of $V_1(t)$ along positive solutions of system (1) yields

$$\begin{aligned}\dot{V}_1(t) &= \left(1 - \frac{i_0}{i(t)}\right)\left[\mu N - \left(\beta_n \frac{n(t)}{k + n(t)} + \beta_p p(t) + \beta_s s(t)\right) i(t) - \mu i(t)\right] \\ &\quad + \left(\beta_n \frac{n(t)}{k + n(t)} + \beta_p p(t) + \beta_s s(t)\right) i(t) - (\gamma + \mu) s(t) \\ &\quad + c_1\big(\xi s(t) - \chi n(t)\big) + c_2\big(\chi n(t) - \delta p(t)\big).\end{aligned}$$

Choose

$$c_1 = N\left(\frac{\beta_n}{k\chi} + \frac{\beta_p}{\delta}\right), \quad c_2 = \frac{N\beta_p}{\delta}.$$

It follows that

$$\dot{V}_1(t) = \mu N\left(2 - \frac{i_0}{i(t)} - \frac{i(t)}{i_0}\right) + (\gamma + \mu)(R_0 - 1)s(t) - \frac{\beta_n N n^2(t)}{k(k + n(t))} \leqslant 0.$$

It follows from the inequality of arithmetic and geometric means that $\dot{V}_1(t) \leqslant 0$ with equality holding if and only if $i = i_0$, $s = n = p = 0$. It can be verified that $\{E_0\} \subset \Omega_1$ is the largest invariant subset of $\{(i(t), s(t), n(t), p(t))\colon \dot{V}_1(t) = 0\}$. It is not difficult to obtain the global asymptotic stability of E_0 from LaSalle's invariance principle. $\square$

Theorem 2. *If $\mathcal{R}_0 > 1$, the message-prevailing equilibrium E^* of system* (1) *is globally asymptotically stable.*

Proof. Let $(i(t), s(t), n(t), p(t)$ be any positive solution of system (1) with initial condition (2). Define

$$\begin{aligned}V_2(t) &= i^*\left(\frac{i(t)}{i^*} - 1 - \ln\frac{i(t)}{i^*}\right) + s^*\left(\frac{s(t)}{s^*} - 1 - \ln\frac{s(t)}{s^*}\right) \\ &\quad + l_1 n^*\left(\frac{n(t)}{n^*} - 1 - \ln\frac{n(t)}{n^*}\right) + l_2 p^*\left(\frac{p(t)}{p^*} - 1 - \ln\frac{p(t)}{p^*}\right),\end{aligned}$$

where positive constants l_1 and l_2 will be determined later. Calculating the derivative of $V_2(t)$ along positive solutions of system (1) yields

$$\begin{aligned}\dot{V}(t) &= \left(1 - \frac{i^*}{i(t)}\right)\left[\mu N - \left(\beta_n \frac{n(t)}{k + n(t)} + \beta_p p(t) + \beta_s s(t)\right) i(t) - \mu i(t)\right] \\ &\quad + \left(1 - \frac{s^*}{s(t)}\right)\left[\left(\beta_n \frac{n(t)}{k + n(t)} + \beta_p p(t) + \beta_s s(t)\right) i(t) - (\gamma + \mu) s(t)\right] \\ &\quad + l_1\left(1 - \frac{n^*}{n(t)}\right)\big(\xi s(t) - \chi n(t)\big) + l_2\left(1 - \frac{p^*}{p(t)}\right)\big(\chi n(t) - \delta p(t)\big).\end{aligned}$$

From Eq. 3 it follows

$$\begin{aligned}
\dot{V}(t) &= (\beta_s s^* + \mu)i^*\left(2 - \frac{i^*}{i(t)} - \frac{i(t)}{i^*}\right) \\
&\quad + \frac{\beta_n n^* i^*}{k + n^*}\left[2 - \frac{i^*}{i(t)} - \frac{i(t)s^*n(t)(k+n^*)}{i^*s(t)n^*(k+n(t))}\right] \\
&\quad + \beta_p p^* i^*\left[2 - \frac{i^*}{i(t)} - \frac{i(t)s^*p(t)}{i^*s(t)p^*}\right] + \left[l_1\xi - \frac{i^*}{s^*}\left(\frac{\beta_n n^*}{k+n^*} + \beta_p p^*\right)\right]s(t) \\
&\quad + \beta_n\frac{n(t)}{k+n(t)}i^* - l_1\frac{\xi n^*}{n(t)}s(t) + \beta_p p(t)i^* - l_2\delta p(t) \\
&\quad + l_2\chi n(t) - l_1\chi n(t) - l_2\frac{p^*}{p(t)}\chi n(t) + l_1\chi n^* + \delta l_2 p^*.
\end{aligned}$$

Choose

$$l_1 = \frac{i^*}{\xi s^*}\left(\frac{\beta_n n^*}{k+n^*} + \beta_p p^*\right), \qquad l_2 = \frac{i^*}{\xi s^*}\beta_p p^*,$$

and we obtain that

$$\begin{aligned}
\dot{V}(t) &= (\beta_s s^* + \mu)i^*\left(2 - \frac{i^*}{i(t)} - \frac{i(t)}{i^*}\right) \\
&\quad + \frac{\beta_n n^* i^*}{k + n^*}\left[4 - \frac{i^*}{i(t)} - \frac{s(t)n^*}{s^*n(t)} - \frac{k+n(t)}{k+n^*} - \frac{i(t)s^*n(t)(k+n^*)}{i^*s(t)n^*(k+n(t))}\right] \\
&\quad + \beta_p p^* i^*\left[4 - \frac{i^*}{i(t)} - \frac{s(t)n^*}{s^*n(t)} - \frac{n(t)p^*}{n^*p(t)} - \frac{i(t)s^*p(t)}{i^*s(t)p^*}\right] \\
&\quad - \frac{\beta_n k i^*(n(t)-n^*)^2}{(k+n^*)^2(k+n(t))} \leqslant 0.
\end{aligned}$$

Thus, it follows from the inequality of arithmetic and geometric means that $\dot{V}_2(t) \leqslant 0$ with equality holding if and only if $i = i^*$, $s = s^*$, $n = n^*$, $p = p^*$. It can be proved that $\{E^*\} \subset \Omega_2$ is the largest invariant subset of $\{(i(t), s(t), n(t), p(t))\colon \dot{V}_2(t) = 0\}$. Then we obtain the global asymptotic stability of E^* from LaSalle's invariance principle.

Thus far, the mathematical completeness of the model has been proved. We get that there are two states in system (1), one is the equilibrium E_0 where messages disappear, and the other is the equilibrium E^* where messages spread wildly, which is consistent with the message spreading trend in a social event. In the message propagation process, we hope that the number of message spreaders and rumor media approach 0, that is, the message-free equilibrium E_0, while staying away from the message-prevailing equilibrium E^*. □

4 Prediction analysis by artificial intelligence

Long short-term memory neural network (LSTM) is a recurrent neural network (RNN) designed to handle the vanishing gradient problem commonly encountered in traditional

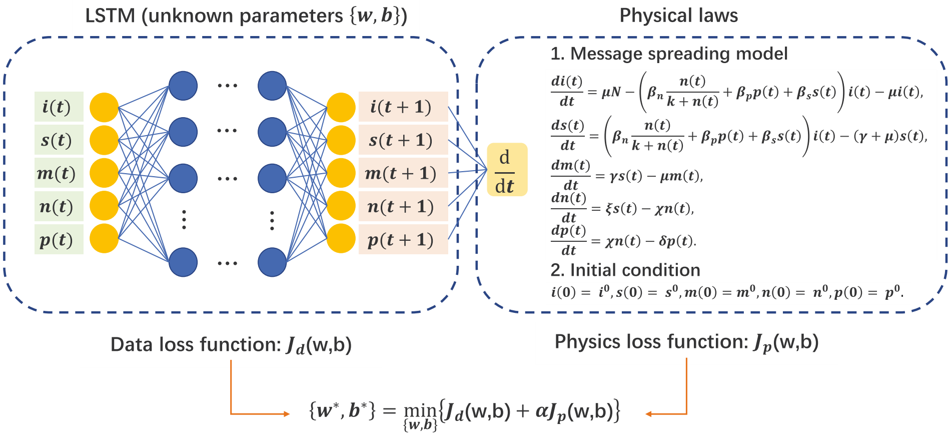

Figure 2. The basic block diagram of message spreading prediction based on PINN.

RNNs. The primary function of an LSTM neural network is to effectively remember and forget information over a lengthy period, unlike traditional RNNs, which suffer from the vanishing gradient problem and tend to ignore information that is too far back in time. The critical advantage of LSTM neural networks is their ability to learn long-term dependencies in data, which makes them particularly suitable for time-series analysis and sequence prediction tasks. This ability to remember long-term dependencies is achieved through specialized memory cells and gates that regulate the flow of information through the network. However, the LSTM neural network prediction can only predict the future trend according to the law of existing data, and the quality and quantity of the data limit its prediction ability, which is a common problem with data-driven modeling.

How to combine the advantages of the efficiency of data-driven modeling with the accuracy of mathematical modeling, physics-informed neural networks (PINN), a new class of universal function approximations that are capable of encoding any underlying physical laws in a given data set, comes into being [11]. The key idea is also fit for ordinary differential equation systems, for instance, message spreading model (1). The basic block diagram of message spreading prediction based on PINN is shown in Fig. 2.

The data-driven DNN model belongs to the black box model, and it is difficult for us to explain its prediction principle. Its advantages are mainly high modeling efficiency, and the model can nicely capture the changing trend of data. In this paper, we primarily use LSTM to predict trends in message propagation by past data. By introducing physical laws into the LSTM, the expected results not only conform to the changing direction of data but also conform to the rule of mathematical model (1) from the perspective of the whole message propagation process. Finally, the residual loss in the model training process is divided into two parts: the loss of prediction data and the loss of mathematical model (1). When both losses converge, the PINN-based prediction model is trained. Therefore, α in Fig. 2 is set through multiple message spreading case data and generally greater than 1.5 in model (1).

5 An message spreading prediction case

To better verify the feasibility and reliability of the PINN model proposed in this paper, we approximately simulate a data set of message spreaders and rumor media over time by analyzing the spreading popularity of a given event in mass media within 20 hours. Model (1)'s parameters are estimated by least-square fitting method according to the data, then model (1) transforms to

$$\begin{aligned}
\frac{\mathrm{d}i(t)}{\mathrm{d}t} &= 0.23 - \frac{0.0075i(t)n(t)}{1430+n(t)} - 0.0125i(t)p(t) - 0.0001i(t)s(t) \\
&\quad - 0.023i(t), \\
\frac{\mathrm{d}s(t)}{\mathrm{d}t} &= \frac{0.0075i(t)n(t)}{1430+n(t)} + 0.0125i(t)p(t) + 0.0001i(t)s(t) \\
&\quad - 0.123s(t), \\
\frac{\mathrm{d}m(t)}{\mathrm{d}t} &= 0.1s(t) - 0.023m(t), \qquad \frac{\mathrm{d}n(t)}{\mathrm{d}t} = 0.16s(t) - 0.2n(t), \\
\frac{\mathrm{d}p(t)}{\mathrm{d}t} &= 0.2n(t) - 0.033p(t).
\end{aligned} \tag{4}$$

Based on the first 20 hours' data, we predict the message spreading trend in the next 30 hours, namely, the values of message spreader $s(t)$ and rumor media $p(t)$. Network parameters of LSTM part in PINN are set in Table 2. As can be seen from Fig. 3, the data-driven LSTM completely captures the changing trend of the simulation data in the past 20 hours, while model-based prediction results are more realistic in terms of the overall trend of message propagation. It is worth mentioning that due to the limited amount of data (only 20 hours' data) used for training, LSTM prediction error is a bit large, but it

Table 2. Network parameters of LSTM part in PINN.

Neural network layer	Number of neurons	Activation function
Shared LSTM layer L1	10 neurons per layer with a total of 1 layer	ReLU
Shared LSTM layer L2	16 neurons per layer with a total of 2 layers	ReLU
Shared LSTM layer L3	32 neurons per layer with a total of 2 layers	ReLU
Shared LSTM layer L4	16 neurons per layer with a total of 2 layers	ReLU
Shared LSTM layer L5	10 neurons per layer with a total of 1 layer	ReLU
Task specific layer S1-1	10 neurons per layer with a total of 2 layers	ReLU
Task specific layer S1-2	5 neurons per layer with a total of 1 layer	ReLU
Task specific layer S2-1	10 neurons per layer with a total of 2 layers	ReLU
Task specific layer S2-2	5 neurons per layer with a total of 1 layer	ReLU
Task specific layer S3-1	10 neurons per layer with a total of 2 layers	ReLU
Task specific layer S3-2	5 neurons per layer with a total of 1 layer	ReLU
Task specific layer S4-1	10 neurons per layer with a total of 2 layers	ReLU
Task specific layer S4-2	5 neurons per layer with a total of 1 layer	ReLU
Task specific layer S5-1	10 neurons per layer with a total of 2 layers	ReLU
Task specific layer S5-2	5 neurons per layer with a total of 1 layer	ReLU
Output layer	5 neurons per layer with a total of 1 layer	Linear

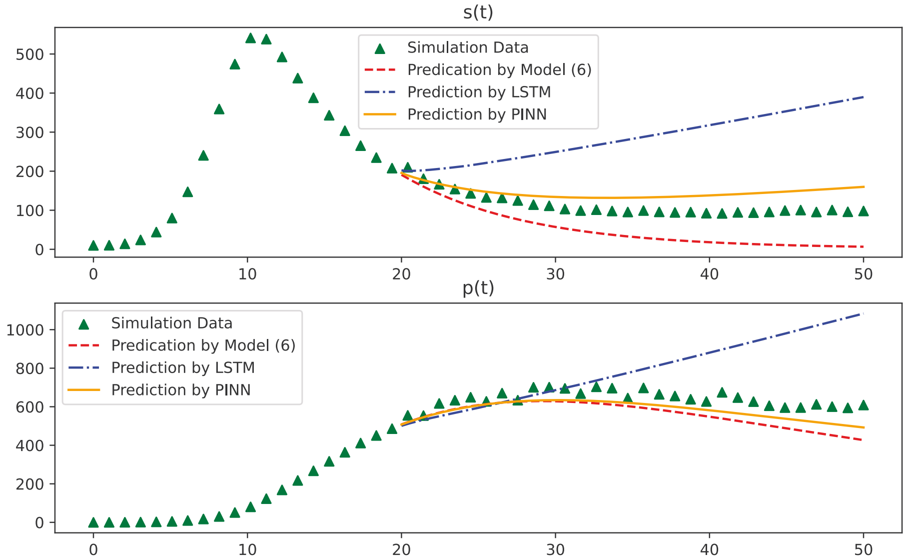

Figure 3. Prediction of message spreading by model 4, LSTM, and PINN.

can learn some rules of data change. Finally, the prediction results based on PINN are closer to the simulation data.

To some extent, the above case shows that in the field of message spreading prediction, the prediction method based on PINN can capture the change rule of data in real time and correct the deviation of data-driven prediction from the perspective of the whole message propagation based on the mathematical model. Hence, this method has strong engineering feasibility and reliability.

6 Conclusion

This paper proposes a mathematical model considering the spread of messages in the crowd and media circles. Integrating the data-driven LSTM and mathematical model predictions makes the final prediction results closer to the actual simulation data. Through a case of message spreading, we explain the new idea of spreading tend prediction based on PINN, which may have specific enlightening significance for the current research on message spreading.

Predicting the law of message propagation is a complicated problem that needs to be analyzed and predicted by combining various factors. Here are some factors that may affect the pattern of message propagation:

(a) *Algorithm of social media platforms*. Most rumors break out on social media platforms, so the algorithm may affect the law of rumor propagation.

(b) *News media reports*. News media reports may affect the spread of rumors. Some actual events become hotbeds of rumors, and news media reports may lead to the rapid spread of rumors.

(c) *Rumor content*. The characteristics and theme of rumor content may affect the law of rumor propagation.
(d) *Spreading time*. Rumor propagation may show different rules in different periods.
(e) *Geographic location*. Rumors may spread differently in different regions.
(f) *Social and cultural background*. Social and cultural background may affect the propagation law of rumors, and some cultures and values may react differently to rumors.

The above are only some factors that may affect the law of message/rumor propagation. In future work, to accurately predict the tend of rumor propagation, the above factors should be considered comprehensively, and in-depth modeling research should be carried out.

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