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On the h-manifolds for impulsive conformable neural networks with reaction–diffusion terms: Practical stability analysis

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Abstract. In this paper, we consider a new class of conformable impulsive reaction–diffusion neural networks. The stable behavior of h-manifolds with respect to the model is investigated, and sufficient conditions are proposed by constructing suitable Lyapunov-like functions. Our results are new and contribute to the development of the theory of impulsive conformable models. Examples are also presented to illustrate the proposed criteria.

Keywords: conformable derivatives, neural networks, reaction–diffusion terms, practical stability, impulses, h-manifolds.

1 Introduction

The dynamical behaviors, such as stability, boundedness, periodicity, almost periodicity, oscillation, and asymptotical behaviors, of different classes of neural networks have attracted the attention of many researchers in science, engineering, and medicine. This is due to the fact that neural network models is one of the artificial intelligence approaches, which is intensively applied in various areas of secure communication, machine learning, optimization, pattern recognition, associative memory, classification, parallel computation, etc. The appropriate applications of such models are highly dependent on their qualitative properties.

Reaction–diffusion neural networks form a very important class of neural networks that are used to model the diffuse displacement trajectory of neurons in a nonuniform electromagnetic field. In fact, diffusion effects exist in many biological and artificial neural networks, and they can affect the high performance of the model. Hence, neural network models with reaction–diffusion terms have been thoroughly studied. See, for example, [14, 35].

In addition, impulsive generalizations of reaction–diffusion neural networks are also well studied, and the effects of some impulsive perturbations on their qualitative properties are investigated [34, 37]. The study of impulsive extensions of such neural network models is motivated by the fact that short-term perturbations are very common during the evolution of the processes modeled, as their dynamic is often subject to momentarily changes of biological or artificial nature. For impulsively extended reaction–diffusion neural network models, the theory of impulsive differential systems [11] is used as a modeling apparatus. In addition, the consideration of appropriate impulsive functions in the impulsive neural network models can be used in the implementation of efficient impulsive control strategies to the nonimpulsive models. Due to its advantages, such a control approach is preferred among the many applied control techniques [38].

Another approach that allows researchers to construct more adequate neural network models is the use of fractional calculus. Fractional-order derivatives generalize the integer-order ones, and their use leads to the development of more flexible models [3, 9, 30]. Hence, the fractional-order approach has been successfully applied in the introduction and analysis of reaction–diffusion neural network models [13, 29]. The presence of some very recent publications on such models evidenced the importance of their study [33]. Very recently, the hybrid fractional-order impulsive approach has also been applied to the development of reaction–diffusion models and the study of their applications [5, 32].

Applying most of the classical fractional derivatives in the development of fractional models leads to some strict limitations that are related to the lack of an easy-to-implement chain rule for the derivatives of compositions of functions. Some of the main difficulties in using classical fractional derivatives are due to their singular properties. In addition, for fractional derivatives, we do not have a corresponding Rolle's theorem or a corresponding Mean Value Theorem. The type of conformable derivatives introduced in [1, 8] attracted research interest since they overcome the above-mentioned difficulties. The definition of a conformable derivative is limit-based, which makes it a very convenient tool for applied problems. The advantages offered by the new concept have drawn the attention of numerous investigators [17]. The conformable calculus is also applied to the modeling of neural networks and the study of real-world phenomena [4, 7], including conformable space–time models [2, 16].

The notion of a conformable fractional derivative is also extended to the impulsive case. Important properties of impulsive conformable differential equations have been investigated, and interesting results have been published recently [22, 31].

Using the generalized impulsive conformable modeling approach, several impulsive conformable neural network models have been created, and their qualitative dynamics have been investigated. In [15], the fractional exponential stability of a nonlinear conformable fractional-order delayed system and the fractional exponential synchronization of conformable fractional-order delayed inertial neural networks, both with delayed impulses, have been studied. The authors in [23] designed an impulsive conformable Hopfield-type neural network and applied an extended stability strategy to the states of interest. An impulsive conformable Cohen–Grossberg-type neural network model has been introduced in [27], and the practical stability with respect to a manifold has been investigated. In [36], sufficient conditions for the exponential stability of a class of conformable fractional-order nonlinear differential systems with time-varying delay and impulses have been established. The authors also applied the condition for exponential stability to conformable fractional-order neural networks.

In all above impulsive conformable neural network models, reaction–diffusion effects are not taken into account. Since neurons are distributed in space and typically interact with the physical environment, it is more reasonable to study a reaction–diffusion version of existing models. A class of impulsive conformable neural network models with reaction–diffusion terms has been studied only in a pioneering paper [24], where the authors established criteria for the almost periodic behavior of the states. Hence, the qualitative theory of conformable impulsive neural network models with reaction–diffusion terms is not developed, and many important stability criteria have not yet been derived. This is the main aim of the proposed research.

Not surprisingly, most of the existing results on neural networks are about the stability of their states [5, 14, 23, 27, 32, 34–36]. In fact, stability is directly related to stabilization, synchronization, and control problems [13, 29, 33, 37]. Due to the fast convergent rate guaranteed, the most investigated stability behavior is the exponential stability for models of integer order [14, 35]. However, no exponential stability criteria have been published yet for impulsive conformable reaction–diffusion neural network models.

Also, in more of the existing stability results, the authors are interested only in the behavior of single states. In this research, we will study not just the stability dynamics of the separate states but also the stability of specific manifolds defined by a function h. This stability is known as h-manifold stability [21]. It is closely related to the general stability of the sets/manifolds notions [28], considering specific manifolds. The notion generalizes the notion of single-state stability. It is of a significant importance in numerous applications, essentially for applied problems in which several states are of interest and their dynamics revolve around a manifold determined by a h-function, as well as in the study of multi-stable systems. That is why it is applied to phenomena studied in engineering design, biology and medicine [28]. The notion is also studied for conformable impulsive neural networks [23, 27] without considering reaction–diffusion terms.

In addition, the practical stability concepts have been introduced in [10] as being more applicable in numerous applied problems when the behavior of the solutions is not ideally mathematically stable by Lyapunov but is acceptable from the practical point of view. This extended stability strategy has recently been very intensively applied in investigations of practical problems of diverse interest [6, 20], including fractional-order models [25, 26] and conformable models without reaction–diffusion terms [18, 23, 27].

In this paper, stimulated by the above analysis, we consider an impulsive conformable reaction–diffusion neural network. The hybrid practical stability with respect to the concept of h-manifolds is proposed in its exponential stability version. The Lyapunov conformable method is applied to study the behavior of the states considering the effects of reaction–diffusion terms and impulsive control functions. The main contributions of our paper can be listed as follows:

(i) Different from the neural network models proposed by the conformable calculus approach [23, 27], reaction–diffusion terms are considered in the model proposed here, which makes it more general and more appropriate for a real system.

(ii) Differently from the almost periodic analysis performed in [24] for impulsive reaction–diffusion conformable neural network models, we introduce the extended notion of practical stability with respect to h-manifolds, and the corresponding stability analysis is done.

(iii) The Lyapunov function conformable strategy and inequality techniques are applied, and several new sufficient criteria for practical stability with respect to the h-manifolds are provided.

(iv) Robust stability analysis is proposed based on the impulsive control mechanism, which allows by means of appropriate control signals, applied only at some fixed time instants, to control the corresponding stability dynamics of the states.

(v) Numerical examples are presented to demonstrate the strength of the established criteria.

The construction plan for the paper is as follows. Section 2 is devoted to the introduction of the impulsive conformable reaction–diffusion model. To this end, the necessary background from the conformable calculus is presented, and some preliminary results are reported. The hybrid extended notion of practical stability with respect to h-manifolds is defined for the proposed model. Some definitions and lemmas related to the Lyapunov's conformable method are also presented. In Section 3, the Lyapunov technique is used to establish the main practical exponential h-manifold stability results. In Section 4, a robust practical stability analysis with respect to h-manifolds is performed considering the uncertain case. In fact, since different neural network models exist within structural uncertainty, it is essential to consider the effect of uncertain parameters on their qualitative behavior [12, 19]. Uncertainty can be due to, for example, measurement inaccuracy, modeling errors, etc. Appropriate examples are discussed in Section 5. The paper concludes in Section 6.

2 The impulsive conformable reaction–diffusion model. Preliminary notes

We consider the Euclidean n-dimensional space with a norm $\|y\| = \sqrt{y_1^2 + y_2^2 + \cdots + y_n^2}$ of $y = (y_1, y_2, \dots, y_n)^{\mathrm{T}} \in \mathbb{R}^n$, and $\mathbb{R}_+ = [0, \infty)$. We will begin with some notations and properties related to conformable derivatives. Let $t_0 \in \mathbb{R}_+$ and $\tilde{t} \geqslant t_0$.

Definition 1. (See [1, 8].) The *conformable derivative* $\mathcal{D}_{\tilde{t}}^{\alpha} x(t)$ *of order* α, $0 < \alpha \leqslant 1$, with the lower limit $\tilde{t}$ for a function $x(t) : [\tilde{t}, \infty) \to \mathbb{R}$ is given as

$$\mathcal{D}_{\tilde{t}}^{\alpha} x(t) = \lim_{\varepsilon \to 0} \frac{x(t + \varepsilon (t - \tilde{t})^{1-\alpha}) - x(t)}{\varepsilon}, \quad t > \tilde{t}.$$

Next, following [22, 23, 27, 31], we consider the discrete times γ_k, $k = 1, 2, \dots$, defined as

$$0 = \gamma_0 < \gamma_1 < \gamma_2 < \cdots, \quad \lim_{k \to \infty} \gamma_k = \infty,$$

and for $\tilde{t} = \gamma_k$, $k = 1, 2, \dots$, we have the *generalized conformable derivative*

$$\mathcal{D}^{\alpha}_{\gamma_k} x(\gamma_k) = \lim_{t \to \gamma_k^+} \mathcal{D}^{\alpha}_{\gamma_k} x(t).$$

The set of all functions $x(t)$ that have α-conformable derivatives for any $t \in (\tilde{t}, \infty)$ is denoted by $C^{\alpha}[(\tilde{t}, \infty), \mathbb{R}]$ [23, 27]. Any function from this class is called α-conformable differentiable at any $t \in (\tilde{t}, \infty)$.

Definition 2. (See [24, 27, 31].) The *conformable integral* of order $0 < \alpha \leqslant 1$ with a lower limit $\tilde{t}$, $\tilde{t} \geqslant t_0$, of the function x is given by

$$I^{\alpha}_{\tilde{t}} x(t) = \int_{\tilde{t}}^{t} (\tau - \tilde{t})^{\alpha - 1} x(\tau) \, \mathrm{d}\tau.$$

We will also need the following main properties of the conformable derivatives.

Lemma 1. (*See* [24, 27, 31].) *Let the function* $x : (\tilde{t}, \infty) \to \mathbb{R}$ *be* α*-conformable differentiable on* $(\tilde{t}, \infty)$ *for* $0 < \alpha \leqslant 1$. *Then the following statements hold*:

(i) $I^{\alpha}_{\tilde{t}}(\mathcal{D}^{\alpha}_{\tilde{t}} x(t)) = x(t) - x(\tilde{t})$, $t > \tilde{t}$.

(ii) *If* $v(x(t)) : (\tilde{t}, \infty) \to \mathbb{R}$ *is differentiable with respect to* $x(t)$, *then for any* $t \in [\tilde{t}, \infty)$ *and* $x(t) \neq 0$, *we have*

$$\mathcal{D}^{\alpha}_{\tilde{t}} v\big(x(t)\big) = v'\big(x(t)\big) \mathcal{D}^{\alpha}_{\tilde{t}} x(t),$$

where v' *is the ordinary derivative of* $v(\cdot)$.

For physical and geometrical interpretations of conformable derivatives, see [17, 39].

Let $\varTheta$ be a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\varTheta$ and a positive measure $\operatorname{mes} \varTheta > 0$ containing the origin. The time conformable derivative is defined as follows.

Definition 3. (See [24].) For a function $z = z(t, y)$, $z : (\tilde{t}, \infty) \times \varTheta \to \mathbb{R}$, the limit

$$\mathcal{D}^{\alpha}_{\tilde{t}} z(t, y) = \frac{\partial^{\alpha} z(t, y)}{\partial t^{\alpha}} \bigg|_{\tilde{t}} = \lim_{\varepsilon \to 0} \frac{z(t + \varepsilon(t - \tilde{t})^{1-\alpha}, y) - z(t, y)}{\varepsilon}, \quad t > \tilde{t}, \ y \in \varTheta,$$

is the *conformable derivative along* t *of order* α, $0 < \alpha \leqslant 1$, for $t > \tilde{t}$, $y \in \varTheta$.

If $\tilde{t} = \gamma_k$, $k = 1, 2, \dots$, then the α-generalized time conformable derivative is

$$\mathcal{D}^{\alpha}_{\gamma_k} z(\gamma_k, y) = \lim_{t \to \gamma_k^+} \mathcal{D}^{\alpha}_{\gamma_k} z(t, y).$$

Remark 1. The α-generalized conformable differentiable with respect to time functions have the same properties as the properties of the α-conformable differentiable functions listed in [24, Lemma 1].

Definition 4. (See [1, 8].) The conformable exponential function $E_\alpha(\nu,\tau)$ for $0<\alpha\leqslant 1$ is defined by

$$E_\alpha(\nu,\tau)=\exp\left(\nu\frac{\tau^\alpha}{\alpha}\right),\quad \nu\in\mathbb{R},\ \tau\in\mathbb{R}_+.$$

Using the above notations, we introduce a conformable network model with reaction–diffusion terms as follows:

$$\begin{aligned}\mathcal{D}_{\tilde{t}}^{\alpha}\bar{z}_q(t,y)={}&\sum_{r=1}^{n}\frac{\partial}{\partial y_r}\left(D_{qr}\frac{\partial\bar{z}_q(t,y)}{\partial y_r}\right)-c_q\bar{z}_q(t,y)\\&+\sum_{j=1}^{m}a_{qj}(t)f_j\big(\bar{z}_j(t,y)\big)+I_q(t),\end{aligned}\tag{1}$$

where $t>0$, $0<\alpha<1$, $q=1,2,\dots,m$, $m\geqslant 2$ corresponds to the number of neurons in the neural network, $\bar{z}=(\bar{z}_1,\bar{z}_2,\dots,\bar{z}_m)^{\mathrm{T}}\in\mathbb{R}^m$, $\bar{z}_q=\bar{z}_q(t,y)$ denotes the state of the qth unit at time $t>0$ and space $y\in\varTheta$, f_j represents the activation functions of the jth unit. The functions $a_{qj}(t)$ are the connection weights in t, and the constant c_q represents the rate with which the qth unit will reset its potential to the resting state in isolation when disconnected from the network and external input, I_q denotes the external input for the qth unit. The smooth functions $D_{qr}=D_{q,r}(t,y)\geqslant 0$ correspond to the diffusion operators of transmission along the qth unit, $q=1,2,\dots,m$, $r=1,2,\dots,n$.

Remark 2. Model (1) extends many existing integer- and fractional-order reaction–diffusion neural network models to the conformable settings [13, 14, 29, 33, 35]. The conformable approach allows for the development of models more flexible than those offered by applying the integer-order approach. At the same time, the computational difficulties inherent in using the fractional-order approach are avoided. In fact, most of the classical fractional-order operators are defined as integrals with different singular kernels, that is, they have a nonlocal structure. Due to this fact, the use of such derivatives leads to complexities in the analysis of fractional-order models. The conformable derivatives introduced in [1, 8] provide computational simplifications as they satisfy the concepts and rules of ordinary derivatives. The simplicity in their applications makes the conformable calculus an exclusive tool in the mathematical modeling [4, 7].

The impulsively controlled model will be defined as

$$\begin{aligned}\mathcal{D}_{\gamma_k}^{\alpha}z_q(t,y)={}&\sum_{r=1}^{n}\frac{\partial}{\partial y_r}\left(D_{qr}\frac{\partial z_q(t,y)}{\partial y_r}\right)-c_qz_q(t,y)\\&+\sum_{j=1}^{m}a_{qj}(t)f_j\big(z_j(t,y)\big)+I_q(t),\quad t\neq\gamma_k,\\z_q(\gamma_k^+,y)={}&z_q(\gamma_k,y)+P_{qk}\big(z_q(\gamma_k,y)\big),\quad k=1,2,\dots,\end{aligned}\tag{2}$$

where $0<\alpha<1$, $t>0$, $q=1,2,\dots,m$, $m\geqslant 2$ represents again the number of neurons in the neural network model, the parameters in the first equation of (2) are the same as in (1), $z_q(t,y)$ is the state of the qth unit at time $t > 0$ and space $y \in \Theta$ in the controlled model (2), the elements of the sequence $\{\gamma_k\} \in \mathbb{R}_+$ are the impulsive moments at which abrupt changes of the node $z_q(t,y)$ from the positions $z_q(\gamma_k^-, y) = \bar{z}_q(\gamma_k, y)$ into the positions $z_q(\gamma_k^+, y)$ are observed, P_{qk} are the impulsive (jump) functions, which measure the magnitudes of the short-term changes in the nodes $\bar{z}_q(t,y)$ in (1) at the instants γ_k, $k = 1, 2, \dots$. The functions $P_{qk}(z_q(\gamma_k, y)) = \Delta z_q(\gamma_k, y) = z_q(\gamma_k^+, y) - z_q(\gamma_k, y)$ can be considered as impulsive controllers, which are applied on the nodes $z_q(t,y)$, $q = 1, 2, \dots, m$, at discrete times γ_k, $k = 1, 2, \dots$.

Remark 3. The designed impulsively controlled model (2) allows control signals to be applied only at the fixed time instants γ_k, $k = 1, 2, \dots$. These signals with magnitudes determined by the functions P_{qk}, $q = 1, 2, \dots, m$, $k = 1, 2, \dots$, can be used to impulsively control the qualitative behavior of model (1).

Remark 4. The impulsive control approach has been applied to numerous integer-order and fractional-order reaction–diffusion neural network models [32, 34, 37]. For conformable models, this approach has been applied to reaction–diffusion neural networks just in [24] not analyzing stability strategies. Thus, our research complements numerous important results in the existing literature. It also contributes to the development of the theory and applications of conformable neural network models.

We will consider the impulsive control conformable model (2) under the following boundary and initial conditions:

$$z_q(t,y) = 0, \quad t \in \mathbb{R},\ y \in \partial\Theta \tag{3}$$

$$z_q(0^+, y) = z_{0q}(y), \quad y \in \Theta, \tag{4}$$

where $z_0 = (z_{01}, z_{02}, \dots, z_{0m})^{\mathrm{T}}$, $z_{0q}(y)$ are real-valued bounded continuous on Θ functions.

The solutions $z(t,y) = z(t,y;z_0)$, $z(t,y) = (z_1(t,y), z_2(t,y), \dots, z_m(t,y))^{\mathrm{T}}$ of the initial boundary-value problem (IBVP) (2)–(4) are piecewise continuous functions with points of discontinuity at the moments γ_k, $k = 1, 2, \dots$ [24, 28, 32, 34, 37].

Next, the notion of h-manifold practical stability will be introduced to the neural network model (2). To this end, we consider a continuous function $h = h(t,z)$, $h : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}^p$, $p \leqslant m$, $h(t,z) = (h_1,(t,z), h_2(t,z), \dots, h_p(t,z))^{\mathrm{T}}$, and define the following sets related to h:

$$M_t(m-p) = \bigl\{z \in \mathbb{R}^m\colon\ h_1(t,z) = h_2(t,z) = \cdots = h_p(t,z) = 0,\ t \in [0,\infty)\bigr\},$$
$$M_t(m-p)(\lambda) = \bigl\{z \in \mathbb{R}^m\colon\ \bigl\|h(t,z)\bigr\| < \lambda,\ t \in [0,\infty)\bigr\}, \quad \lambda > 0.$$

We will also suppose that the set $M_t(m-p)(\lambda)$ is an $m-p$-dimensional manifold in $\mathbb{R}^m$ and each solution $z(t,y)$ of the IBVP (2)–(4) satisfying

$$\bigl\|h\bigl(t, z(t,y)\bigr)\bigr\| \leqslant H < \infty$$

is defined on $\mathbb{R}_+ \times \Theta$.

Definition 5. Let $0 < \alpha < 1$. The impulsive control conformable reaction–diffusion neural network model (2) is:

(i) (λ, A)-*practically stable with respect to the function* h if, for given (λ, A) with $0 < \lambda < A$, the condition $z_0 \in M_{0^+}(m-p)(\lambda)$ implies

$$z(t,y) \in M_t(m-p)(A), \quad t \geqslant 0;$$

(ii) (λ, A)-*practically globally exponentially stable with respect to the function* h if, for given (λ, A) with $0 < \lambda < A$, the condition $z_0 \in M_{0^+}(m-p)(\lambda)$ implies the existence of constants $\kappa, \mu > 0$ such that

$$z(t,y) \in M_t(m-p)\bigl(A + \kappa \bigl\| h(0^+, z_0) \bigr\| E_\alpha(-\mu, t)\bigr), \quad t \geqslant 0.$$

Remark 5. Note that the practical stability of model (2) with respect to the function h is equivalent to the practical stability of an entire manifold of states $M_t(m-p)$ defined by this function. Thus, the h-manifold practical stability notion is more general than the single state stability, and as such it is preferred in the study of many important applied problems [28]. For particular values of the function h, Definition 5 can be reduced to the practical stability definitions of specific states or manifolds of states. For example, if $h(t,z) = z$, then Definition 5 reduces to the definitions of practical stability of the zero state of model (2). Also, for $\alpha = 1$, Definition 5 reduces to the definition of the h-manifold practical stability for neural network models with ordinary derivative, which are investigated in [28].

Remark 6. It is also seen from Definition 5 that the practical stability notion does not completely match with the Lyapunov stability. However, in some cases, the practical global exponential stability may imply global exponential stability in the sense of Lyapunov for $A = 0$ [10, 27, 28].

Remark 7. The practical stability with respect to a function h (see Definition 6) can also be used in the study of practical synchronization issues between the master system (1) and the impulsive control system (2). To this end, the researchers have to consider the error system defined by

$$\begin{aligned}
\mathcal{D}^{\alpha}_{\gamma_k} e_q(t,y) &= \sum_{r=1}^{n} \frac{\partial}{\partial y_r}\left(D_{qr}\frac{\partial e_q(t,y)}{\partial y_r}\right) - c_q e_q(t,y) \\
&\quad + \sum_{j=1}^{m} a_{qj}(t) g_j\bigl(e_j(t,y)\bigr), \quad t \neq \gamma_k, \\
e_q(\gamma_k^+, y) &= e_q(\gamma_k, y) + U_{qk}\bigl(e_q(\gamma_k, y)\bigr), \quad k = 1, 2, \dots,
\end{aligned}$$

where $0 < \alpha < 1$, $t > 0$, $y \in \Theta$, $e_q(t,z) = z_q(t,z) - \bar{z}_q(t,z)$, $q = 1, 2, \dots, m$, $g_j(e_j) = f_j(e_j + \bar{z}_j) - f_j(\bar{z}_j)$, $j = 1, 2, \dots, m$, $U_{qk}(e_q) = P_{qk}(z_q)$, $q = 1, 2, \dots, m$, $k = 1, 2, \dots$. Thus, the impulses can be used to practically synchronize the behaviors of models (1) and (2).

Also, for a bounded continuous function g defined on $J \subseteq \mathbb{R}$, we set

$$\overline{g} = \sup_{t\in J}\bigl|g(t)\bigr|, \qquad \underline{g} = \inf_{t\in J}\bigl|g(t)\bigr|.$$

The existence and uniqueness of the solutions of model (2) [24, 28] and the further analysis require the following assumptions:

($\mathscr{A}_1$) There exist constants $L_q > 0$ such that

$$\bigl|f_q(\chi_1) - f_q(\chi_2)\bigr| \leqslant L_q|\chi_1 - \chi_2|,$$

and $f_q(0) = 0$ for all $\chi_1, \chi_2 \in \mathbb{R}$, $\chi_1 \neq \chi_2$, $q = 1, 2, \dots, m$.

($\mathscr{A}_2$) The functions a_{qj} and I_q are continuous on $\mathbb{R}_+$, $q, j = 1, 2, \dots, m$.

($\mathscr{A}_3$) For the diffusion coefficients, there exist constants $d_{qr} \geqslant 0$ such that

$$D_{qr}(t, y) \geqslant d_{qr}$$

for $q = 1, 2, \dots, m$, $r = 1, 2, \dots, n$, $t > 0$, and $y \in \Theta$.

($\mathscr{A}_4$) The impulsive functions P_{qk} are continuous on $\mathbb{R}$ and $P_{qk}(0) = 0$ for all $q = 1, 2, \dots, m$ and $k = 1, 2, \dots$.

We will also need the next result.

Lemma 2. (*See* [14, 28].) *Let Θ be a cube $|y_r| < l_r$, $r = 1, 2, \dots, n$, and let $v(y)$ be a real-valued function belonging to $C^1(\Theta)$, which vanishes on the boundary $\partial\Theta$ of Θ, i.e., $v(y)|_{\partial\Theta} = 0$. Then*

$$\int\limits_{\Theta} v^2(y)\,\mathrm{d}y \leqslant l_r^2 \int\limits_{\Theta} \left|\frac{\partial v(z)}{\partial z_r}\right|^2 \mathrm{d}y.$$

In the last part of Section 2, we will present some notations, definitions, and a comparison lemma from the conformable Lyapunov function method using piecewise continuous Lyapunov-type functions.

Define the sets

$$\mathcal{G}_k = \bigl\{(t, z)\colon\ t \in (\gamma_{k-1}, \gamma_k),\ z \in \mathbb{R}^m\bigr\}, \quad k = 1, 2, \dots,$$
$$\mathcal{G} = \bigcup_{k=1}^{\infty} \mathcal{G}_k$$

and the class $\mathcal{V}^{\alpha}_{\gamma_k}$ of nonnegative Lyapunov functions $\mathcal{V}$ [24, 28] for any $\gamma_k \in \mathbb{R}_+$, $k = 1, 2, \dots$, such that $\mathcal{V}(t, 0) = 0$ for $t \in \mathbb{R}_+ \times \Theta$, $t \geqslant \gamma_k$, $\mathcal{V}$ is continuous in $\mathcal{G}$, α-generalized conformable differentiable in t and locally Lipschitz continuous with respect to z on each of the sets $\mathcal{G}_k$, and for each $k = 1, 2, \dots$ and $z \in \mathbb{R}^m$, there exist the finite limits

$$\mathcal{V}(\gamma_k^-, z) = \lim_{t\to\gamma_k,\, t<\gamma_k} \mathcal{V}(t, z) = \mathcal{V}(\gamma_k, z),$$
$$\mathcal{V}(\gamma_k^+, z) = \lim_{t\to\gamma_k,\, t>\gamma_k} \mathcal{V}(t, z).$$

Now, for a given function $\mathcal{V} \in \mathcal{L}^{\alpha}_{\gamma_k}$, $t \in \mathbb{R}_+$, $t \neq \gamma_k$, $k = 1, 2, \dots$, we define the following derivative with respect to system (2):

$$
\begin{aligned}
&{}^{+}\mathcal{D}^{\alpha}_{\gamma_k}\mathcal{V}\big(t, z(t,\cdot)\big)\\
&\quad = \lim_{\varepsilon \to 0^+} \sup \frac{\mathcal{V}(t + \varepsilon(t-\gamma_k)^{1-\alpha},\, z(t + \varepsilon(t-\gamma_k)^{1-\alpha}, \cdot)) - \mathcal{V}(t, z(t,\cdot))}{\varepsilon}.
\end{aligned}
$$

The following lemma from [24, 28] will also be useful.

Lemma 3. *If for the function $\mathcal{V} \in \mathcal{V}^{\alpha}_{\gamma_k}$ and for $t \in \mathbb{R}_+$, $z \in \mathbb{R}^m$, we have*

(i) $\mathcal{V}(\gamma_k^+, z(\gamma_k,\cdot) + \Delta z(\gamma_k,\cdot)) \leqslant \mathcal{V}(\gamma_k, z(\gamma_k,\cdot))$, $k = 1, 2, \dots$, *and*

(ii) ${}^{+}\mathcal{D}^{\alpha}_{\gamma_k}\mathcal{V}(t, z(t,\cdot)) \leqslant -\zeta\mathcal{V}(t, z(t,\cdot)) + o(t)$, $t \neq \gamma_k$, $k = 0, 1, \dots$, *for* $\zeta = \mathrm{const} > 0$, $o \in C^{\alpha}[\mathbb{R}_+, \mathbb{R}_+]$,

then

$$
\begin{aligned}
\mathcal{V}(t, z(t,\cdot)) \leqslant \mathcal{V}(0^+, z_0)E_\alpha(-\zeta, t) &+ \int\limits_{\gamma_k}^{t} \frac{W^{\alpha}(t-\gamma_k,\, \sigma-\gamma_k)o(\sigma)}{(\sigma-\gamma_k)^{1-\alpha}}\,\mathrm{d}\sigma\\
&+ \sum_{j=1}^{k} \prod_{l=k-j+1}^{k} E_\alpha(-\zeta,\, \gamma_l - \gamma_{l-1})\\
&\times \int\limits_{\gamma_{k-j}}^{\gamma_{k-j+1}} \frac{W^{\alpha}(t-\gamma_k,\, \sigma-\gamma_{k-j})o(\sigma)}{(\sigma-\gamma_{k-j})^{1-\alpha}}\,\mathrm{d}\sigma, \quad t \geqslant 0,
\end{aligned}
$$

where

$$
W^{\alpha}(t-\gamma_k,\, \sigma-\gamma_k) = E_\alpha(-\zeta,\, t-\gamma_k)E_\alpha(\zeta,\, \sigma-\gamma_k).
$$

3 Main practical stability results

In this section, we will apply the conformable Lyapunov function approach to establish practical stability criteria for the impulsive control conformable reaction–diffusion neural network (2).

Firstly, we will prove a (λ, A)-practical stability result.

Theorem 1. *Assume that* $0 < \lambda < A$, *assumptions* $(\mathscr{A}_1)$–$(\mathscr{A}_4)$ *hold, and*:

1. $\Theta = \{y\colon\, y = (y_1, y_2, \dots, y_n)^{\mathrm{T}},\ |y_r| < l_r\}$, l_r, $r = 1, 2, \dots, n$, *are positive constants.*
2. *The model's parameters and the impulsive control functions of the neural network* (2) *satisfy*:
   (i) $\min_{1\leqslant q\leqslant m}((c_q + \bar{D}_q) - \sum_{j=1}^{m}(L_j\bar{a}_{qj} + L_q\bar{a}_{jq})) > 0$ *with* $\bar{D}_q = \sum_{r=1}^{n} d_{qr}/l_r^2$, $r = 1, 2, \dots, n$;
   (ii) $I_q(t) = 0$, $q = 1, 2, \dots, m$, $t \geqslant 0$;

(iii) $P_{qk}(z_q(\gamma_k, y)) = -\eta_{qk} z_q(\gamma_k, y)$, $0 < \eta_{qk} < 2$, $q = 1, 2, \dots, m$, $k = 1, 2, \dots$, $y \in \Theta$.

3. *The function* $h = h(t, z)$, $h : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}^p$ *is such that*

$$\|h(t,z)\| < \int\limits_{\Theta} \sum_{q=1}^{m} z_q^2(t,y)\,\mathrm{d}y \leqslant \Lambda(H)\|h(t,z)\|, \quad t \in [0,\infty),$$

where $\Lambda(H) \geqslant 1$ *exists for any* $0 < H \leqslant \infty$ *and* $\Lambda(H)\lambda < A$.

Then the impulsively controlled model (2) *is* (λ, A)*-practically stable with respect to the function* h*, i.e., the* h*-manifold* $M_t(m-p)$ *is* (λ, A)*-practically stable.*

Proof. For $0 < \lambda < A$, let $z_0 \in M_{0^+}(m-p)(\lambda)$, and let $z(t, y) = z(t, y; z_0)$,

$$z(t,y) = \big(z_1(t,y), z_2(t,y), \dots, z_m(t,y)\big)^{\mathrm{T}}$$

be the solution of the IBVP (2)–(4).

We consider a Lyapunov function defined as

$$\mathcal{V}\big(t, z(t,\cdot)\big) = \int\limits_{\Theta} \sum_{q=1}^{m} z_q^2(t,y)\,\mathrm{d}y. \tag{5}$$

From $(\mathscr{A}_4)$ and condition 2(iii) of Theorem 1, at the impulsive control instants $t = \gamma_k$, $k = 1, 2, \dots$, we have

$$\begin{aligned}
&\mathcal{V}\big(\gamma_k^+, z(\gamma_k,\cdot) + \Delta z(\gamma_k,\cdot)\big) \\
&\quad = \int\limits_{\Theta} \sum_{q=1}^{m} \Big(z_q(t,y) + P_{qk}(z_q(\gamma_k,y))\Big)^2 \mathrm{d}y = \int\limits_{\Theta} \sum_{q=1}^{m} (1-\eta_{qk})^2 z_q^2(t,y)\,\mathrm{d}y \\
&\quad < \int\limits_{\Theta} \sum_{q=1}^{m} z_q^2(t,y)\,\mathrm{d}y = \mathcal{V}(\gamma_k, z(\gamma_k,\cdot)), \quad k = 1, 2, \dots,
\end{aligned} \tag{6}$$

i.e., condition (i) of Lemma 3 is satisfied for the Lyapunov-type function $\mathcal{V} \in \mathcal{V}^{\alpha}_{\gamma_k}$.

Next, for function (5), using the definition and properties of the conformable derivative ${}^{+}\mathcal{D}^{\alpha}_{\gamma_k}\,\mathcal{V}(t, z(t,\cdot))$, we obtain for $t \neq \gamma_k$, $k = 0, 1, 2, \dots$,

$$\begin{aligned}
{}^{+}\mathcal{D}^{\alpha}_{\gamma_k}\mathcal{V}\big(t, z(t,\cdot)\big) &= 2\sum_{q=1}^{m} \int\limits_{\Theta} z_q(t,y) \frac{\partial^{\alpha} z_q(t,y)}{\partial \gamma_k}\,\mathrm{d}y \\
&= 2\sum_{q=1}^{m} \int\limits_{\Theta} z_q(t,y) \Bigg(\sum_{r=1}^{n} \frac{\partial}{\partial y_r}\bigg(D_{qr}\frac{\partial z_q(t,y)}{\partial y_r}\bigg) \\
&\qquad - c_q z_q(t,y) + \sum_{j=1}^{m} a_{qj}(t) f_j\big(z_j(t,y)\big)\Bigg)\mathrm{d}y.
\end{aligned} \tag{7}$$

Applying the Green formula and using the boundary condition (3), we get

$$\sum_{r=1}^{n} \int_{\Theta} z_q(t,y) \frac{\partial}{\partial y_r}\left(D_{qr} \frac{\partial z_q(t,y)}{\partial y_r}\right) \mathrm{d}y = -\sum_{r=1}^{n} \int_{\Theta} D_{qr}\left(\frac{\partial z_q(t,y)}{\partial y_r}\right)^2 \mathrm{d}y.$$

From the above equality, $(\mathscr{A}_3)$, condition 2(i) of Theorem 1, and Lemma 2 we obtain

$$\begin{aligned}
&\sum_{r=1}^{n} \int_{\Theta} z_q(t,y) \frac{\partial}{\partial y_r}\left(D_{qr} \frac{\partial z_q(t,y)}{\partial y_r}\right) \mathrm{d}y \\
&\quad \leqslant -\sum_{r=1}^{n} \int_{\Theta} d_{qr}\left(\frac{\partial z_q(t,y)}{\partial y_r}\right)^2 \mathrm{d}y \leqslant -\sum_{r=1}^{n} \int_{\Theta} \frac{d_{qr}}{l_r^2} z_q^2(t,y)\, \mathrm{d}y \\
&\quad = -\bar{D}_q \int_{\Theta} z_q^2(t,y)\, \mathrm{d}y.
\end{aligned} \tag{8}$$

Also, using $(\mathscr{A}_1)$ and $(\mathscr{A}_2)$, we have

$$\begin{aligned}
&\sum_{j=1}^{m} a_{qj}(t) \int_{\Theta} z_q(t,y) f_j\big(z_j(t,y)\big)\, \mathrm{d}y \\
&\quad \leqslant \sum_{j=1}^{m} L_j \big|a_{qj}(t)\big| \int_{\Theta} \big|z_q(t,y)\big|\big|z_j(t,y)\big|\, \mathrm{d}y \\
&\quad \leqslant \sum_{j=1}^{m} L_j \overline{a}_{qj} \int_{\Theta} \big(z_q^2(t,y) + z_j^2(t,y)\big)\, \mathrm{d}y.
\end{aligned} \tag{9}$$

From (7)–(9) and condition 2(i) of Theorem 1 we obtain for $t \neq \gamma_k$, $k = 0, 1, 2 \ldots,$

$$\begin{aligned}
&{}^{+}\mathcal{D}_{\gamma_k}^{\alpha} \mathcal{V}\big(t, z(t,\cdot)\big) \\
&\quad \leqslant 2\sum_{q=1}^{m} \left[\left((c_q + \bar{D}_q) - \sum_{j=1}^{m} (L_j \overline{a}_{qj} L_q \overline{a}_{jq})\right) \int_{\Theta} z_q^2(t,y)\, \mathrm{d}y\right] \\
&\quad \leqslant -\min_{1 \leqslant q \leqslant m} \left((c_q + \bar{D}_q) - \sum_{j=1}^{m} (L_j \overline{a}_{qj} + L_q \overline{a}_{jq})\right) 2\sum_{q=1}^{m} \int_{\Theta} z_q^2(t,y)\, \mathrm{d}y < 0.
\end{aligned} \tag{10}$$

Then, using (6), (10), and Lemma 3, we get

$$\mathcal{V}\big(t, z(t,\cdot)\big) < \mathcal{V}\big(0^+, z_0\big), \quad t \geqslant 0.$$

Hence, from condition 3 of Theorem 1 we have

$$\begin{aligned}
\big\|h(t,z)\big\| < \mathcal{V}\big(t, z(t,\cdot)\big) < \mathcal{V}(0^+, z_0) \leqslant \Lambda(H)\big\|h(0^+, z_0)\big\| \\
< \Lambda(H)\lambda < A, \quad t \in [0, \infty).
\end{aligned}$$

From the last estimate it follows that the impulsively controlled model (2) is (λ, A)-practically stable with respect to the function h, and the proof is complete. □

Next, we will propose criteria for the practical exponential stability of the impulsive control conformable reaction–diffusion neural network model (2).

Theorem 2. *Assume that* $0 < \lambda < A$, *assumptions* $(\mathscr{A}_1)$–$(\mathscr{A}_4)$, *conditions* 1, 3, *and* 2(iii) *of Theorem* 1 *hold, and conditions* 2(i) *and* 2(ii) *in Theorem* 1 *are replaced by*:

2(i*) *There exists* $\mu > 0$ *such that*

$$\min_{1\leqslant q\leqslant m}\left((c_q + \bar{D}_q) - \sum_{j=1}^{m}\left(L_j\overline{a}_{qj} + L_q\overline{a}_{jq}\right)\right) \geqslant \mu.$$

2(ii*) *For* $t \geqslant 0$, *we have*

$$\begin{aligned}\bar{G}(t) = &\int_{\gamma_k}^{\infty}\frac{W^{\alpha}(t-\gamma_k,\,\sigma-\gamma_k)}{(\sigma-\gamma_k)^{1-\alpha}}2\sum_{q=1}^{m}\left|I_q(\sigma)\right|\mathrm{d}\sigma\\ &+\sum_{j=1}^{k}\prod_{l=k-j+1}^{k}E_\alpha(-\zeta,\,\gamma_l-\gamma_{l-1})\int_{t_{k-j}}^{\gamma_{k-j+1}}\frac{W^{\alpha}(t-\gamma_k,\,\sigma-\gamma_{k-j})}{(\sigma-\gamma_{k-j})^{1-\alpha}}\\ &\times 2\sum_{q=1}^{m}\left|I_q(\sigma)\right|\mathrm{d}\sigma\\ &< A.\end{aligned}$$

Then the impulsive control conformable reaction–diffusion neural network model (2) *is* (λ, A)*-practically globally exponentially stable with respect to the function* h*, i.e., the* h*-manifold* $M_t(m-p)$ *is* (λ, A)*-practically globally exponentially stable.*

Proof. Let $0 < \lambda < A$, $z_0 \in M_{0^+}(m-p)(\lambda)$, and consider again the Lyapunov function $\mathcal{V}(t, z(t,\cdot))$ defined by (5).

From (7)–(9) and condition 2(i*) of Theorem 2 we obtain

$$^{+}\mathcal{D}^{\alpha}_{\gamma_k}\mathcal{V}\big(t, z(t,\cdot)\big) \leqslant -2\mu\mathcal{V}\big(t, z(t,\cdot)\big) + 2\sum_{q=1}^{m}\left|I_q(t)\right|, \quad t \neq \gamma_k,\ k = 0, 1, 2\dots. \tag{11}$$

Then from (6) and (11), according to Lemma 3 for $\zeta = 2\mu$, $o(t) = 2\sum_{q=1}^{m}|I_q(t)|$, $t \geqslant 0$, we get

$$\mathcal{V}\big(t, z(t,\cdot)\big) \leqslant \mathcal{V}(0^+, z_0)E_\alpha(-2\mu, t) + \bar{G}(t), \quad t \geqslant 0.$$

Hence, from condition 3 of Theorem 1 and condition 2(ii*) of Theorem 2 we obtain for $t \geqslant 0$,

$$\begin{aligned}\left\|h\big(t, z(t, y; z_0)\big)\right\| &< \mathcal{V}\big(t, z(t,\cdot)\big) \leqslant \mathcal{V}\big(0^+, z_0\big)E_\alpha(-2\mu, t) + \bar{G}(t)\\ &\leqslant \Lambda(H)\left\|h\big(0^+, z_0\big)\right\|E_\alpha(-2\mu, t) + A, \quad t \geqslant 0.\end{aligned}$$

Therefore,

$$z(t, y; z_0) \in M_t(m-p)\big(A + \Lambda(H)\left\|h\big(0^+, z_0\big)\right\|E_\alpha(-2\mu, t)\big), \quad t \geqslant 0,$$

i.e., system (2) is (λ, A)-practically globally exponentially stable with respect to h. □

Remark 8. Due to its advantages, the notion of practical stability has been investigated for various applied systems [6, 20]. The recent practical stability results for fractional-order models are additional evidences of its importance [25, 26]. The concept has been also applied to a very few conformable models [18, 23, 27], not considering reaction–diffusion terms. In Theorems 1 and 2, we realized our idea to apply the concept to an impulsive conformable reaction–diffusion neural network. In addition, instead of considering separate states of the model, the practical stability is extended to sets defined by a function h. The practical stability concept of h-manifolds extends the stability and practical stability of a single state and includes many particular cases [21,23,27,28]. Thus, the hybrid practical stability of h-manifolds approach offers a high-powered mechanism, which can be applied not only for single solutions, but also in the cases when manifolds of solutions are attractors for the models, as well as in the cases when these manifolds are not mathematically stable, but they oscillate around acceptable trajectories.

Remark 9. The results established in this section also offer an impulsive control strategy for model (1) that can be reached in a set time and greatly improve functionality in real application. In fact, Theorems 1 and 2 can be applied as impulsive control mechanisms to guarantee the practical stability and practical global exponential stability of a manifold of states of (1) determined by a function h. The proposed criteria are in the inequality forms for the impulse control functions and are therefore convenient for applications.

Remark 10. In most impulsive control mechanisms, some upper bounds of the impulsive intervals and restrictions on the distance between the impulsive moments are constrained [15, 36, 37]. One advantage of the proposed stability criteria is that they do not include restrictions on the distance between the impulsive control instances. Hence, the proposed mechanism improves the impulsive control strategies proposed to similar models in [15, 36, 37].

4 Robust practical stability analysis

In this section, we will consider the impulsive conformable reaction–diffusion neural network model (2) as a "nominal" model for the following conformable neural network model with reaction–diffusion terms and uncertain parameters:

$$\begin{aligned}
\mathcal{D}^{\alpha}_{\gamma_k} z_q(t,y) &= \sum_{r=1}^{n} \frac{\partial}{\partial y_r}\left(D_{qr}\frac{\partial z_q(t,y)}{\partial y_r}\right) - (c_q+\tilde{c}_q)z_q(t,y) \\
&\quad + \sum_{j=1}^{m}\big(a_{qj}(t)+\tilde{a}_{qj}(t)\big)f_j\big(z_j(t,y)\big) + I_q(t)+\tilde{I}_q(t), \quad t\neq\gamma_k, \\
z_q(\gamma_k^+,y) &= z_q(\gamma_k,y) + P_{qk}\big(z_q(\gamma_k,y)\big) + \tilde{P}_{qk}\big(z_q(\gamma_k,y)\big), \quad k=1,2,\dots,
\end{aligned} \tag{12}$$

where $0<\alpha<1$, $t>0$, $q=1,2,\dots,m$, $m\geqslant 2$, and $\tilde{c}_q$ are constant uncertain parameters, $\tilde{a}_{qj}$ are uncertainties in the connection coefficients, $\tilde{I}_q$ represents the uncertainty in the external input of the q-th unit, and $\tilde{P}_{qk}$, $q, j = 1, 2, \dots, m$, $k = 1, 2, \dots$, are uncertainties in the impulsive control functions.

In fact, uncertain parameters often exist in the real-world neural network models, and the study of their effect on the qualitative behavior of the models is crucial [12, 19].

We will use the following definition in our robust practical stability analysis with respect to the function $h = h(t, z)$, $h : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}^p$, $p \leqslant m$.

Definition 6. The impulsive control conformable reaction–diffusion neural network model (2) is (λ, A)-*practically globally robustly exponentially stable with respect to the function* h if, for given (λ, A) with $0 < \lambda < A$ and $z_0 \in M_{0^+}(m-p)(\lambda)$, the uncertain model (12) is (λ, A)-practically globally exponentially stable with respect to the function h for uncertain parameters $\tilde{c}_q$, $\tilde{a}_{qj}$ and $\tilde{P}_{qk}$, $q, j = 1, 2, \dots, m$, $k = 1, 2, \dots$, taking values in some bounded sets.

In this section, we will present global robust stability criteria for the equilibrium z^* of model (2) with respect to the function h.

The following robust practical result is a direct corollary of Theorem 2.

Theorem 3. *Assume that conditions of Theorem* 2 *hold,* 2(i*), 2(ii*) *of Theorem* 2 *and* 2(iii) *of Theorem* 1 *are respectively replaced by*:

2(i**) *There exists $\mu > 0$ such that*

$$\min_{1 \leqslant q \leqslant m} \left((c_q + \bar{c}_q + \bar{D}_r) - \sum_{j=1}^{m} \left(L_j(\overline{a}_{qj} + \overline{\overline{a}}_{qj}) + L_q(\overline{a}_{jq} + \overline{\overline{a}}_{jq}) \right) \right) \geqslant \mu.$$

2(ii**) *For $t \geqslant 0$, we have*

$$\begin{aligned}
\bar{G}^*(t) = {} & \int_{\gamma_k}^{\infty} \frac{W^\alpha(t-\gamma_k,\, \sigma-\gamma_k)}{(\sigma-\gamma_k)^{1-\alpha}} 2 \sum_{q=1}^{m} \left(\left| I_q(\sigma) \right| + \left| \tilde{I}_q(\sigma) \right| \right) \mathrm{d}\sigma \\
& + \sum_{j=1}^{k} \prod_{l=k-j+1}^{k} E_\alpha(-\zeta,\, \gamma_l - \gamma_{l-1}) \int_{\gamma_{k-j}}^{\gamma_{k-j+1}} \frac{W^\alpha(t-\gamma_k,\, \sigma-\gamma_{k-j})}{(\sigma-\gamma_{k-j})^{1-\alpha}} \\
& \times 2 \sum_{q=1}^{m} \left(\left| I_q(\sigma) \right| + \left| \tilde{I}_q(\sigma) \right| \right) \mathrm{d}\sigma \\
< {} & A.
\end{aligned}$$

2(iii**) *The uncertainties in the impulsive controls $\tilde{P}_{qk}(z_q(\gamma_k, y)) = -\tilde{\nu}_{qk} z_q(\gamma_k, y)$ and the unknown constants $\tilde{\nu}_{qk}$ are such that $0 < \tilde{\nu}_{qk} < 2 - \eta_{qk}$, $q = 1, 2, \dots, m$, $k = 1, 2, \dots$.*

Then the impulsive control conformable reaction–diffusion neural network model (2) *is (λ, A)-practically globally robustly exponentially stable with respect to the function h, i.e., the h-manifold $M_t(m-p)$ is (λ, A)-practically globally robustly exponentially stable.*

Remark 11. In many previous works, the robust stability have been investigated for numerous systems. All these works considered the effects of some bounded uncertain parameters on the stability performance of a system. Theorem 3 pioneers the robust stability analysis for the impulsive conformable reaction–diffusion model (2).

5 Representative examples

Example 1. In this example, we consider the following conformable network model with reaction–diffusion terms as a drive system:

$$\begin{aligned}\mathcal{D}^{\alpha}_{\gamma_k}\bar{z}_q(t,y) = {}& \sum_{r=1}^{n}\frac{\partial}{\partial y_r}\left(D_{qr}\frac{\partial \bar{z}_q(t,y)}{\partial y_r}\right) - c_q\bar{z}_q(t,y)\\ &+ \sum_{j=1}^{m} a_{qj}(t)f_j\big(\bar{z}_j(t,y)\big) + I_q(t),\end{aligned} \tag{13}$$

where $n = m = 2$, $t > 0$, $0 < \alpha < 1$, $c_1 = 2$, $c_2 = 3$, $f_j(\bar{z}_j) = (|\bar{z}_j + 1| - |\bar{z}_j - 1|)/2$, $j = 1, 2$, $\Theta = \{y\colon\ y = (y_1, y_2)^{\mathrm{T}},\ |y_r| < 1\}$, $r = 1, 2$, $I_q(t)$ are continuous on $\mathbb{R}_+$, $q = 1, 2$,

$$(a_{qj})_{2\times 2} = \begin{pmatrix} a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix} = \begin{pmatrix} 0.4\sin t + 1 & 0.2\cos t\\ 0.9\cos t & 1 + 0.2\sin t\end{pmatrix},$$
$$(D_{qr})_{2\times 2} = \begin{pmatrix} D_{11} & D_{12}\\ D_{21} & D_{22}\end{pmatrix} = \begin{pmatrix} 5 + \cos t & 0\\ 0 & 4 + \sin t\end{pmatrix}.$$

The impulsively controlled conformable model is

$$\begin{aligned}\mathcal{D}^{\alpha}_{\gamma_k} z_q(t,y) = {}& \sum_{r=1}^{n}\frac{\partial}{\partial y_r}\left(D_{qr}\frac{\partial z_q(t,y)}{\partial y_r}\right) - c_q z_q(t,y)\\ &+ \sum_{j=1}^{m} a_{qj}(t)f_j\big(z_j(t,y)\big) + I_q(t), \quad t \neq \gamma_k,\\ z_q(\gamma_k^+, y) = {}& z_q(\gamma_k, y) + P_{qk}\big(z_q(\gamma_k, y)\big), \quad k = 1, 2, \dots,\end{aligned} \tag{14}$$

where $0 < \gamma_1 < \gamma_2 < \cdots$, $\gamma_k \to \infty$ as $k \to \infty$, the impulsive functions are $P_{qk}(z_q(\gamma_k, y)) = -z_q(\gamma_k, y)/5$, $q = 1, 2$, $k = 1, 2, \dots$.

System (14) satisfies assumption ($\mathscr{A}_1$) for $L_q = 1$. Also, the constants d_{qr}, $q = 1, 2$, $r = 1, 2$, in ($\mathscr{A}_3$) are

$$(d_{qr})_{2\times 2} = \begin{pmatrix} d_{11} & d_{12}\\ d_{21} & d_{22}\end{pmatrix} = \begin{pmatrix} 4 & 0\\ 0 & 3\end{pmatrix}.$$

Consider a continuous function $h : \mathbb{R}^3 \to \mathbb{R}^2$, which satisfies condition 3 of Theorem 1, and the h-manifold

$$M_t(0) = \big\{z \in \mathbb{R}^2\colon\ h_1(t,z) = h_2(t,z) = 0,\ t \geqslant 0\big\}.$$

We can also check that condition 2(i) of Theorem 1 is satisfied. Hence, for $I_q(t) = 0$, $q = 1, 2$, $t \geqslant 0$, by Theorem 1 we can conclude that if $0 < \lambda < A$ are such that $z_0 \in M_{0^+}(0)(\lambda)$, then the impulsively controlled model (14) is (λ, A)-practically stable with respect to the function h.

Example 2. Consider model (13) as a drive system with $I_1(t) = \cos t$, $I_2(t) = 1.3$, $t \geqslant 0$, and the corresponding impulsive control model (14) with $0 < \gamma_1 < \gamma_2 < \cdots$, $\gamma_k \to \infty$ as $k \to \infty$, the impulsive functions are $P_{qk}(z_q(\gamma_k, y)) = -z_q(\gamma_k, y)/5$, $q = 1, 2$, $k = 1, 2, \dots$.

For the parameters of the impulsive conformable reaction–diffusion neural network model (14), condition 2(i*) of Theorem 2 is satisfied for any $0 < \mu \leqslant 2.1$ since

$$\min_{1 \leqslant q \leqslant m} \left((c_q + \bar{D}_q) - \sum_{j=1}^{m} (L_j \overline{a}_{qj} + L_q \overline{a}_{jq}) \right) = 2.1.$$

Hence, for $0 < \lambda < A$, Theorem 2 guarantees that the impulsive control conformable reaction–diffusion neural network model (14) is (λ, A)-practically globally exponentially stable with respect to the function h for $\bar{G}(t) < A$.

Example 3. Consider the conformable impulsive reaction–diffusion model (14) as a "nominal" system and the corresponding uncertain model

$$\begin{aligned}
\mathcal{D}^{\alpha}_{\gamma_k} z_q(t, y) = {} & \sum_{r=1}^{n} \frac{\partial}{\partial y_r} \left(D_{qr} \frac{\partial z_q(t, y)}{\partial y_r} \right) - (c_q + \tilde{c}_q) z_q(t, y) \\
& + \sum_{j=1}^{m} \big(a_{qj}(t) + \tilde{a}_{qj}(t) \big) f_j \big(z_j(t, y) \big) + I_q(t) + \tilde{I}_q(t), \quad t \neq \gamma_k, \\
z_q(\gamma_k^+, y) = {} & z_q(\gamma_k, y) + P_{qk}(z_q(\gamma_k, y)) + \tilde{P}_{qk}(z_q(\gamma_k, y)), \quad k = 1, 2, \dots,
\end{aligned}$$

where $t > 0$, $q = 1, 2$, with uncertain parameters: $\tilde{c}_q$, $\tilde{a}_{qj}$, $\tilde{I}_q$, and $\tilde{P}_{qk}$, $q, j = 1, 2$, $k = 1, 2, \dots$.

If the uncertain parameters satisfy the boundedness condition

$$\min_{1 \leqslant q \leqslant m} \left((c_q + \bar{c}_q + \bar{D}_r) - \sum_{j=1}^{m} \big(L_j (\overline{a}_{qj} + \overline{\tilde{a}}_{qj}) + L_q (\overline{a}_{jq} + \overline{\tilde{a}}_{jq}) \big) \right) \geqslant \mu$$

and the uncertainties in the impulsive controls $\tilde{P}_{qk}(z_q(\gamma_k, y)) = -\nu_{qk} z_q(\gamma_k, y)$ are such that $0 < \tilde{\nu}_{qk} < 9/5$, $q = 1, 2$, $k = 1, 2, \dots$, then, according to Theorem 3, the impulsive control conformable reaction–diffusion neural network model (14) is (λ, A)-practically globally robustly exponentially stable with respect to the function h.

If one of the above conditions is not satisfied, for example, if $\tilde{\nu}_{qk} > 9/5$, $q = 1, 2$, $k = 1, 2, \dots$, we cannot make a conclusion about the (λ, A)-practical global robust exponential stable behavior of system (14) with respect to the function h.

Remark 12. The presented examples demonstrate the efficiency of the proposed results. In addition, we demonstrate how the impulses can affect the stability behavior of the states. Hence, suitable impulsive control strategies can be applied. In addition, Example 2 shows that the external input has a significant impact on the h-manifold stability.

6 Conclusions

In this paper, a conformable approach is applied to extend the class of impulsive reaction–diffusion neural networks. The extended model has all the advantages of impulsive reaction–diffusion models applied as frameworks for modeling real-world processes in science, engineering, biology, and medicine. In addition, it has the flexibility provided by the use of conformable derivatives. The hybrid concept of practical stability with respect to h-manifolds is introduced into the model, and new criteria that guarantee the practical stability of h-manifold and the practical global exponential stability of h-manifold are established. The conformable Lyapunov function method is applied to derive our results. Robust stability analysis is also conducted considering the effect of uncertain parameters. Three examples are presented to demonstrate the results derived. An area of future development of the theory involves the consideration of delay effects on the stability behavior. It is also interesting to extend the conformable approach and the proposed results to fuzzy models.

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