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Exploring fixed points via admissibility criteria for fuzzy θ_f-weak contractions

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Abstract. In this study, we introduce a new class of fuzzy contractions, called fuzzy α-η-θ_f-weak contractions, and establish fixed point results within the framework of complete fuzzy metric spaces. A fuzzy metric space generalizes the concept of a metric space by defining the "distance" between two points ω and υ using a function $\vartheta(\omega, \upsilon, \varsigma)$ that quantifies the degree of nearness between these points for a parameter $\varsigma > 0$. This parameter ς reflects various factors influencing the closeness of the points, making fuzzy metric spaces a powerful tool for modeling uncertainty and imprecision in mathematical contexts. Based on this framework, we prove several fixed point theorems addressing the existence and uniqueness of fixed points for such contractions. By carefully selecting specific forms of the functions θ_f, α, and η, our primary results can be adapted to yield a variety of significant corollaries. Furthermore, our findings leverage admissible and auxiliary functions to provide a broader framework that consolidates, extends, and refines existing results in fixed point theory.

Keywords: fixed point theory, fuzzy metric, contractions mapping, admissible functions, θ_f-weak contraction.

1 Introduction

The study of fixed points holds a central position in modern functional analysis, providing a versatile framework of mathematical principles, techniques, and a powerful set of mathematical tools that address diverse challenges encountered in mathematics and its numerous applications. Beyond mathematics, fixed point theory also finds extensive utility in scientific modeling, optimization, and engineering problems. Over the last sixty years, this area has evolved considerably, establishing itself as a dynamic and continually expanding research domain, particularly in the investigation of nonlinear systems. A key strength of fixed point theory lies in its ability to reformulate many traditional problems, enabling the discovery of solutions in a more structured manner. For instance, challenges involving operator equations such as $\mathcal{L}(\omega) = 0$ can often be equivalently transformed into fixed point equations of the form $\mathcal{G}(\omega) = \omega$, where $\mathcal{G}$ is a self-mapping defined on an appropriate space. This transformation not only simplifies the analysis but also facilitates the use of well-established fixed point theorems to confirm the existence and, in some cases, the uniqueness of solutions.

Since Zadeh's pioneering work on fuzzy sets in 1965 [27], the area has seen sustained growth, with notable developments in logic, topology, and analysis, leading to numerous applications in computer science and engineering. The concept of fuzzy metric spaces was introduced by Kramosil and Michaelek [12], followed by refinements from George and Veeramani [1], who demonstrated that every fuzzy metric generates a Hausdorff topology. In a fuzzy metric space, the distance between two points, ω and υ, is not a simple numerical value but is instead defined by the degree of nearness between them, which depends on a parameter $\varsigma > 0$. This parameter ς can represent various factors that influence the closeness between the points. For example, imagine traveling by train from Casablanca (ω) to Marrakesh (υ). The degree of nearness between these two cities can be quantified by a factor (ς) such as the time required to complete the journey or the fuel consumed by the train during the trip. The value of ς depends on the train's efficiency. In line with the axiom ($\mathcal{FM}_2$), when $\omega = \upsilon$, the degree of proximity between ω and υ is perfect, that is, equal to 1. Consequently, $\vartheta(\omega, \omega, \varsigma) = 1$ for each $\omega \in \mathcal{F}$ and for all $\varsigma > 0$. This fuzzy measurement highlights the nonabsolute nature of distance in real-world scenarios, where factors like time or fuel usage can serve as key indicators of proximity (as illustrated in Fig. 1).

A significant theoretical advancement was the extension of contractive mapping concepts to fuzzy metric spaces. Grabiec [3] extended the classical Banach and Edelstein theorems to these spaces in 1988. Further exploration into fuzzy contractive mappings was conducted by Gregori and Sapena, who established several fixed point results for these mappings. Mihet [14] contributed by broadening the concept to include fuzzy ψ-contractive mappings. More recently, Wardowski [26] introduced fuzzy $\mathcal{H}$-contractive mappings and applied them to derive new fixed point results.

In an effort to unify and extend various traditional fuzzy contraction types, Moussaoui et al. [17] (also see [21]) utilized the simulation function approach to introduce new classes of fuzzy contractive principles, demonstrating their applications through new fixed point theorems. Additionally, Saleh et al. [6] presented novel fixed point results by defining a new class of auxiliary functions, $\theta_f : (0,1) \to (0,1)$, inspired by the work of Jleli et al. [11]. For a detailed review of recent advances in metric and fuzzy metric fixed point theory and related methodologies, readers can refer to works such as [5, 7, 8, 11, 13, 15, 16, 18–20, 25, 26].

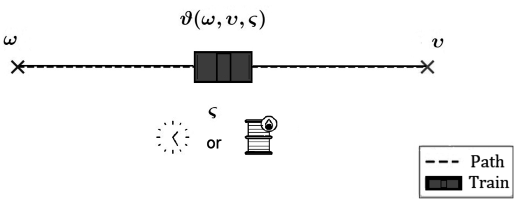

Figure 1. Example illustrating the degree of proximity between ω and υ with respect to ς.

2 Preliminaries

To ensure the self-contained nature of our study, this section provides an overview of essential concepts. For the entirety of this paper, $\mathbb{N}$ and $\mathbb{R}$ will denote the set of positive integers and the set of real numbers, respectively.

Definition 1. (See [23].) A continuous t-norm is a binary function $\curlywedge : [0,1] \times [0,1] \to [0,1]$ that satisfies the following properties:

($\mathcal{C}_1$) $\curlywedge$ is both commutative and associative,
($\mathcal{C}_2$) $\curlywedge$ is a continuous operation,
($\mathcal{C}_3$) $\hbar \curlywedge 1 = \hbar$ for all $\hbar \in [0,1]$,
($\mathcal{C}_4$) $\hbar \curlywedge \Im \leqslant \beta \curlywedge \varrho$ whenever $\hbar \leqslant \beta$ and $\Im \leqslant \varrho$ for any $\hbar, \Im, \beta, \varrho \in [0,1]$.

Example 1. Below are well-known examples of continuous t-norms:

(i) $\hbar \curlywedge_P \Im = \hbar \cdot \Im$,
(ii) $\hbar \curlywedge_L \Im = \max\{0, \hbar + \Im - 1\}$,
(iii) $\hbar \curlywedge_Z \Im = \min\{\hbar, \Im\}$.

We now proceed by recalling from the literature the definition of a fuzzy metric space and surveying notable results and concepts pertaining to it.

Definition 2. (See [1].) A fuzzy metric space is defined as a triple $(\mathcal{F}, \vartheta, \curlywedge)$, where $\mathcal{F}$ is a nonempty set, $\curlywedge$ is a continuous t-norm, and ϑ is a fuzzy relation on $\mathcal{F}^2 \times (0, +\infty)$ satisfying the following conditions:

($\mathcal{FM}_1$) $\vartheta(\omega, \upsilon, \varsigma) > 0$,
($\mathcal{FM}_2$) $\vartheta(\omega, \upsilon, \varsigma) = 1$ if and only if $\omega = \upsilon$,
($\mathcal{FM}_3$) $\vartheta(\omega, \upsilon, \varsigma) = \vartheta(\upsilon, \omega, \varsigma)$,
($\mathcal{FM}_4$) $\vartheta(\omega, z, \varsigma + t) \geqslant \vartheta(\omega, \upsilon, \varsigma) \curlywedge \vartheta(\upsilon, z, t)$,
($\mathcal{FM}_5$) $\vartheta(\omega, \upsilon, \cdot) : (0, +\infty) \to [0,1]$ is continuous

for all $\omega, \upsilon, z \in \mathcal{F}$ and $\varsigma, t > 0$.

The value $\vartheta(\omega, \upsilon, \varsigma)$ represents the level of closeness between ω and υ for the parameter ς.

Axiom ($\mathcal{FM}_1$) is supported by the reasoning that, just as a classical metric cannot assume the value ∞, the function ϑ is constrained to avoid taking the value 0. Axiom ($\mathcal{FM}_2$) can be expressed equivalently as follows: $\vartheta(\omega, \omega, \varsigma) = 1$ for all $\omega \in \mathcal{F}$ and $\varsigma > 0$, while $\vartheta(\omega, \upsilon, \varsigma) < 1$ when $\omega \neq \upsilon$ and $\varsigma > 0$. This formulation suggests that when $\omega = \upsilon$, the degree of nearness between ω and υ reaches its maximum value of 1, which leads to $\vartheta(\omega, \omega, \varsigma) = 1$ for every $\omega \in \mathcal{F}$ and for all $\varsigma > 0$.

Additionally, axiom ($\mathcal{FM}_5$) presumes that the variable ς behaves appropriately. In other words, for fixed ω and υ, the function $\varsigma \to \vartheta(\omega, \upsilon, \varsigma)$ is continuous. From this point forward, we will refer to a fuzzy metric space as a fuzzy metric space in the context defined by George and Veeramani.

In [1], George and Veeramani demonstrated that any fuzzy metric ϑ defined on the set $\mathcal{F}$ gives rise to a topology τ_ϑ on $\mathcal{F}$. The basis for this topology is the collection of open sets described by

$$B_\vartheta(\omega, \varepsilon, \varsigma) = \bigl\{\upsilon \in \mathcal{F}\colon\ \vartheta(\omega, \upsilon, \varsigma) > 1 - \varepsilon\bigr\},$$

where $\omega \in \mathcal{F}$, $0 < \varepsilon < 1$, and $\varsigma > 0$. This topology τ_ϑ is often referred to as the topology induced by ϑ.

A topological space $(\mathcal{F}, \tau)$ is termed fuzzy metrizable if there exists a fuzzy metric ϑ on $\mathcal{F}$ such that $\tau_\vartheta = \tau$.

Furthermore, when $(\mathcal{F}, d)$ is a metric space, the topology induced by the metric d aligns with the topology τ_{ϑ_d} associated with the corresponding fuzzy metric ϑ_d, as shown in [1]. Hence, every metrizable topological space can also be characterized as fuzzy metrizable.

Lemma 1. (*See* [3].) *For any $\omega, \upsilon \in \mathcal{F}$, the function $\vartheta(\omega, \upsilon, \cdot)$ is nondecreasing.*

Example 2.

(i) Let $\varphi : \mathbb{R} \to [0, 1[$ be an increasing continuous function. Define the function ϑ as follows:

$$\vartheta(\omega, \upsilon, \varsigma) = \begin{cases} 1 & \text{if } x = y, \\ \varphi(\varsigma) & \text{if } x \neq y. \end{cases}$$

In particular, if φ is a constant function, i.e., $\varphi(\varsigma) = k \in (0, 1)$, then the expression becomes

$$\vartheta(\omega, \upsilon, \varsigma) = \begin{cases} 1 & \text{if } x = y, \\ k & \text{if } x \neq y. \end{cases}$$

This is referred to as a discrete fuzzy metric due to its similarity to the classical discrete metric [4].

(ii) Let $\vartheta : \mathcal{F} \to \mathbb{R}^+$ be a one-to-one mapping, $\chi : \mathbb{R}^+ \to [0, +\infty)$ be a continuous increasing function, and let $\tau, \sigma > 0$. Define the function $\vartheta(\omega, \upsilon, \varsigma)$ as follows:

$$\vartheta(\omega, \upsilon, \varsigma) = \left(\frac{(\min\{\zeta(\omega), \zeta(\upsilon)\})^\tau + \chi(\varsigma)}{(\max\{\zeta(\omega), \zeta(\upsilon)\})^\tau + \chi(\varsigma)}\right)^\sigma.$$

Then the pair $(\vartheta, \curlywedge_P)$ forms a fuzzy metric [4].

(iii) Let $(\mathcal{F}, d)$ be a metric space, and define $\chi \curlywedge \phi = \min(\chi, \phi)$ for any $\chi, \phi \in [0, 1]$. Consider the function $\vartheta(\omega, \upsilon, \varsigma)$ defined as

$$\vartheta(\omega, \upsilon, \varsigma) = \frac{\lambda\varsigma^\lambda}{\lambda\varsigma^\lambda + \beta d(\omega, \upsilon)}, \quad \lambda, \beta \in \mathbb{R}^+.$$

Then the triple $(\mathcal{F}, \vartheta, \curlywedge)$ forms a fuzzy metric space.

When we set $\lambda = \beta = 1$, we obtain:

$$\vartheta(\omega, \upsilon, \varsigma) = \frac{\varsigma}{\varsigma + d(\omega, \upsilon)}.$$

This fuzzy metric, which is induced by the metric d, is referred to as the standard fuzzy metric [1].

(iv) Let $\mathcal{F} = \mathbb{R}$ and define the operation $\chi \curlywedge \phi = \chi \cdot \phi$ for all $\chi, \phi \in [0,1]$. Consider the mapping $\vartheta : \mathcal{F} \times \mathcal{F} \times (0, +\infty) \to [0,1]$ given by

$$\vartheta(\omega, \upsilon, \varsigma) = \left[\exp\left(\frac{|\omega - \upsilon|}{\varsigma}\right)\right]^{-1}$$

for all $\omega, \upsilon \in \mathcal{F}$, $\varsigma > 0$. Then the triple $(\mathcal{F}, \vartheta, \curlywedge)$ is a fuzzy metric space [1].

Definition 3. (See [1].) Let $(\mathcal{F}, \vartheta, \curlywedge)$ be a fuzzy metric space.

- (i) A sequence $\{\omega_s\} \subseteq \mathcal{F}$ is called convergent or said to converge to $\omega \in \mathcal{F}$ if $\lim_{s \to +\infty} \vartheta(\omega_s, \omega, \varsigma) = 1$ for all $\varsigma > 0$.
- (ii) A sequence $\{\omega_s\} \subseteq \mathcal{F}$ is termed a Cauchy sequence if, for any $\varepsilon \in (0,1)$ and $\varsigma > 0$, there exists $n_0 \in \mathbb{N}$ such that $\vartheta(\omega_s, \omega_p, \varsigma) > 1 - \varepsilon$ for all $n, p \geqslant n_0$.
- (iii) A fuzzy metric space in which every Cauchy sequence converges is referred to as a complete fuzzy metric space.

Definition 4. (See [5].) Consider $(\mathcal{F}, \vartheta, \curlywedge)$ as a fuzzy metric space. A function $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ is called a fuzzy contractive mapping if there exists a constant $\delta \in (0,1)$ such that

$$\frac{1}{\vartheta(\mathcal{G}(\omega), \mathcal{G}(\upsilon), \varsigma)} - 1 \leqslant \delta\left(\frac{1}{\vartheta(\omega, \upsilon, \varsigma)} - 1\right) \tag{1}$$

for all $\omega, \upsilon \in \mathcal{F}$ and $\varsigma > 0$.

Definition 5. (See [5].) A sequence $\{\omega_s\}$ in a fuzzy metric space $(\mathcal{F}, \vartheta, \curlywedge)$ is called a fuzzy contractive sequence if there exists a constant $\delta \in (0,1)$ such that

$$\frac{1}{\vartheta(\omega_{s+1}, \omega_{s+2}, \varsigma)} - 1 \leqslant \delta\left(\frac{1}{\vartheta(\omega_s, \omega_{s+1}, \varsigma)} - 1\right)$$

for all $n \in \mathbb{N}$ and $\varsigma > 0$.

Gregori and Sapena then proved the following fixed point theorem.

Theorem 1. (*See* [5].) *Let $(\mathcal{F}, \vartheta, \curlywedge)$ be a complete fuzzy metric space in which fuzzy contractive sequences are Cauchy. If $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ is a fuzzy contractive mapping, then $\mathcal{G}$ has a unique fixed point.*

Tirado [24] introduced the following theorem grounded in the application of Tirado's contraction.

Theorem 2. (*See* [24].) *Consider* $(\mathcal{F}, \vartheta, \curlywedge_L)$ *as a complete fuzzy metric space, and let* $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ *be a mapping satisfying the inequality*

$$1 - \vartheta\big(\mathcal{G}(\omega), \mathcal{G}(\upsilon), \varsigma\big) \leqslant \delta\big(1 - \vartheta(\omega, \upsilon, \varsigma)\big)$$

for all $\omega, \upsilon \in \mathcal{F}$, $\varsigma > 0$ *and some* $\delta \in (0, 1)$. *Then* $\mathcal{G}$ *admits a unique fixed point.*

Gopal and Vetro generalized the notion of α-admissible mappings to fuzzy metric spaces in the following manner.

Definition 6. (See [2].) Let $(\mathcal{F}, \vartheta, \curlywedge)$ be a fuzzy metric space, and let $\alpha : \mathcal{F} \times \mathcal{F} \times (0, +\infty) \to [0, +\infty)$ be a function. The mapping $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ is called α-admissible if, for every $\omega, \upsilon \in \mathcal{F}$,

$$\alpha(\omega, \upsilon, \varsigma) \geqslant 1 \quad \Longrightarrow \quad \alpha\big(\mathcal{G}(\omega), \mathcal{G}(\upsilon), \varsigma\big) \geqslant 1$$

for all $\varsigma > 0$.

In accordance with [9,22], we define the concept of an admissible mapping as follows.

Definition 7. (See [9,22].) Let $(\mathcal{F}, \vartheta, \curlywedge)$ be a fuzzy metric space, and let $\alpha, \eta : \mathcal{F} \times \mathcal{F} \times (0, +\infty) \to [0, +\infty)$ be two functions. We say that $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ is α-admissible with respect to η if, for all $\omega, \upsilon \in \mathcal{F}$,

$$\alpha(\omega, \upsilon, t) \geqslant \eta(\omega, \upsilon, \varsigma) \quad \Longrightarrow \quad \alpha\big(\mathcal{G}(\omega), \mathcal{G}(\upsilon), \varsigma\big) \geqslant \eta\big(\mathcal{G}(\omega), \mathcal{G}(\upsilon), \varsigma\big)$$

for all $\varsigma > 0$. If we define $\alpha(\omega, \upsilon, \varsigma) = 1$ for all $\omega, \upsilon \in \mathcal{F}$ and $\varsigma > 0$, we then say that $\mathcal{G}$ is an η-subadmissible mapping.

Definition 8. (See [10].) Let $\alpha, \eta : \mathcal{F} \times \mathcal{F} \times (0, +\infty) \to [0, +\infty)$ be two functions. We say that $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ is an α-η-continuous mapping if, for a given $\omega \in \mathcal{F}$ and a sequence $\{\omega_s\}$ such that $\omega_s \to \omega \in \mathcal{F}$ as $s \to +\infty$,

$$\alpha(\omega_s, \omega_{s+1}, \varsigma) \geqslant \eta(\omega_s, \omega_{s+1}, \varsigma) \quad \Longrightarrow \quad \mathcal{G}(\omega_s) \to \mathcal{G}(\omega) \quad \text{as } s \to +\infty.$$

In 2020, inspired by the work of Jleli et al. [11], Saleh et al. [6] introduced the notion of fuzzy θ_f-contractive mappings by utilizing an auxiliary function $\theta_f : (0, 1) \to (0, 1)$, which satisfies the following properties:

- (Ω_1) θ_f is a nondecreasing function,
- (Ω_2) θ_f is continuous,
- (Ω_3) $\lim_{p\to+\infty} \theta_f(\psi_p) = 1$ if and only if $\lim_{p\to+\infty} \psi_p = 1$, where $\{\psi_p\}$ is a sequence in $(0, 1)$.

Example 3. (See [6].) $\theta_f : (0, 1) \to (0, 1)$ is defined by

$$\theta_f(\psi) = \mathrm{e}^{1-1/\psi}, \quad \psi \in (0, 1).$$

Example 4. (See [6].) $\theta_f : (0, 1) \to (0, 1)$ is defined by

$$\theta_f(\psi) = 1 - \cos\frac{\pi\psi}{2}, \quad \psi \in (0, 1).$$

3 Main results

In this section, we introduce the notion of a fuzzy α-η-θ_f-weak contraction and demonstrate several fixed-point results for this class of mappings in the context of complete fuzzy metric spaces.

Definition 9. Let $(\mathcal{F}, \vartheta, \curlywedge)$ be a fuzzy metric space, and let $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ be a self-mapping. Suppose $\alpha, \eta : \mathcal{F} \times \mathcal{F} \times (0, +\infty) \to [0, +\infty)$ are two functions. We say that $\mathcal{G}$ is a fuzzy α-η-θ_f-weak contraction with respect to $\theta_f \in \mathcal{F}$ if there exists $\delta \in (0, 1)$ such that the following holds:

$$\begin{aligned}&\vartheta\bigl(\mathcal{G}(\omega), \mathcal{G}(\upsilon), \varsigma\bigr) < 1 \quad \text{and} \quad \eta\bigl(\omega, \mathcal{G}(\omega), \varsigma\bigr) \leqslant \alpha(\omega, \upsilon, \varsigma) \\ &\quad \implies \quad \bigl[\theta_f\bigl(\Lambda(\omega, \upsilon, \varsigma)\bigr)\bigr]^\delta \leqslant \theta_f\bigl(\vartheta\bigl(\mathcal{G}(\omega), \mathcal{G}(\upsilon), \varsigma\bigr)\bigr)\end{aligned}$$

for all $\omega, \upsilon \in \mathcal{F}$ and $\varsigma > 0$, where $\Lambda(\omega, \upsilon, \varsigma) = \min\{\vartheta(\omega, \upsilon, \varsigma), \vartheta(\omega, \mathcal{G}(\omega), \varsigma), \vartheta(\upsilon, \mathcal{G}(\upsilon), \varsigma)\}$.

Theorem 3. *Let $(\mathcal{F}, \vartheta, \curlywedge)$ be a complete fuzzy metric space. Assume that $\alpha, \eta : \mathcal{F} \times \mathcal{F} \times (0, +\infty) \to [0, +\infty)$ are two functions, and let $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ satisfy the following conditions*:

(i) *$\mathcal{G}$ is α-admissible concerning η*;
(ii) *$\mathcal{G}$ is a fuzzy α-η-θ_f-weak contraction*;
(iii) *there exists an element $\omega_0 \in \mathcal{F}$ such that $\alpha(\omega_0, \mathcal{G}(\omega_0), \varsigma) \geqslant \eta(\omega_0, \mathcal{G}(\omega_0), \varsigma)$*;
(iv) *the mapping $\mathcal{G}$ is α-η-continuous.*

Under these assumptions, the mapping $\mathcal{G}$ has a fixed point.

Proof. Let $\omega_0 \in \mathcal{F}$ satisfy $\alpha(\omega_0, \mathcal{G}(\omega_0), \varsigma) \geqslant \eta(\omega_0, \mathcal{G}(\omega_0), \varsigma)$, and define the sequence $\{\omega_s\}$ as follows:

$$\mathcal{G}^s\omega_0 = \mathcal{G}(\omega_{s-1})$$

for all $s \geqslant 1$. Since $\mathcal{G}$ is α-admissible with respect to η, it follows that

$$\alpha(\omega_0, \omega_1, \varsigma) = \alpha\bigl(\omega_0, \mathcal{G}(\omega_0), \varsigma\bigr) \geqslant \eta\bigl(\omega_0, \mathcal{G}(\omega_0), \varsigma\bigr) = \eta(\omega_0, \omega_1, \varsigma).$$

By induction, we derive

$$\alpha\bigl(\omega_s, \mathcal{G}(\omega_s), \varsigma\bigr) = \alpha(\omega_s, \omega_{s+1}, \varsigma) \geqslant \eta(\omega_s, \omega_{s+1}, \varsigma)$$

for all $s \in \mathbb{N}$.

If there exists $p_0 \in \mathbb{N}$ such that $\omega_{p_0} = \omega_{p_0+1}$, then ω_{p_0} is a fixed point of $\mathcal{G}$. Now, assume that $\omega_s \neq \omega_{s+1}$ for all $s \in \mathbb{N}$. Using the fact that $\mathcal{G}$ satisfies the fuzzy α-η-θ_f-weak contraction condition, we obtain

$$\begin{aligned}1 &> \theta_f\bigl(\vartheta(\mathcal{G}\omega_{s-1}, \mathcal{G}\omega_s, \varsigma)\bigr) \\ &\geqslant \bigl[\theta_f\bigl(\min\bigl\{\vartheta(\omega_{s-1}, \omega_s, \varsigma), \vartheta(\omega_{s-1}, \mathcal{G}\omega_{s-1}, \varsigma), \vartheta(\omega_s, \mathcal{G}\omega_s, \varsigma)\bigr\}\bigr)\bigr]^\delta \\ &= \bigl[\theta_f\bigl(\min\bigl\{\vartheta(\omega_{s-1}, \omega_s, \varsigma), \vartheta(\omega_s, \omega_{s+1}, \varsigma)\bigr\}\bigr)\bigr]^\delta. \end{aligned} \tag{2}$$

If, for some $s \in \mathbb{N}$,

$$\min\{\vartheta(\omega_{s-1}, \omega_s, \varsigma), \vartheta(\omega_s, \omega_{s+1}, \varsigma)\} = \vartheta(\omega_s, \omega_{s+1}, \varsigma),$$

then by (2), it follows that

$$\theta_f\big(\vartheta(\omega_s, \omega_{s+1}, \varsigma)\big) \geqslant \big[\theta_f(\vartheta(\omega_s, \omega_{s+1}, \varsigma))\big]^\delta > \theta_f\big(\vartheta(\omega_s, \omega_{s+1}, \varsigma)\big),$$

which is a contradiction. This implies that

$$\min\{\vartheta(\omega_{s-1}, \omega_s, \varsigma), \vartheta(\omega_s, \omega_{s+1}, \varsigma)\} = \vartheta(\omega_{s-1}, \omega_s, \varsigma)$$

for all $s \in \mathbb{N}$. Consequently, we have

$$\begin{aligned} 1 > \theta_f\big(\vartheta(\omega_s, \omega_{s+1}, \varsigma)\big) &\geqslant \big[\theta_f\big(\vartheta(\omega_{s-1}, \omega_s, \varsigma)\big)\big]^\delta \\ &\geqslant \big[\theta_f\big(\vartheta(\omega_{s-2}, \omega_{s-1}, \varsigma)\big)\big]^{\delta^2} \geqslant \cdots \geqslant \big[\theta_f\big(\vartheta(\omega_0, \omega_1, \varsigma)\big)\big]^{\delta^s}. \end{aligned}$$

Taking the limit as $s \to +\infty$, it follows that

$$\lim_{s \to +\infty} \theta_f\big(\vartheta(\omega_s, \omega_{s+1}, \varsigma)\big) = 1.$$

Employing (Ω_3), we get

$$\lim_{s \to +\infty} \vartheta(\omega_s, \omega_{s+1}, \varsigma) = 1. \tag{3}$$

We now establish that the sequence $\{\omega_s\}$ is Cauchy. Assume, for the sake of contradiction, that $\{\omega_s\}$ is not a Cauchy sequence. Then there exist $\epsilon \in (0, 1)$, $\varsigma_0 > 0$, and two subsequences $\{\omega_{s_p}\}$ and $\{\omega_{t_p}\}$ of $\{\omega_s\}$, where $t_p > s_p \geqslant p$ for all $p \in \mathbb{N}$, such that

$$\vartheta(\omega_{t_p}, \omega_{s_p}, \varsigma_0) \leqslant 1 - \epsilon. \tag{4}$$

By applying Lemma 1, we have

$$\vartheta\left(\omega_{t_p}, \omega_{s_p}, \frac{\varsigma_0}{2}\right) \leqslant 1 - \epsilon. \tag{5}$$

Suppose s_p is the smallest index fulfilling (5). Consequently, we have

$$\vartheta\left(\omega_{t_p - 1}, \omega_{s_p}, \frac{\varsigma_0}{2}\right) > 1 - \epsilon \tag{6}$$

with $\omega = \omega_{t_p - 1}$ and $\upsilon = \omega_{s_p - 1}$, the subsequent inequality results

$$\begin{aligned} \theta_f\big(\vartheta(\omega_{t_p}, \omega_{s_p}, \varsigma_0)\big) &\geqslant \big[\theta_f\big(\Lambda(\omega_{t_p - 1}, \omega_{s_p - 1}, \varsigma_0)\big)\big]^\delta \\ &> \theta_f\big(\Lambda(\omega_{t_p - 1}, \omega_{s_p - 1}, \varsigma_0)\big), \end{aligned} \tag{7}$$

where

$$\begin{aligned}
&\Lambda\big(\omega_{t_p-1}, \omega_{s_p-1}, \varsigma_0\big) \\
&\quad = \min\big\{\vartheta(\omega_{t_p-1}, \omega_{s_p-1}, \varsigma_0), \vartheta(\omega_{t_p-1}, \mathcal{G}\omega_{t_p-1}, \varsigma_0), \vartheta(\omega_{s_p-1}, \mathcal{G}\omega_{s_p-1}, \varsigma_0)\big\}, \\
&\quad = \min\big\{\vartheta(\omega_{t_p-1}, \omega_{s_p-1}, \varsigma_0), \vartheta(\omega_{t_p-1}, \omega_{t_p}, \varsigma_0), \vartheta(\omega_{s_p-1}, \omega_{s_p}, \varsigma_0)\big\}. \qquad (8)
\end{aligned}$$

Taking the limit as $p \to +\infty$ in the above and using (3), we get

$$\begin{aligned}
&\lim_{p\to+\infty} \Lambda(\omega_{t_p-1}, \omega_{s_p-1}, \varsigma_0) \\
&\quad = \lim_{p\to+\infty} \min\big\{\vartheta(\omega_{t_p-1}, \omega_{s_p-1}, \varsigma_0), \vartheta(\omega_{t_p-1}, \mathcal{G}\omega_{t_p-1}, \varsigma_0), \vartheta(\omega_{s_p-1}, \mathcal{G}\omega_{s_p-1}, \varsigma_0)\big\}, \\
&\quad = \min\Big\{\lim_{p\to+\infty} \vartheta(\omega_{t_p-1}, \omega_{s_p-1}, \varsigma_0), 1, 1\Big\} = \lim_{p\to+\infty} \vartheta(\omega_{t_p-1}, \omega_{s_p-1}, \varsigma_0). \qquad (9)
\end{aligned}$$

Additionally, from (4), (6), and condition $(\mathcal{FM}_4)$ we infer

$$\begin{aligned}
1-\epsilon \geqslant \vartheta(\omega_{t_p}, \omega_{s_p}, \varsigma_0) &> \vartheta(\omega_{t_p-1}, \omega_{s_p-1}, \varsigma_0) \\
&\geqslant \vartheta\left(\omega_{t_p-1}, \omega_{s_p}, \frac{\varsigma_0}{2}\right) \curlywedge \vartheta\left(\omega_{s_p}, \omega_{s_{p-1}}, \frac{\varsigma_0}{2}\right) \\
&> (1-\epsilon) * \vartheta\left(\omega_{s_p}, \omega_{s_p-1}, \frac{\varsigma_0}{2}\right).
\end{aligned}$$

Allowing $p \to +\infty$ in the above inequality and applying (3), we deduce

$$\lim_{k\to+\infty} \vartheta(\omega_{t_p}, \omega_{s_p}, \varsigma_0) = \lim_{k\to+\infty} \vartheta(\omega_{t_p-1}, \omega_{s_p-1}, \varsigma_0) = 1-\epsilon. \qquad (10)$$

Taking the limit in (8), considering the continuity of θ_f, and using (10), we find

$$\big[\theta_f(1-\epsilon)\big]^\delta \leqslant \theta_f(1-\epsilon),$$

which results in a contradiction. Thus, $\{\omega_s\}$ is a Cauchy sequence. Since $(\mathcal{F}, \vartheta, \curlywedge)$ is a complete fuzzy metric space, there exists $\omega \in \mathcal{F}$ such that $\omega_s \to \omega$ as $s \to +\infty$. Finally, as $\mathcal{G}$ is α-η-continuous, with $\alpha(\omega_{s-1}, \omega_s, \varsigma) \geqslant \eta(\omega_{s-1}, \omega_s, \varsigma)$, we obtain

$$\begin{aligned}
\vartheta(\omega, \mathcal{G}\omega, \varsigma) &= \lim_{s\to+\infty} \vartheta(\omega_s, \mathcal{G}\omega_s, \varsigma) = \lim_{s\to+\infty} \vartheta(\omega_s, \omega_{s+1}, \varsigma) \\
&= \vartheta(\omega, \omega, \varsigma) = 1.
\end{aligned}$$

Thus, ω is a fixed point of $\mathcal{G}$. □

Theorem 4. *Let $(\mathcal{F}, \vartheta, \curlywedge)$ be a complete fuzzy metric space, $\alpha, \eta : \mathcal{F} \times \mathcal{F} \times (0, +\infty) \to [0, +\infty)$ two given functions, and let $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ be such that*:

(i) *$\mathcal{G}$ is α-admissible with respect to η*;
(ii) *$\mathcal{G}$ is a fuzzy α-η-θ_f-weak contraction*;

(iii) *there exists* $\omega_0 \in \mathcal{F}$ *such that* $\alpha(\omega_0, \mathcal{G}\omega_0, \varsigma) \geqslant \eta(\omega_0, \mathcal{G}\omega_0, \varsigma)$;

(iv) *if* $\{\omega_s\}$ *is a sequence in* $\mathcal{F}$ *such that* $\alpha(\omega_s, \omega_{s+1}, \varsigma) \geqslant \eta(\omega_s, \omega_{s+1}, \varsigma)$ *for all* $s \in \mathbb{N}$, $\varsigma > 0$ *and* $\omega_s \to \omega \in \mathcal{F}$ *as* $s \to +\infty$, *then either* $\alpha(\mathcal{G}\omega_s, \omega, \varsigma) \geqslant \eta(\mathcal{G}\omega_s, \mathcal{G}^2\omega_s, \varsigma)$ *or* $\alpha(\mathcal{G}^2\omega_s, \omega, \varsigma) \geqslant \eta(\mathcal{G}^2\omega_s, \mathcal{G}^3\omega_s, \varsigma)$ *for all* $s \in \mathbb{N}$.

Then $\mathcal{G}$ *has a fixed point.*

Proof. Let $\omega_0 \in \mathcal{F}$ such that $\alpha(\omega_0, \mathcal{G}\omega_0, \varsigma) \geqslant \eta(\omega_0, \mathcal{G}\omega_0, \varsigma)$. By following the same lines of the proof of Theorem 3, we obtain

$$\alpha(\omega_s, \mathcal{G}\omega_s, \varsigma) = \alpha(\omega_s, \omega_{s+1}, \varsigma) \geqslant \eta(\omega_s, \omega_{s+1}, \varsigma)$$

for all $s \in \mathbb{N}$, where $\mathcal{G}\omega_s = \omega_{s+1}$, and $\omega_s \to \omega \in \mathcal{F}$ as $s \to +\infty$. From (iii) we have $\alpha(\mathcal{G}\omega_s, \omega, \varsigma) \geqslant \eta(\mathcal{G}\omega_s, \mathcal{G}^2\omega_s, \varsigma)$ or $\alpha(\mathcal{G}^2\omega_s, \omega, \varsigma) \geqslant \eta(\mathcal{G}^2\omega_s, \mathcal{G}^3\omega_s, \varsigma)$ for all $n \in \mathbb{N}$, which means $\alpha(\omega_{s+1}, \omega, \varsigma) \geqslant \eta(\omega_{s+1}, \omega_{s+2}, \varsigma)$ or $\alpha(\omega_{s+2}, \omega, \varsigma) \geqslant \eta(\omega_{s+2}, \omega_{s+3}, \varsigma)$. Thus, there exist a subsequence $\{\omega_{s_p}\}$ of $\{\omega_s\}$ such that

$$\alpha(\omega_{s_p}, \omega, \varsigma) \geqslant \eta(\omega_{s_p}, \omega_{s_p+1}, \varsigma) = \eta(\omega_{s_p}, \mathcal{G}\omega_{s_p}, \varsigma). \tag{11}$$

From (2) we have

$$\begin{aligned} &\theta_f\big(\vartheta(\omega_{s_p+1}, \mathcal{G}\omega, \varsigma)\big) \\ &\quad = \theta_f\big(\vartheta(\mathcal{G}\omega_{s_p}, \mathcal{G}\omega, \varsigma)\big) \geqslant \big[\theta_f\big(\Lambda\big(\vartheta(\omega_{s_p}, \omega, \varsigma)\big)\big)\big]^\delta \\ &\quad = \big[\theta_f\big(\min\big\{\vartheta(\omega_{s_p}, \omega, \varsigma), \vartheta(\omega_{s_p}, \omega_{s_p+1}, \varsigma), \vartheta(\omega, \vartheta\omega, \varsigma)\big\}\big)\big]^\delta. \end{aligned} \tag{12}$$

If $\vartheta(\omega, \mathcal{G}\omega, \varsigma) < 1$, then we obtain

$$\lim_{s\to+\infty} \Lambda\big(\vartheta(\omega_{s_p}, \omega, \varsigma)\big) = \min\big\{1, 1, \vartheta(\omega, \mathcal{G}\omega, \varsigma)\big\} = \vartheta(\omega, \mathcal{G}\omega, \varsigma).$$

By taking the limit in (12), utilizing the result in (13), and applying the continuity of θ_f, we obtain

$$\theta_f\big(\vartheta(\omega, \mathcal{G}\omega, \varsigma)\big) > \big[\theta_f\big(\vartheta(\omega, \mathcal{G}\omega, \varsigma)\big)\big]^\delta. \tag{13}$$

This leads to a contradiction as $\delta \in (0, 1)$. Consequently, it follows that $\vartheta(\omega, \mathcal{G}\omega, \varsigma) = 1$, which implies $\mathcal{G}\omega = \omega$. □

To ensure the uniqueness of the fixed point for an fuzzy α-η-θ_f-weak contraction, the following condition will be examined:

($\mathcal{C}$) For every pair of points $\omega, \upsilon \in \mathcal{FP}(\mathcal{G})$, the relation $\alpha(\omega, \upsilon, \varsigma) \geqslant \eta(\omega, \upsilon, \varsigma)$ must hold, where $\mathcal{FP}(\mathcal{G})$ represents the set of fixed points of $\mathcal{G}$.

Theorem 5. *Incorporating condition* ($\mathcal{C}$) *into hypotheses of Theorems* 3 *and* 4 *ensures the uniqueness of the fixed point for* $\mathcal{G}$.

Proof. The proof proceeds by contradiction. Assume that $\omega, \mu \in \mathcal{F}$ are two distinct fixed points. Consequently, $\vartheta(\omega, \mu, \varsigma) < 1$ for all $\varsigma > 0$. Using condition ($\mathcal{C}$), it follows that

$$\alpha(\omega, \mu, \varsigma) \geqslant \eta(\omega, \mu, \varsigma).$$

Thus, we deduce

$$\theta_f\big(\vartheta(\mathcal{G}\omega, \mathcal{G}\mu, \varsigma)\big) = \theta_f\big(\vartheta(\omega, \mu, \varsigma)\big) \geqslant \big[\theta_f\big(\varLambda(\omega, \mu, \varsigma)\big)\big]^\delta,$$

where

$$\varLambda(\omega, \mu, \varsigma) = \min\big\{\vartheta(\omega, \mu, \varsigma), \vartheta(\omega, \mathcal{G}\omega, \varsigma), \vartheta(\mu, \mathcal{G}\mu, \varsigma)\big\}.$$

If $\vartheta(\omega, \mu, \varsigma) < 1$, then

$$\min\big\{\vartheta(\omega, \mu, \varsigma), \vartheta(\omega, \mathcal{G}\omega, \varsigma), \vartheta(\mu, \mathcal{G}\mu, \varsigma)\big\} = \vartheta(\omega, \mu, \varsigma).$$

As a result, we have

$$\theta_f\big(\vartheta(\mathcal{G}\omega, \mathcal{G}\mu, \varsigma)\big) = \theta_f\big(\vartheta(\omega, \mu, \varsigma)\big) \geqslant \big[\theta_f\big(\vartheta(\omega, \mu, \varsigma)\big)\big]^\delta,$$

which contradicts the fact that $\delta < 1$. Therefore, the fixed point of $\mathcal{G}$ must be unique. □

In this part, we aim to show that a range of implications and fixed-point results can be systematically derived from our main findings through the use of appropriate admissible and auxiliary functions.

Corollary 1. *Let $(\mathcal{F}, \vartheta, \curlywedge)$ be a complete fuzzy metric space, and let $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ be a self-mapping satisfying the following condition*:

$$\alpha(\omega, \upsilon, \varsigma) \geqslant \eta(\omega, \upsilon, \varsigma) \quad \textit{and} \quad 1 > \vartheta(\mathcal{G}\omega, \mathcal{G}\upsilon, \varsigma) \\ \implies \quad \big[\varLambda(\omega, \upsilon, \varsigma)\big]^\delta \leqslant \vartheta\big(\mathcal{G}\omega, \mathcal{G}\upsilon, \varsigma\big)$$

for all $\omega, \upsilon \in \mathcal{F}$, $\varsigma > 0$ and for some $\delta \in (0, 1)$, where $\varLambda(\omega, \upsilon, \varsigma) = \min\{\vartheta(\omega, \upsilon, \varsigma), \vartheta(\omega, \mathcal{G}\omega, \varsigma), \vartheta(\upsilon, \mathcal{G}\upsilon, \varsigma)\}$. Suppose further that:

(i) *$\mathcal{G}$ is α-admissible*;
(ii) *there exists $\omega_0 \in \mathcal{F}$ such that $\alpha(\omega_0, \mathcal{G}\omega_0, \varsigma) \geqslant \eta(\omega_0, \mathcal{G}\omega_0, \varsigma)$*;
(iii) *$\mathcal{G}$ is α-η-continuous.*

Then $\mathcal{G}$ has a fixed point.

Proof. The conclusion can be drawn from Theorem 3 by defining $\theta_f(\psi) = \psi$ for all $\psi \in (0, 1)$. □

Corollary 2. *Let $(\mathcal{F}, \vartheta, \curlywedge)$ be a complete fuzzy metric space and $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ be a self mapping such that*

$$\alpha(\omega, \upsilon, \varsigma) \geqslant 1 \quad \textit{and} \quad 1 > \vartheta(\mathcal{G}\omega, \mathcal{G}\upsilon, \varsigma) \\ \implies \quad \big[\theta_f\big(\varLambda(\omega, \upsilon, \varsigma)\big)\big]^\delta \leqslant \theta_f\big(\vartheta(\mathcal{G}\omega, \mathcal{G}\upsilon, \varsigma)\big)$$

for all $\omega, \upsilon \in \mathcal{F}$, $\varsigma > 0$ and for some $\delta \in (0, 1)$, where $\theta_f \in \varOmega$ and $\varLambda(\omega, \upsilon, \varsigma) = \min\{\vartheta(\omega, \upsilon, \varsigma), \vartheta(\omega, \mathcal{G}\omega, \varsigma), \vartheta(\upsilon, \mathcal{G}\upsilon, \varsigma)\}$. Suppose further that:

(i) $\mathcal{G}$ *is* α*-admissible*;
(ii) *there exists* $\omega_0 \in \mathcal{F}$ *such that* $\alpha(\omega_0, \mathcal{G}\omega_0, \varsigma) \geqslant \eta(\omega_0, \mathcal{G}\omega_0, \varsigma)$;
(iii) $\mathcal{G}$ *is* α-η*-continuous.*

Then $\mathcal{G}$ *has a fixed point.*

Proof. The result follows from Theorem 3 by setting $\eta(\omega, \upsilon, \varsigma) = 1$ for all $\omega, \upsilon \in \mathcal{F}$. □

Corollary 3. (*See* [6].) *Let* $(\mathcal{F}, \vartheta, \curlywedge)$ *be a complete fuzzy metric space, and let* $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ *be a mapping such that for all* $\omega, \upsilon \in \mathcal{F}$, $\varsigma > 0$ *and for some* $\delta \in (0, 1)$*, with* $\vartheta(\mathcal{G}\omega, \mathcal{G}\upsilon, \varsigma) < 1$*, we have*

$$\left[\theta_f\left(\Lambda(\omega, \upsilon, \varsigma)\right)\right]^\delta \leqslant \theta_f\left(\vartheta(\mathcal{G}\omega, \mathcal{G}\upsilon, \varsigma)\right).$$

Then $\mathcal{G}$ *has a fixed point.*

Proof. The result is obtained by using Theorem 3 with the definition $\theta_f(\psi) = \psi$ for every $\psi \in (0, 1)$. □

Corollary 4. *Let* $(\mathcal{F}, \vartheta, \curlywedge)$ *be a complete fuzzy metric space and* $\mathcal{G} : \mathcal{F} \to \mathcal{F}$ *be a mapping satisfying*

$$\begin{aligned} &1 > \vartheta(\mathcal{G}\omega, \mathcal{G}\upsilon, \varsigma) \quad \textit{and} \quad \alpha(\omega, \upsilon, \varsigma) \geqslant \eta(\omega, \upsilon, \varsigma) \\ &\quad \Longrightarrow \quad \left[1 - \cos\left(\frac{\pi}{2}\Lambda(\omega, \upsilon, \varsigma)\right)\right]^\delta \leqslant 1 - \cos\left(\frac{\pi}{2}\vartheta(\mathcal{G}\omega, \mathcal{G}\upsilon, \mathcal{F})\right), \end{aligned}$$

and

(i) $\mathcal{G}$ *is* α*-admissible with respect to* η;
(ii) *there exists* $\omega_0 \in \mathcal{F}$ *such that* $\alpha(\omega_0, \mathcal{G}\omega_0, \varsigma) \geqslant \eta(\omega_0, \mathcal{G}\omega_0, \varsigma)$;
(iii) $\mathcal{G}$ *is* α-η*-continuous.*

Then $\mathcal{G}$ *has a fixed point.*

Proof. The proof follows from Theorem 3 by taking $\theta_f(\psi) = 1 - \cos(\pi\psi/2)$ for all $\psi \in (0, 1)$. □

4 Conclusion

We introduced the concept of fuzzy α-η-θ_f-weak contractions within the framework of fuzzy metric spaces, incorporating the notion of α-admissibility alongside the control function θ_f. Based on this framework, we established several fixed point theorems addressing the existence and uniqueness of fixed points for such contractions. It is worth emphasizing that by carefully selecting specific forms of the functions θ_f, α, and η, our primary findings can be adapted to yield a variety of significant corollaries. These results not only pave the way for further advancements in the study of fixed points in fuzzy metric spaces but also encourage exploration into broader contexts, including fuzzy b-metric spaces, partially ordered fuzzy metric spaces, and other generalized structures.

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