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Adaptive event-triggered output-feedback stabilization for nonlinear systems with function control coefficients*

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Abstract. This article presents an adaptive event-triggered control scheme for global stabilization of uncertain nonlinear systems with function control coefficients, input matching uncertainty, and unknown polynomial-function rates. A single dynamic gain observer is designed to handle these complexities, replacing the traditional dual-gain approach. An extended state observer asymptotically estimates the input matching uncertainty. An adaptive output-feedback controller, based on a time-varying threshold event-triggering mechanism, ensures closed-loop system stabilization and prevents Zeno behavior. Simulation results validate the proposed strategy.

Keywords: uncertain nonlinear systems, function control coefficients, input matching uncertainty, event-triggered output feedback.

1 Introduction

Output-feedback control for nonlinear systems has gained considerable attention in recent research, as demonstrated by [13, 18, 27, 33] and related references. Nonlinear systems are widely used in practical life, including power systems [4], mechanical systems [5], biomedical systems [22], among others. Furthermore, these systems not only possess inherent nonlinear characteristics but also encounter operational challenges due to external disturbances, which are often challenging to accurately model mathematically. To overcome these challenges, control theory researchers have been actively pursuing effective control design strategies. These strategies aim to design adaptable nonlinear system controllers capable of handling diverse situations, thereby ensuring the achievement of control objectives for uncertain nonlinear systems.

In fact, most control schemes are based on sampling control methods [12, 23, 30, 32], which operate at fixed and unchanging intervals. The time-triggered control approach

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performs sampling and data transmission at a conservative, fixed rate, regardless of whether the system actually requires it. While this method is relatively straightforward to implement, it can lead to inefficient use of communication resources and may fail to respond promptly to changes in system performance. With the increasing adoption of networked control systems, event-triggered control, an evolution of traditional sampling control, has garnered significant attention in recent years [1,24,25,31,34]. In contrast to time-triggered control, event-triggered control ensures robust system performance, while making more efficient use of computational and communication resources. Additionally, it offers enhanced feedback capabilities to address nonlinearities and compensate for uncertainties in the system.

Significant progress has recently been made in output-feedback stabilization of nonlinear systems [6,10,15–17,19–21,29]. In [19], the system has nonlinearities only present in the system output, while in [20], the system exhibits linearly dependent growth on unmeasured states. However, these systems allow only certain weak uncertainties rather than any large uncertainty. In [29], the system allows not only large parametric uncertainties but also general nonlinear functions that are not required to be globally Lipschitz. In [15, 16], the systems under investigation allow large parametric uncertainties and linearly dependent growth on unmeasured states. In [17], unknown polynomial-function rates are allowed in the growth dependent on unmeasurable states. In [21], the system growth rates can be arbitrary unknown nonpolynomial functions of the system output. In [10], the system has two types of significant uncertainties: growth dependent on unknown unmeasured states and input matching uncertainty. In [6], the nonlinear time-delay system is studied with input matching uncertainty, and its nonlinearities are bounded by linear unmeasured states multiplied by unknown constants, polynomial functions of output, and polynomial functions of input. However, the control schemes in [6, 10] are based on sampling control. The articles mentioned above are devoted to global stabilization via output feedback for uncertain nonlinear systems, but the control coefficients of the systems are known constants.

This article is devoted to establishing an adaptive event-triggered output-feedback scheme to achieve global stabilization for a class of nonlinear systems. We first introduce an adaptive observer with a dynamic gain compensating the extra nonlinearities from the function control coefficients and counteracting the polynomial-function rate. Then, inspired by the idea and method in [6,10,16], an adaptive output-feedback event-triggered controller is successfully designed to guarantee the global stabilization. Particularly, we design an event-triggering mechanism with time-varying threshold, which is strictly positive and gradually decaying. This mechanism is proposed to reduce communication frequency and avoid Zeno phenomenon by selecting appropriate event-triggered threshold parameter. Compared with the related literatures [16,17,21,24,25], the main contributions of this article are summarized in the following aspects.

(i) Within the event-triggered framework, the systems under investigation exhibit a high degree of generality. Specifically, the system simultaneously incorporates function control coefficients, input matching uncertainty, and unmeasured state-dependent growth with unknown dynamics.

(ii) In contrast to related studies [12, 14, 17], our approach employs an adaptive technique that introduces only a single dynamic gain (rather than dual gains), to both compensate for the additional nonlinearities induced by function control coefficients and counteract the polynomial-function growth rate. Utilizing this dynamic gain, we design an adaptive extended state observer with dual objectives: reconstructing the unmeasured states and estimating the input matching uncertainty.

The remainder of this paper is arranged as follows. Section 2 presents the detailed formulation of the system model and control objective. Section 3 establishes an adaptive output-feedback event-triggered stabilization scheme for uncertain nonlinear systems. The main results are provided in Section 4, which conducts the performance analysis of the closed-loop system. Section 5 presents two numerical examples, and Section 6 concludes the paper.

2 Problem formulation

We consider the global stabilization via event-triggered output feedback for the following uncertain nonlinear system with function control coefficients:

$$
\begin{aligned}
\dot{\xi}_i &= g_i(y)\xi_{i+1} + \phi_i(t,\xi), \quad i = 1,\dots,n-1,\\
\dot{\xi}_n &= g_n(y)(u+\nu) + \phi_n(t,\xi),\\
y &= \xi_1,
\end{aligned} \tag{1}
$$

where $\xi = [\xi_1,\dots,\xi_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the system state vector with the initial condition $\xi(0) = \xi_0$; $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the control input and system output, respectively; $\nu \in \mathbb{R}$ is an unknown constant, called the input matching uncertainty; $\phi_i(t,\xi)$, $i = 1,\dots,n$, are unknown functions that are continuous in the first argument and locally Lipschitz in the second argument; and $g_i(y)$, $i = 1,\dots,n$, are known and locally Lipschitz functions. Since $g_i(y)$, $i = 1,\dots,n$, are functions of the system output rather than constants, system (1) has control coefficients in a more general sense.

In the following, we assume that only the system output y is measurable. To achieve the desired research objectives, system (1) satisfies the following assumptions:

Assumption 1. Let $\underline{c}$, $\overline{c}$, $\underline{f}$, $\overline{f}$, and p be known positive constants such that for any $y \in \mathbb{R}$,

$$
\begin{aligned}
&0 < \underline{c} \leqslant \left|g_i(y)\right| \leqslant \overline{c}\left(1 + |y|^p\right), \quad i = 1,\dots,n,\\
&\underline{f} \leqslant \left|g_i(y)/g_{i-1}(y)\right| \leqslant \overline{f}, \quad i = 2,\dots,n.
\end{aligned}
$$

Assumption 2. There exist an unknown positive constant θ_s and a positive integer q such that for any $t \in \mathbb{R}^+$ and any $\xi \in \mathbb{R}^n$,

$$
\left|\phi_i(t,\xi)\right| \leqslant \theta_s\left(1 + |y|^q\right)\sum_{j=1}^{i}|\xi_j|, \quad i = 1,\dots,n.
$$

As indicated by Assumption 1, system (1) possesses function-based control coefficients, which is different from other relevant literature [15–17, 21], where the systems possess constant control coefficients. Assumption 1 indicates that not only should the control coefficients be bounded by the polynomial-of-output, but the control coefficients between different subsystems must also satisfy a multiple relationship. Assumption 2 states that the system nonlinearities exhibit growth dependent on unmeasurable states with a polynomial-of-output rate. More importantly, Assumption 2 shows that the system nonlinearities allow large uncertainties (reflected by the unknown θ_s) and are expressed as the product of unmeasured states, a constant, and the polynomial-of-output, which is significantly differing from those in [19, 20, 23, 32]. In [23, 32], the systems only have a known constant growth rate or a known polynomial growth rate. In [19], the system nonlinearities are only present in the system output, while in [20], the system exhibits linearly dependent growth on unmeasured states.

3 Adaptive event-triggered control scheme

Motivated by [6,10] and letting $\nu = \xi_{n+1}$, we design the following dynamic-gain observer for system (1) to reconstruct the unmeasurable states:

$$\begin{aligned} \dot{\hat{\xi}}_i &= g_i(y)\hat{\xi}_{i+1} + \gamma^i(t)h_i(y)(\xi_1 - \hat{\xi}_1), \quad i = 1, \dots, n-1, \\ \dot{\hat{\xi}}_n &= g_n(y)\hat{\xi}_{n+1} + g_n(y)u + \gamma^n(t)h_n(y)(\xi_1 - \hat{\xi}_1), \\ \dot{\hat{\xi}}_{n+1} &= h_{n+1}(y)\gamma^{n+1}(\xi_1 - \hat{\xi}_1). \end{aligned} \tag{2}$$

Here $\hat{\xi} = [\hat{\xi}_1, \dots, \hat{\xi}_n]^{\mathrm{T}}$ with the initial value $\hat{\xi}(0) = \hat{\xi}_0$, $h_i(y)$, $i = 1, \dots, n+1$, will be determined later, and γ is called the dynamic gain satisfying the following updating law:

$$\dot{\gamma} = \max\left\{\left(1 + \frac{|y|^p}{\gamma^{pb}}\right)\frac{(\xi_1 - \hat{\xi}_1)^2}{\gamma^{2b}}, -\beta_1\gamma^2 + \beta_2\big(1 + |y|^q\big)^2\gamma\right\}, \quad \gamma(0) = 1, \tag{3}$$

where p and q are the same as in Assumptions 1–2, and b satisfies $0 < b < 1/(2q)$. β_1 and β_2 are to-be-determined design parameters, which satisfy $0 < \beta_1 \leqslant \beta_2$. In addition, we define $\hat{\xi}_{[i]} = [\hat{\xi}_1, \dots, \hat{\xi}_i]^{\mathrm{T}}$, $i = 1, \dots, n$, for later use.

The event-triggered output-feedback controller is designed as

$$\begin{aligned} u(t) &= \omega(t_k), \quad t \in [t_k, t_{k+1}), \\ t_{k+1} &= \inf\big\{t > t_k \mid \big|g_n(y)\big(\omega(t) - \omega(t_k)\big)\big| \geqslant \mathrm{e}^{-\alpha t}\big\}, \quad t_1 = 0,\ \alpha > 0, \\ \omega(\gamma, \hat{\xi}, y) &= -\frac{1}{g_n(y)}\sum_{i=1}^{n}\gamma^{n-i+1}k_i(y)\hat{\xi}_i - \hat{\xi}_{n+1}, \end{aligned} \tag{4}$$

where $k_i(y)$, $i = 1, \dots, n$, will be determined later, and threshold parameter α satisfies $\alpha < \lambda/2 = \min\{1/(2\lambda_{\max}(P)), 1/(2\lambda_{\max}(Q))\}$.

Remark 1. There are two main approaches to handling the unknown term θ_s: one is the estimation-based method [8, 11], and the other is the high-gain-based method [10, 17]. A single dynamic gain γ is introduced through an adaptive technique to simultaneously compensate for the extra nonlinearities arising from the function control coefficients and counteract the unknown polynomial-of-output growth rate associated with the system nonlinearities. The updating law of γ involves two components:

(i) The component
$$\left(1 + \frac{|y|^p}{\gamma^{pb}}\right)\frac{(\xi_1 - \hat{\xi}_1)^2}{\gamma^{2b}}$$
is designed to compensate for the unknown constant θ_s and the extra nonlinearities from the function control coefficients $g_i(y)$, $i = 1, \dots, n$, which will be demonstrated in Proposition 1 and inequality (14) of Proposition 2, respectively.

(ii) The component
$$-\beta_1\gamma^2 + \beta_2\bigl(1 + |y|^q\bigr)^2\gamma$$
is designed to handle the polynomial-of-output term $(1 + |y|^q)$.

Remark 2. Compared to the dual-gain approach [17, 28], the single dynamic gain offers a more compact structure. Specifically, in the subsequent proof of Proposition 2, only the boundedness of this single gain needs to be demonstrated, significantly enhancing the conciseness and readability of the proof process. Although we only adopt a single dynamic gain, it effectively serves the purpose of a dual-gain structure. On one hand, it can compensate the extra nonlinearities from the function control coefficients; on the other hand, it can counteract the polynomial-function rate.

Remark 3. $\omega(t)$ denotes $\omega(\gamma(t), \hat{\xi}(t), y(t))$ for convenience. Event-triggering mechanisms are generally classified by threshold into four types: absolute (fixed) [2, 9], (static) relative, dynamic, and time-varying [3, 16]. Sampling/execution times t_k are generated by the second relation in controller (4), which is called time-varying event-triggering mechanism. The threshold $|g_n(y)(\omega(t) - \omega(t_k))| \geqslant \mathrm{e}^{-\alpha t}$ in controller (4), achieved by choosing the function $\mathrm{e}^{-\alpha t}$, offers two advantages:

(i) for any time t, it is strictly positive;
(ii) it decays to zero as time goes on.

Building upon this time-varying event-triggering mechanism, we design an adaptive output-feedback controller (4). By carefully selecting design parameters, this controller ensures the convergence of both the original system states and their corresponding observer states, effectively preventing the Zeno phenomenon. Additionally, by incorporating $\hat{\xi}_{n+1}$ into the designed controller, it guarantees that the estimation of the input matching uncertainty accurately converges to its true value.

Remark 4. Under the architecture depicted in Fig. 1, we design an adaptive event-triggering output-feedback controller based on a dynamic-gain observer, which guarantees that the system states globally converge to the origin. As shown in Fig. 1, the information

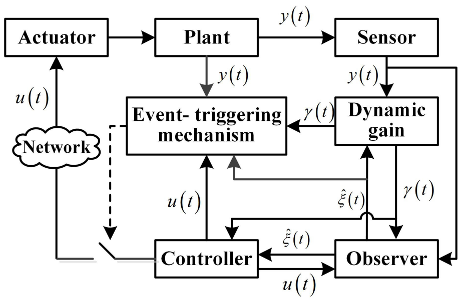

Figure 1. Event-triggered control architecture.

flow from the controller to the actuator is determined by the event-triggering mechanism. We design a triggering condition derived from the estimated execution error, enabling the system to persistently track state variables. Consequently, relevant data is dispatched to the actuator. Thus, the control input u is updated to $\omega(\gamma(t_k), \hat{\xi}(t_k))$ only at each sampling instant t_k and remains constant between two consecutive sampling instants.

We next introduce the following scaling transformation:

$$\begin{aligned} \eta_i &= \frac{\xi_i - \hat{\xi}_i}{\gamma^{b+i-1}}, \quad i = 1, \dots, n+1, \\ \epsilon_i &= \frac{\hat{\xi}_i}{\gamma^{b+i-1}}, \quad i = 1, \dots, n. \end{aligned} \tag{5}$$

Then, by (1), (2), and (4), we have

$$\begin{aligned} D^+\epsilon &= \gamma(t)B(y)\epsilon + \gamma(t)H_n(y)\eta_1 - \frac{\dot{\gamma}(t)}{\gamma(t)}D_n\epsilon + \frac{Cg_n(y)(u-\omega)}{\gamma^{b+n-1}}, \\ D^+\eta &= \gamma(t)A(y)\eta + \varPhi(t,\xi,\gamma) - \frac{\dot{\gamma}(t)}{\gamma(t)}D_{n+1}\eta, \end{aligned} \tag{6}$$

where $\eta = [\eta_1, \dots, \eta_{n+1}]^{\mathrm{T}}$, $\epsilon = [\epsilon_1, \dots, \epsilon_n]^{\mathrm{T}}$, $\varPhi(t,\xi,\gamma) = [\phi_1(t,\xi)/\gamma^b, \dots, \phi_n(t,\xi)/\gamma^{n-1+b}, 0]^{\mathrm{T}}$, $k(y) = [k_1(y), \dots, k_n(y)]^{\mathrm{T}}$, $C = [0, \dots, 0, 1]^{\mathrm{T}} \in \mathbb{R}^n$, $H_i(y) = [h_1(y), \dots, h_i(y)]^{\mathrm{T}}$, $i = n,\ n+1$, $D_n = \mathrm{diag}\{b,\ 1+b,\ \dots,\ n-1+b\}$, $D_{n+1} = \mathrm{diag}\{b,\ 1+b, \dots, n+b\}$, and

$$A(y) = \begin{bmatrix} -h_1(y) & g_1(y) & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ -h_n(y) & 0 & \cdots & g_n(y) \\ -h_{n+1}(y) & 0 & \cdots & 0 \end{bmatrix},$$

$$B(y) = \begin{bmatrix} 0 & g_1(y) & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & g_{n-1}(y) \\ -k_1(y) & -k_2(y) & \cdots & -k_n(y) \end{bmatrix}.$$

Table 1. Common symbols of upper and lower bounds.

Category	Mathematical expression
Function control coefficients	$0 < c \leqslant \lvert g_i(y)\rvert \leqslant \bar{c}(1+\lvert y\rvert^p),\ i=1,\dots,n$ $\lvert g_1(y)\rvert \leqslant \tilde{c}\gamma^{pb}(1+\lvert\eta_1+\epsilon_1\rvert^p)$
Matrix inequalities	$PA(y)+A^{\mathrm{T}}(y)P \leqslant -\nu_o\lvert g_1(y)\rvert I_{n+1}$ $QB(y)+B^{\mathrm{T}}(y)Q \leqslant -\nu_c\lvert g_1(y)\rvert I_n$ $D_{n+1}P+PD_{n+1} \geqslant \underline{\nu}_o I_{n+1}$ $QD_n+D_nQ \geqslant \underline{\nu}_c I_n$
Nonlinearity bounds	$\lvert\phi_i(t,\xi)\rvert \leqslant \theta_s(1+\lvert y\rvert^q)\sum_{j=1}^{i}\lvert\xi_j\rvert,\ i=1,\dots,n$ $\lVert H_i(y)\rVert = \sqrt{\sum_{j=1}^{i} h_j^2(y)} \leqslant \bar{h}\lvert g_1(y)\rvert,\ i=n,n+1$

Based on Assumption 1 and [13, Remark 4], we choose $h_i(y)$, $i=1,\dots,n+1$, and $k_i(y)$, $i=1,\dots,n$, which are locally Lipschitz functions and linear combinations with constant coefficients of $g_i(y)$, $i=1,\dots,n$. There exist known symmetric positive definite matrices P and Q and positive constants ν_o, ν_c, $\underline{\nu}_o$, $\underline{\nu}_c$ such that for all $y\in\mathbb{R}$,

$$\begin{aligned} D_{n+1}P+PD_{n+1} &\geqslant \underline{\nu}_o I_{n+1}, \qquad & PA(y)+A^{\mathrm{T}}(y)P &\leqslant -\nu_o\bigl|g_1(y)\bigr|I_{n+1}, \\ QD_n+D_nQ &\geqslant \underline{\nu}_c I_n, \qquad & QB(y)+B^{\mathrm{T}}(y)Q &\leqslant -\nu_c\bigl|g_1(y)\bigr|I_n. \end{aligned} \tag{7}$$

Moreover, by Assumption 1, there exists a known positive constant $\bar{h}$, and we have

$$\lVert H_i(y)\rVert = \sqrt{\sum_{j=1}^{i} h_j^2(y)} \leqslant \bar{h}\bigl|g_1(y)\bigr|, \quad i=n,\ n+1. \tag{8}$$

In order to enhance the readability of the paper, we have organized the above formulas and presented the upper and lower bounds of commonly used symbols in subsequent proofs in Table 1.

4 Performance analysis of event-triggered control

This section presents the stability analysis of closed-loop system (1)-(3) under the event-triggered controller (4), and the main results are summarized later.

Noting the local Lipschtizness of $\phi_i(t,\xi)$'s, the right-hand side of resulting closed-loop system (1)–(4) is locally Lipschitz in $(\xi,\hat{\xi},\gamma)$. By [7, p. 18, Thm. 3.1], for any initial value $(\xi(0),\hat{\xi}(0),\gamma(0))$, the entire system with constant-valued $u(t)\equiv u(t_1)$ has a unique solution on a small existence interval $[0,T_{m_1})$, where $t_1=0$ is the first execution time. By [7, p. 17, Thm. 2.1], interval $[0,T_{m_1})$ can be right maximized to $[0,T_{m_2})$. If the next execution time arises at $t_2\in[0,T_{m_2})$, we regard t_2 as the new initial time, and the controller is switched to another constant-valued $u(t_2)$. Then we can establish the existence and uniqueness of solution $(\xi(t),\hat{\xi}(t),\gamma(t))$ with initial value $(\xi(t_2),\hat{\xi}(t_2),\gamma(t_2))$ and $u(t)\equiv u(t_2)$. This process would be iterated in a consistent manner, provided that a larger execution time exists after each execution. Hence, by

[7, p. 17, Thm. 2.1], the unique solution can be extended to maximal interval $[0, T_m)$ and $0 < T_m \leqslant +\infty$.

Next, we give Proposition 1 to characterize the dynamic behavior of the closed-loop system via Lyapunov functions. This proposition is very important for guaranteeing the boundedness of γ, as will be shown in the proof of Proposition 2.

Proposition 1. *Define* $V(\epsilon) = \epsilon^{\mathrm{T}} Q \epsilon$ *and* $V(\eta) = \eta^{\mathrm{T}} P \eta$, *where* P *and* Q *have been defined in* (7). *Note that* $V(\eta, \epsilon) = \mu V(\eta) + V(\epsilon)$ *with* $\mu = 8\bar{h}^2 \|Q\|^2/(\nu_o \nu_c) + 1$. *Along the trajectories of* (6), *there holds*

$$D^+ V(\eta, \epsilon) \leqslant -\beta(\gamma - \Theta)\big(\mu\|\eta\|^2 + \|\epsilon\|^2\big) + \frac{4\|QC\|^2}{\nu_c \underline{c} \gamma^{2b+2n-1} \mathrm{e}^{2\alpha t}}, \tag{9}$$

where $D^+ V(\eta, \epsilon) = \limsup_{h \to 0^+} (V(t+h) - V(t))/h$ *denotes the upper right-hand derivative of* $V(\eta, \epsilon)$, $\beta = \underline{c} \min\{\nu_o, \nu_c\}/4$, *and* $\Theta = 2\mu n^3 \theta^2 \|P\|^2 \max\{1/(\nu_o \underline{c}), 1/(\nu_c \underline{c})\}$.

Proof. From (6) and (7) there holds

$$\begin{aligned} &D^+ V(\eta, \epsilon) \\ &\quad = \mu\bigg(\gamma\big(\eta^{\mathrm{T}} A^{\mathrm{T}}(y) P \eta + \eta^{\mathrm{T}} P A(y) \eta\big) + 2\eta^{\mathrm{T}} P \Phi - \frac{\dot{\gamma}}{\gamma} \eta^{\mathrm{T}} (D_{n+1} P + P D_{n+1}) \eta\bigg) \\ &\qquad + \gamma\big(\epsilon^{\mathrm{T}} B^{\mathrm{T}}(y) Q \epsilon + \epsilon^{\mathrm{T}} Q B(y) \epsilon\big) + 2\gamma\eta_1 \epsilon^{\mathrm{T}} Q H_n(y) \\ &\qquad - \frac{\dot{\gamma}}{\gamma} \epsilon^{\mathrm{T}} (D_n Q + Q D_n) \epsilon + \frac{2\epsilon^{\mathrm{T}} Q C g_n(y)(u - \omega)}{\gamma^{b+n-1}} \\ &\quad \leqslant -\mu\nu_o \gamma \big|g_1(y)\big| \cdot \|\eta\|^2 + \mu \underline{\nu}_o \beta_1 \gamma \|\eta\|^2 - \mu \underline{\nu}_o \beta_2 \big(1 + |y|^q\big)^2 \|\eta\|^2 \\ &\qquad - \nu_c \gamma \big|g_1(y)\big| \cdot \|\epsilon\|^2 + \underline{\nu}_c \beta_1 \gamma \|\epsilon\|^2 - \underline{\nu}_c \beta_2 \big(1 + |y|^q\big)^2 \|\epsilon\|^2 \\ &\qquad + 2\mu \eta^{\mathrm{T}} P \Phi + 2\gamma \eta_1 \epsilon^{\mathrm{T}} Q H_n(y) + \frac{2 g_n(y)(u - \omega)}{\gamma^{b+n-1}} \epsilon^{\mathrm{T}} Q C. \end{aligned} \tag{10}$$

By (8) and the method of completing squares, the last three terms of (10) satisfy

$$\begin{aligned} &2\mu \eta^{\mathrm{T}} P \Phi \leqslant 2\mu^2 n^3 \theta^2 \|P\|^2 \|\eta\|^2 + \big(1 + |y|^q\big)^2 \big(\|\eta\|^2 + \|\epsilon\|^2\big), \\ &2\gamma \eta_1 \epsilon^{\mathrm{T}} Q H_n(y) \leqslant \frac{\nu_c}{4} \gamma \big|g_1(y)\big| \cdot \|\epsilon\|^2 + \frac{4\bar{h}^2 \|Q\|^2}{\nu_c} \gamma \big|g_1(y)\big| \cdot \|\eta\|^2, \\ &\frac{2 g_n(y)(u - \omega)}{\gamma^{b+n-1}} \epsilon^{\mathrm{T}} Q C \leqslant \frac{\nu_c}{4} \gamma \big|g_1(y)\big| \cdot \|\epsilon\|^2 + \frac{4\|QC\|^2 |g_n(y)(u - \omega)|^2}{\nu_c |g_1(y)| \gamma^{2b+2n-1}}. \end{aligned} \tag{11}$$

Next, we choose parameters β_1 and β_2 that satisfy

$$\begin{cases} \beta_1 \leqslant \min\{\frac{\underline{c}\nu_c}{4\underline{\nu}_c}, \frac{\underline{c}\nu_o}{4\underline{\nu}_o}\}, \\ \beta_2 \geqslant \max\{\frac{1}{\underline{\nu}_c}, \frac{1}{\underline{\nu}_o}\}. \end{cases} \tag{12}$$

By Assumption 1 and controller (4), noting that $V(\eta,\epsilon) = \mu V(\eta) + V(\epsilon)$ with $\mu = 8\bar{h}^2\|Q\|^2/(\nu_o\nu_c) + 1$ and substituting (11), (12) into (10), we have that

$$\begin{aligned}
&D^+V(\eta,\epsilon)\\
&\quad\leqslant -\frac{\nu_o\underline{c}}{4}\gamma\mu\|\eta\|^2 - \frac{\nu_c\underline{c}}{4}\gamma\|\epsilon\|^2 + 2\mu^2n^3\theta^2\|P\|^2\|\eta\|^2 + \frac{4\|QC\|^2}{\nu_c\underline{c}\gamma^{2b+2n-1}\mathrm{e}^{2\alpha t}}\\
&\quad\leqslant \frac{\underline{c}\min\{\nu_o,\nu_c\}}{4}\gamma\big(\mu\|\eta\|^2+\|\epsilon\|^2\big) + 2\mu n^3\theta^2\|P\|^2\big(\mu\|\eta\|^2+\|\epsilon\|^2\big) + \frac{4\|QC\|^2d^2}{\nu_c\underline{c}\gamma^{2b+2n-1}}\\
&\quad\leqslant -\beta(\gamma-\Theta)\big(\mu\|\eta\|^2+\|\epsilon\|^2\big) + \frac{4\|QC\|^2}{\nu_c\underline{c}\gamma^{2b+2n-1}\mathrm{e}^{2\alpha t}}
\end{aligned}$$

with $\beta = \underline{c}\min\{\nu_o,\nu_c\}/4$, $\Theta = 2\mu n^3\theta^2\|P\|^2\max\{1/(\nu_o\underline{c}), 1/(\nu_c\underline{c})\}$.

The proof of Proposition 1 is completed. □

The following Propositions 2–3 present the boundedness of the closed-loop system states and exclude Zeno phenomenon.

Proposition 2. *For the closed-loop system consisting of* (1)–(4)*, states* $\epsilon(t)$*,* $\eta(t)$ *and dynamic gain* $\gamma(t)$ *are bounded on* $[0,T_m)$*, and moreover,*

$$\lim_{t\to\infty}\int_0^t \big\|\epsilon(s)\big\|^2\,\mathrm{d}s < +\infty, \qquad \lim_{t\to\infty}\int_0^t \big\|\eta(s)\big\|^2\,\mathrm{d}s < +\infty.$$

Proof. For contradiction, let $\gamma(t)$ be unbounded on $[0,T_m)$. Then, noting $\gamma(t)\geqslant 1$, we have $\lim_{t\to T_m}\gamma(t) = +\infty$, and hence there exists a finite time $t_1\in[0,T_m)$ such that

$$\gamma(t)\geqslant \Theta + 1 \quad \forall t\in[t_1,T_m).$$

Together with (9), this yields that

$$\begin{aligned}
D^+V(\eta,\epsilon) &\leqslant -\beta\big(\mu\|\eta\|^2+\|\epsilon\|^2\big) + \frac{4\|QC\|^2}{\nu_c\underline{c}\gamma^{2b+2n-1}\mathrm{e}^{2\alpha t}}\\
&\leqslant -\frac{1}{\lambda_{\max}(P)}\mu\eta^{\mathrm{T}}P\eta - \frac{1}{\lambda_{\max}(Q)}\epsilon^{\mathrm{T}}Q\epsilon + \frac{4\|QC\|^2}{\nu_c\underline{c}\gamma^{2b+2n-1}\mathrm{e}^{2\alpha t}}\\
&\leqslant -\lambda V + \frac{4\|QC\|^2}{\nu_c\underline{c}\mathrm{e}^{2\alpha t}},
\end{aligned}\tag{13}$$

where $\lambda = \min\{1/\lambda_{\max}(P), 1/\lambda_{\max}(Q)\}$.

From (13) it can be seen that

$$D^+V(\eta,\epsilon)\leqslant -\lambda V + \frac{4\|QC\|^2}{v_c\underline{c}\mathrm{e}^{2\alpha t}} \leqslant \frac{4\|QC\|^2}{v_c\underline{c}}.$$

Thus, we obtain

$$D^{+}\left(\mathrm{e}^{\lambda t}V\right) \leqslant \frac{4\|QC\|^2\mathrm{e}^{\lambda t}}{v_c\underline{c}}.$$

By integrating over $[T_1, t)$, it follows that

$$\mathrm{e}^{\lambda t}V(t) - \mathrm{e}^{\lambda T_1}V(T_1) \leqslant \frac{4\|QC\|^2}{v_c\underline{c}}\int_{T_1}^{t}\mathrm{e}^{\lambda\tau}\,\mathrm{d}\tau \leqslant \frac{4\|QC\|^2\mathrm{e}^{\lambda t}}{v_c\underline{c}\lambda}.$$

Thus,

$$\mathrm{e}^{\lambda t}V(t) \leqslant \mathrm{e}^{\lambda T_1}V(T_1) + \frac{4\|QC\|^2\mathrm{e}^{\lambda t}}{v_c\underline{c}\lambda}.$$

So we have

$$V(t) \leqslant \mathrm{e}^{\lambda T_1}V(T_1) + \frac{4\|QC\|^2}{v_c\underline{c}\lambda},$$

which implies that $V(\eta, \epsilon)$ is bounded on $[t_1, T_m)$.

From this it is easy to show that η_1 and ϵ_1 are bounded on $[t_1, T_m)$. Since $0 < b < 1/(2q)$, there exists a finite time $t_2 \in [t_1, T_m)$ such that

$$\begin{aligned}
&-\beta_1\gamma^2 + \beta_2\gamma\left(1 + |y|^q\right)^2 \\
&\quad \leqslant -\beta_1\gamma^2 + \beta_2\gamma^{1+2bq}\left(1 + |\eta_1 + \epsilon_1|^q\right)^2 < 0 \quad \forall t \in [t_2, T_m).
\end{aligned}$$

Hence, by dynamic gain updating law (3), we have

$$\dot{\gamma} = \left(1 + |\eta_1 + \epsilon_1|^p\right)\eta_1^2 \quad \forall t \in [t_2, T_m),$$

and there exists a positive constant N such that $0 < \sup_{t\to T_m}(1 + |\eta_1 + \epsilon_1|^p) < N$.

However, integrating both side of (13) over $[t_2, t] \subset [t_2, T_m)$ and noting that $\alpha < \lambda/2 = \min\{1/(2\lambda_{\max}(P)), 1/(2\lambda_{\max}(Q))\}$, we obtain

$$\begin{aligned}
+\infty &= \lim_{t\to T_m}\gamma(t) - \gamma(t_2) = \lim_{t\to T_m}\int_{t_2}^{t}\dot{\gamma}(s)\,\mathrm{d}s \leqslant \lim_{t\to T_m}\frac{N}{\lambda_{\min}(P)}\int_{t_2}^{t}V(s)\,\mathrm{d}s \\
&\leqslant \lim_{t\to T_m}\frac{N}{\lambda_{\min}(P)}\int_{t_2}^{t}\left(V(t_2)\mathrm{e}^{-\lambda(t-t_2)} + \frac{4\|QC\|^2}{\nu_c\underline{c}}\int_{t_2}^{t}\mathrm{e}^{-\lambda t}\mathrm{e}^{(\lambda-2\alpha)s}\,\mathrm{d}s\right)\mathrm{d}t \\
&\leqslant \frac{N}{\lambda_{\min}(P)}\left(\frac{\lambda}{V(t_1)} + \frac{4\|QC\|^2}{\nu_c\underline{c}(\lambda\alpha - 2\alpha^2)}\mathrm{e}^{-2\alpha t_2}\right) \\
&< +\infty,
\end{aligned}$$

which is a contradiction for some $t_2 \in [0, T_m)$. Therefore, $\gamma(t)$ is bounded on $[0, T_m)$.

Let $\gamma_m = \sup_{t\in[0,T_m)} \gamma(t)$. We next show that ϵ is bounded on $[0, T_m)$. Consider the function $V(\epsilon) = \epsilon^{\mathrm{T}} Q \epsilon$. From (7) and (11), noting that $\gamma \geqslant 1$, it clearly follows that

$$D^+V(\epsilon) \leqslant -\frac{\nu_c \underline{c}}{4}\gamma\|\epsilon\|^2 + \frac{4\bar{h}^2\|Q\|^2}{\nu_c}\gamma\bigl|g_1(y)\bigr|\eta_1^2 + \frac{4\|QC\|^2}{\nu_c \underline{c}\gamma^{2b+2n-1}\mathrm{e}^{2\alpha t}}.$$

In addition, by Assumption 1, (5), and $\gamma \geqslant 1$, we know that there exists an unknown positive constant $\tilde{c}$ such that

$$\begin{aligned}\bigl|g_1(y)\bigr| &\leqslant \bar{c}\bigl(1 + |y|^p\bigr) = \bar{c}\bigl(1 + \bigl|\gamma^b(\eta_1 + \epsilon_1)\bigr|^p\bigr)\\ &\leqslant \bar{c}\max\bigl\{1, 2^{p-1}\bigr\}\bigl(1 + \gamma^{pb}|\eta_1 + \epsilon_1|^p\bigr)\\ &\leqslant \bar{c}\max\bigl\{1, 2^{p-1}\bigr\}\gamma^{pb}\bigl(1 + |\eta_1 + \epsilon_1|^p\bigr)\\ &\leqslant \tilde{c}\gamma^{pb}\bigl(1 + |\eta_1 + \epsilon_1|^p\bigr).\end{aligned} \tag{14}$$

Then using the definition of $\dot{\gamma}$ and the fact that $\gamma \geqslant 1$, we have

$$\begin{aligned}D^+V(\epsilon) &\leqslant -\frac{\nu_c \underline{c}}{4}\gamma\|\epsilon\|^2 + \frac{4\bar{h}^2\|Q\|^2}{\nu_c}\gamma_m^{pb+1}\dot{\gamma} + \frac{4\|QC\|^2}{\nu_c \underline{c}\gamma^{2b+2n-1}\mathrm{e}^{2\alpha t}}\\ &\leqslant -\frac{\nu_c \underline{c}}{4\lambda_{\max}(Q)}V(\epsilon) + \frac{4\bar{h}^2\|Q\|^2\tilde{c}}{\nu_c}\gamma_m^{pb+1}\dot{\gamma} + \frac{4\|QC\|^2}{\nu_c \underline{c}\gamma^{2b+2n-1}\mathrm{e}^{2\alpha t}}\\ &\leqslant -l_1 V(\epsilon) + l_2\dot{\gamma} + \frac{l_3}{\mathrm{e}^{2\alpha t}},\end{aligned} \tag{15}$$

where

$$l_1 = \frac{\nu_c \underline{c}}{4\lambda_{\max}(Q)}, \qquad l_2 = \frac{4\bar{h}^2\|Q\|^2\tilde{c}}{\nu_c}\gamma_m^{pb+1}, \qquad l_3 = \frac{4\|QC\|^2}{\nu_c \underline{c}\gamma^{2b+2n-1}}$$

are positive constants. From this it is easy to see that on $[0, T_m)$,

$$D^+\bigl(\mathrm{e}^{l_1 t}V(\epsilon)\bigr) \leqslant l_2\mathrm{e}^{l_1 t}\dot{\gamma} + l_3\mathrm{e}^{l_1 t}.$$

Hence, for any $t \in [0, T_m)$, we have

$$\begin{aligned}V\bigl(\epsilon(t)\bigr) &\leqslant \frac{V(\epsilon(0))}{\mathrm{e}^{l_1 t}} + l_2\int_0^t \mathrm{e}^{l_1(s-t)}\,\mathrm{d}\gamma(s) + l_3\int_0^t \mathrm{e}^{l_1(s-t)}\,\mathrm{d}s\\ &\leqslant V\bigl(\epsilon(0)\bigr) + l_2\gamma_m + \frac{l_3}{l_1},\end{aligned}$$

which shows that ϵ is bounded on $[0, T_m)$.

Moreover, integrating both sides of (15) over $[0, t] \subset [0, T_m)$ directly yields

$$V\bigl(\epsilon(t)\bigr) - V\bigl(\epsilon(0)\bigr) \leqslant -l_1\int_0^t V\bigl(\epsilon(s)\bigr)\,\mathrm{d}s + l_2\int_0^t \dot{\gamma}(s)\,\mathrm{d}s + l_3\int_0^t \frac{1}{\mathrm{e}^{2\alpha s}}\,\mathrm{d}s,$$

which implies that

$$l_1 \int_0^t V\big(\epsilon(s)\big)\,\mathrm{d}s \leqslant V\big(\epsilon(0)\big) + l_2\gamma_m + \frac{l_3}{2\alpha} < +\infty.$$

From this and noting $V(\epsilon) = \epsilon^{\mathrm{T}} Q\epsilon$, hence $\lim_{t\to T_m} \int_0^t \|\epsilon(s)\|^2\,\mathrm{d}s < +\infty$.

We next prove that η is bounded on $[0, T_m)$ as well. We introduce the following scaling transformation:

$$\zeta_i = \frac{\eta_i}{(\gamma^*)^{b+i-1}}, \quad i = 1, 2, \dots, n+1,$$

where γ^* is a positive constant satisfying $\gamma^* = \max\{8n^3\theta^2\|p\|^2/(\nu_o\underline{c} + 1, \gamma_m)\}$. Then, by (6), we derive that

$$\begin{aligned} D^+\zeta &= \gamma^*\gamma A(y)\zeta + \gamma^*\gamma H_{n+1}(y)\zeta_1 - r\varGamma H_{n+1}(y)\zeta_1 \\ &\quad - \frac{\dot\gamma}{\gamma} D_{n+1}\zeta + \varPsi(t, \xi, \gamma^*, \gamma), \end{aligned} \tag{16}$$

where $\zeta = [\zeta_1, \dots, \zeta_{n+1}]^{\mathrm{T}}$, $\varGamma = \operatorname{diag}\{1, 1/\gamma^*, \dots, 1/(\gamma^*)^n\}$, $H_{n+1}(y)$, $A(y)$, D_{n+1} are the same as before, and

$$\varPsi(t, \xi, \gamma^*, \gamma) = \left[\frac{\phi_1(t,\xi)}{(\gamma^*\gamma)^b}, \dots, \frac{\phi_n(t,\xi)}{(\gamma^*\gamma)^{b+n-1}}, 0\right]^{\mathrm{T}}.$$

Define $V_3(\zeta) = \zeta^{\mathrm{T}} P\zeta$. By (3), (7), and (16), we have

$$\begin{aligned} D^+V(\zeta) &\leqslant -\nu_o\gamma^*\gamma\big|g_1(y)\big| \cdot \|\zeta\|^2 + 2\gamma^*\gamma\xi^{\mathrm{T}} PH_{n+1}(y)\zeta_1 - 2\gamma\zeta^{\mathrm{T}} P\varGamma H_{n+1}(y)\zeta_1 \\ &\quad + \beta_1\underline{\nu}_o\gamma\|\zeta\|^2 - \beta_2\underline{\nu}_o\big(1 + |y|^q\big)^2\|\zeta\|^2 + 2\zeta^{\mathrm{T}} P\varPsi. \end{aligned} \tag{17}$$

Then, by the method of completing square, we obtain

$$\begin{aligned} 2\gamma^*\gamma\zeta^{\mathrm{T}} PH_{n+1}(y)\zeta_1 &\leqslant \frac{\nu_o}{4}\gamma^*\gamma\big|g_1(y)\big| \cdot \|\zeta\|^2 + \frac{4\bar h^2\|P\|^2(\gamma^*)^{1-2b}}{\nu_o}\big|g_1(y)\big|\gamma\eta_1^2, \\ -2\gamma\zeta^{\mathrm{T}} P\varGamma H_{n+1}(y)\zeta_1 &\leqslant \frac{\nu_o}{4}\gamma^*\gamma\big|g_1(y)\big| \cdot \|\zeta\|^2 + \frac{4\bar h^2\|P\|^2\|\varGamma\|^2}{\nu_o(\gamma^*)^{1+2b}}\big|g_1(y)\big|\gamma\eta_1^2, \\ 2\zeta^{\mathrm{T}} P\varPsi &\leqslant 2n^3\theta^2\|P\|^2\big(\|\zeta\|^2 + \|\epsilon\|^2\big) + \big(1 + |y|^q\big)^2\|\zeta\|^2. \end{aligned} \tag{18}$$

Substituting (12) and (18) into (17), then using Assumption 1, (3), and the definition of γ^*, we obtain

$$\begin{aligned} D^+V(\zeta) &\leqslant \frac{\nu_o\underline{c}}{4}\gamma^*\gamma\|\zeta\|^2 + 2n^3\theta^2\|P\|^2(\|\zeta\|^2 + \|\epsilon\|^2) \\ &\quad + \left(\frac{4\bar h^2\|P\|^2(\gamma^*)^{1-2b}}{\nu_o} + \frac{4\bar h^2\|P\|^2\|\varGamma\|^2}{\nu_o(\gamma^*)^{1+2b}}\right)|g_1(y)|\gamma\eta_1^2 \end{aligned}$$

$$
\begin{aligned}
&\leqslant -\left(\frac{\nu_o\underline{c}}{4}\gamma^* - 2n^3\theta^2\|P\|^2\right)\gamma\|\zeta\|^2 + 2n^3\theta^2\|P\|^2\|\epsilon\|^2 \\
&\quad + \left(\frac{4\bar{h}^2\|P\|^2(\gamma^*)^{1-2b}}{\nu_o} + \frac{4\bar{h}^2\|P\|^2\|\Gamma\|^2}{\nu_o(\gamma^*)^{1+2b}}\right)\tilde{c}\gamma^{1+pb}\dot{\gamma} \\
&\leqslant -\frac{\nu_o\underline{c}}{4\lambda_{\max}(P)}V(\zeta) + 2n^3\theta^2\|P\|^2\|\epsilon\|^2 \\
&\quad + \left(\frac{4\bar{h}^2\|P\|^2(\gamma^*)^{1-2b}}{\nu_o} + \frac{4\bar{h}^2\|P\|^2\|\Gamma\|^2}{\nu_o(\gamma^*)^{1+2b}}\right)\tilde{c}\gamma_m^{1+pb}\dot{\gamma} \\
&= -c_1V(\zeta) + c_2\dot{\gamma} + c_3\|\epsilon\|^2,
\end{aligned} \tag{19}
$$

where

$$
\begin{aligned}
c_1 &= \frac{\nu_o\underline{c}}{4\lambda_{\max}(P)}, \\
c_2 &= \left(\frac{4\bar{h}^2\|P\|^2(\gamma^*)^{1-2b}}{\nu_o} + \frac{4\bar{h}^2\|P\|^2\|\Gamma\|^2}{\nu_o(\gamma^*)^{1+2b}}\right)\tilde{c}\gamma_m^{1+pb},
\end{aligned}
$$

and

$$c_3 = 2n^3\theta^2\|P\|^2.$$

From this it follows that

$$D^+\left(\mathrm{e}^{c_1t}V(\zeta)\right) \leqslant c_2\dot{\gamma}\mathrm{e}^{c_1t} + c_3\|\epsilon\|^2\mathrm{e}^{c_1t}.$$

Integrating both side of the above inequality over $[0,t] \subset [0,T_m)$, we obtain

$$
\begin{aligned}
V\left(\zeta(t)\right) &\leqslant \frac{V(\zeta(0))}{\mathrm{e}^{c_1t}} + c_2\int_0^t \mathrm{e}^{c_1(s-t)}\,\mathrm{d}\gamma(s) + c_3\|\epsilon\|^2\int_0^t \mathrm{e}^{c_1(s-t)}\,\mathrm{d}s \\
&\leqslant V\left(\zeta(0)\right) + c_2\gamma_m + \frac{c_3\|\epsilon\|^2}{c_1},
\end{aligned}
$$

which shows the boundedeness of ϵ. It is deduced that ζ is bounded on $[0,T_m)$. Recalling that $\zeta_i = \eta_i/(\gamma^*)^{b+i-1}$, we can easily show that η is bounded on $[0,T_m)$. Moreover, (19) implies that for any $t \in [0,T_m)$,

$$c_1\int_0^t V\left(\zeta(s)\right)\mathrm{d}s \leqslant V\left(\zeta(0)\right) + c_2\int_0^t \dot{\gamma}(s)\,\mathrm{d}s + c_3\int_0^t \left\|\epsilon(s)\right\|^2\mathrm{d}s.$$

Hence, together with $V(\zeta) = \zeta^{\mathrm{T}}Q\zeta$, we can know $\lim_{t\to T_m}\int_0^t \|\zeta(s)\|^2\,\mathrm{d}s < +\infty$. Noting that $\zeta_i = \eta_i/(\gamma^*)^{b+i-1}$, we can conclude $\lim_{t\to T_m}\int_0^t \|\eta(s)\|^2\,\mathrm{d}s < +\infty$.

The proof of Proposition 2 is thus completed. □

Proposition 3. *If all the signals of the resulting closed-loop system are bounded on* $[0,T_m)$*, then* $T_m = +\infty$*, and Zeno phenomenon does not occur.*

Proof. From Assumption 1 and the event-triggered controller (4), for any $t \in [t_k, t_{k+1})$, we have

$$\left|\underline{c}\big(\omega(t) - \omega(t_k)\big)\right| \leqslant \left|g_n(y)\right|\left|\omega(t) - \omega(t_k)\right|.$$

Suppose for contradiction that $T_m < +\infty$. Then, by the hypothesis of Proposition 3, we see that Zeno phenomenon indeed occurs, and moreover, $\lim_{k\to+\infty} t_k = T_m$.

By event-triggered controller (4), we know that

$$\left|\underline{c}\big(u(t) - \omega(t)\big)\right| \leqslant \left|g_n(y)\right|\left|u(t) - \omega(t)\right| \leqslant \mathrm{e}^{-\alpha t}.$$

There exists a positive constant M such that on each $[t_k, t_{k+1})$,

$$\begin{aligned}
&\left|\frac{\mathrm{d}}{\mathrm{d}t}\Big(\underline{c}^2\big(u(t) - \omega(t)\big)^2 \mathrm{e}^{2\alpha t}\Big)\right| \leqslant M\mathrm{e}^{\alpha t},\\
&\lim_{t\to t_k} \underline{c}^2\big(u(t) - \omega(t)\big)^2 = \mathrm{e}^{-2\alpha t_k},\\
&\underline{c}^2\big(u(t_k) - \omega(t_k)\big)^2 = 0.
\end{aligned} \tag{20}$$

Then, by (20), we have

$$\begin{aligned}
1 &= \lim_{t\to t_{k+1}} \underline{c}^2\big(u(t) - \omega(t)\big)^2 \mathrm{e}^{2\alpha t} - \underline{c}^2\big(u(t_k) - \omega(t_k)\big)^2 \mathrm{e}^{2\alpha t_k}\\
&= \lim_{t\to t_{k+1}} \int_{t_k}^{t} \frac{\mathrm{d}}{\mathrm{d}t}\Big(\underline{c}^2\big(u(s) - \omega(s)\big)^2 \mathrm{e}^{2\alpha s}\Big)\,\mathrm{d}s\\
&\leqslant \frac{M}{\alpha}\big(\mathrm{e}^{\alpha t_{k+1}} - \mathrm{e}^{\alpha t_k}\big).
\end{aligned}$$

Let $f_1(x) = M\mathrm{e}^{\alpha x}$ and $f_2(x) = x$. By Cauchy's mean value theorem[†], there exists $\rho_k \in [t_k, t_{k+1}) \subset [0, T_m)$ such that

$$M\mathrm{e}^{\alpha\rho_k}(t_{k+1} - t_k) \geqslant 1,$$

which concludes $t_{k+1} - t_k \geqslant 1/(M\mathrm{e}^{\alpha\rho_k}) > 1/(M\mathrm{e}^{\alpha T_m}) > 0$. This means that inter-execution intervals δ_k in event-triggered controller (4) satisfy $\delta_k = t_{k+1} - t_k > 0$. Thus we get $\lim_{k\to+\infty} t_k = +\infty$, which results in a contradiction and, in turn, implies $T_m = +\infty$, and Zeno phenomenon does not occur.

The proof of Proposition 3 is completed. □

With Propositions 1–3 in hand, based on the above controller (4), we have proved the boundedness of the closed-loop system in turn. In addition, the presented control scheme has satisfied the demand of avoiding the Zeno behavior. These outcomes are illustrated in the following theorem.

[†]Cauchy's mean value theorem: "Let f_1 and f_2 be functions that are continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Suppose that $\dot{f}_2(x) \neq 0$ for all $x \in (a, b)$. Then there exists at least one point $c \in (a, b)$ such that $\dot{f}_1(c)/\dot{f}_2(c) = (f_1(b) - f_1(a))/(f_2(b) - f_2(a))$."

Theorem 1. *Consider system* (1) *under Assumptions* 1 *and* 2*. The event-triggered output-feedback controller* (4)*, combined with the observer* (2) *and the updating law* (3)*, ensures that for any initial condition* $(\xi_0, \hat{\xi}_0)$*, the closed-loop system admits a unique and bounded solution on* $[0, +\infty)$*. Moreover, the system states satisfy*

$$\lim_{t \to +\infty} \big(\xi(t), \hat{\xi}_{[n]}(t), \hat{\xi}_{n+1}(t), u(t)\big) = (0, 0, \nu, -\nu). \tag{21}$$

Proof. We only need to prove the convergence of $(\xi(t), \hat{\xi}_{[n]}(t), \hat{\xi}_{n+1}(t), u(t))$. By (5), we can obtain that

$$\begin{aligned} \xi_i &= \gamma^{b+i-1}(\eta_i + \epsilon_i), \quad i = 1, \dots, n, \\ \hat{\xi}_i &= \gamma^{b+i-1}\epsilon_i, \quad i = 1, \dots, n, \\ \hat{\xi}_{n+1} &= \nu - \gamma^{b+n}\eta_{n+1}. \end{aligned} \tag{22}$$

Then, by Proposition 2 and (22), it can be verified that $\xi(t)$ and $\hat{\xi}(t)$ are bounded on $[0, +\infty)$. In fact, by (6) and Proposition 2, we can see that $\dot{\eta}(t)$ and $\dot{\epsilon}(t)$ are bounded on $[0, +\infty)$. In addition, from Proposition 2 it follows that

$$\int_0^{+\infty} \big\|\eta(s)\big\|^2 \,\mathrm{d}s < +\infty, \qquad \int_0^{+\infty} \big\|\epsilon(s)\big\|^2 \,\mathrm{d}s < +\infty.$$

Using Barbalat lemma [26], we have $\lim_{t \to +\infty}(\eta(t), \epsilon(t)) = (0, 0)$. From (4) and (22) we can see that (21) holds.

The proof is thus completed. □

5 Simulation examples

In this section, two examples are given to illustrate the validity of the theoretical results.

Example 1. Consider the following two-dimensional uncertain nonlinear system:

$$\begin{aligned} \dot{\xi}_1 &= 1.1\big(1 + 0.5y^2\big)\xi_2, \\ \dot{\xi}_2 &= 0.8\big(1 + 0.5y^2\big)(u + \nu) + \theta_s \xi_1^2 \xi_2, \\ y &= \xi_1. \end{aligned} \tag{23}$$

It can be verified that this system satisfies Assumptions 1 and 2 with $\underline{c} = 0.8$, $\bar{c} = 1.2$, $p = q = 2$, $\theta_s = 0.5$, $\lambda = 12.84$, and matrices P and Q satisfy

$$P = \begin{bmatrix} 0.0747 & -0.0089 & -0.0054 \\ -0.0089 & 0.0258 & -0.0117 \\ -0.0054 & -0.0117 & 0.0687 \end{bmatrix}, \qquad Q = \begin{bmatrix} 0.0687 & 0.0093 \\ 0.0093 & 0.0607 \end{bmatrix}.$$

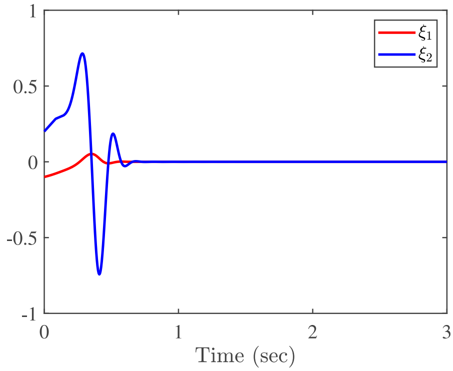

Figure 2. The trajectories of states ξ_1 and ξ_2.

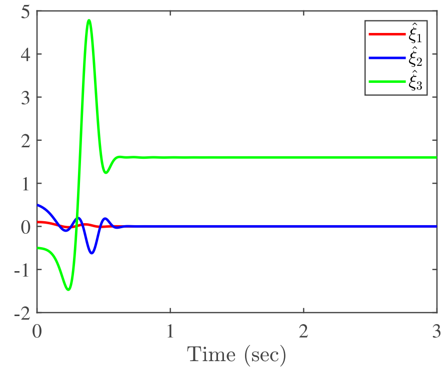

Figure 3. The trajectories of states $\hat{\xi}_1, \hat{\xi}_2$ and $\hat{\xi}_3$.

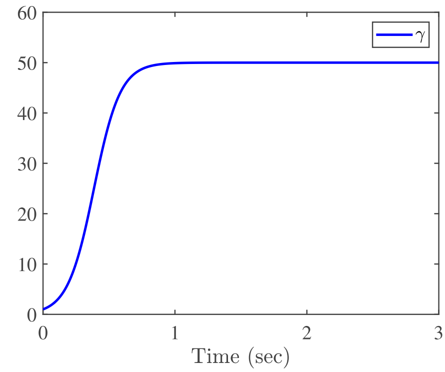

Figure 4. The trajectory of gain γ.

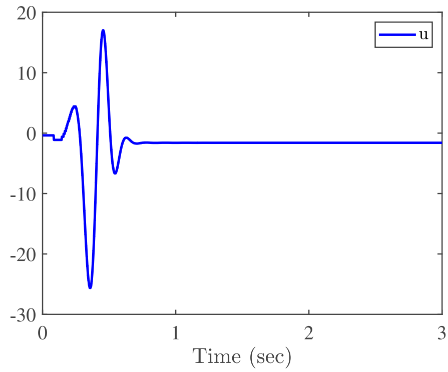

Figure 5. The trajectory of u.

By (2) and (3), we design the following observer and dynamic gain:

$$\begin{aligned}
\dot{\hat{\xi}}_1 &= 1.1\bigl(1+0.5y^2\bigr)\hat{\xi}_2 + 2.5\bigl(1+0.5y^2\bigr)\gamma(\xi_1-\hat{\xi}_1),\\
\dot{\hat{\xi}}_2 &= 0.8\bigl(1+0.5y^2\bigr)\hat{\xi}_3 + 0.8\bigl(1+0.5y^2\bigr)u + 2\bigl(1+0.5y^2\bigr)\gamma^2(\xi_1-\hat{\xi}_1),\\
\dot{\hat{\xi}}_3 &= \bigl(1+0.5y^2\bigr)\gamma^3(\xi_1-\hat{\xi}_1),\\
\dot{\gamma}(t) &= \max\left\{-0.2\gamma^2 + 10\gamma\bigl(1+y^2\bigr)^2, \left(1+\frac{\xi_1^2}{\gamma^{1/5}}\right)\frac{(\xi_1-\hat{\xi}_1)^2}{\gamma^{1/5}}\right\}, \qquad \gamma(0)=1.
\end{aligned}$$

Then, by the previous design procedure, we can design the time-varying event-triggered controller with $k_1 = (1+0.5y^2)$, $k_2 = 1.2(1+0.5y^2)$, and $\alpha = 6$. For simulation, we choose $\nu = 1.6$ and set the initial conditions as $\xi(0) = [-0.1, 0.2]^{\mathrm{T}}$ and $\hat{\xi}(0) = [0.1, 0.5, -0.5]^{\mathrm{T}}$. The simulation results are shown in Figs. 2–6. Figures 2–5 are obtained to exhibit the trajectories of all the signals of the closed-loop system and show the expected convergence of the closed-loop system. Figure 6 provides the inter-execution interval δ_k, and we can see that Zeno phenomenon can be excluded.

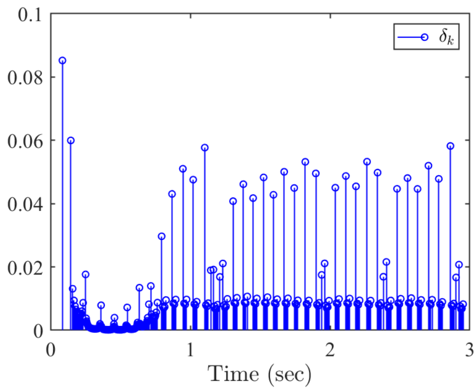

Figure 6. The inter-execution interval δ_k.

Example 2. Based on the output-feedback scheme proposed, we consider the nonzero set-point regulation of the following uncertain nonlinear system:

$$\ddot{v} + \alpha_1\big(1 - v^2\big)\dot{v} + \alpha_2 v = u, \tag{24}$$

where α_1, α_2 are unknown constants. Moreover, suppose that only v is measurable.

Specifically, the desired regulation means that signal v converges to a prescribed nonzero constant v_0. Let $\xi_1 = v - v_0$ and $\xi_2 = \dot{v}$. Then system (24) becomes

$$\begin{aligned} \dot{\xi}_1 &= \xi_2, \\ \dot{\xi}_2 &= (u - \alpha_2 v_0) - \alpha_2\eta_1 - \alpha_1(1 - (\xi_1 + v_0)^2)\xi_2, \\ y &= \xi_1, \end{aligned} \tag{25}$$

where $-\alpha_2 v_0$ is the input matching uncertainty of system (25). We can easily see that system (25) satisfies Assumptions 1–2 with $p = 2$, $\theta = \max\{|\alpha_1|, |\alpha_2|\}\cdot\max\{1+2v_0^2, 2\}$ and $q = 2$, $\lambda = 12.03$. Matrices P and Q satisfy:

$$P = \begin{bmatrix} 0.0750 & -0.0106 & -0.0078 \\ -0.0106 & 0.0212 & -0.0115 \\ -0.0078 & -0.0115 & 0.0755 \end{bmatrix}, \qquad Q = \begin{bmatrix} 0.0673 & 0.0172 \\ 0.0172 & 0.0580 \end{bmatrix}.$$

Thus, we design the following observer and dynamic gain:

$$\begin{aligned} &\dot{\hat{\xi}}_1 = \hat{\xi}_2 + 2\gamma(\xi_1 - \hat{\xi}_1), \qquad \dot{\hat{\xi}}_2 = \hat{\eta}_3 + u + 3\gamma^2(\xi_1 - \hat{\xi}_1), \qquad \dot{\hat{\xi}}_3 = \gamma^3(\xi_1 - \hat{\xi}_1), \\ &\dot{\gamma}(t) = \max\left\{-0.25\gamma^2 + 10\gamma\big(1 + y^2\big)^2, \left(1 + \frac{\xi_1^2}{\gamma^{1/4}}\right)\frac{(\xi_1 - \hat{\xi}_1)^2}{\gamma^{1/4}}\right\}, \qquad \gamma(0) = 1. \end{aligned}$$

Then, by the previous design procedure, we can design the time-varying event-triggered controller with $k_1 = 2$, $k_2 = 3$, and $\alpha = 5$. We select $\alpha_1 = 2$, $\alpha_2 = 2$. Let $v_0 = 1$

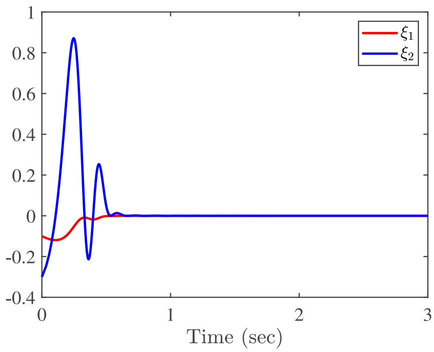

Figure 7. The trajectories of states ξ_1 and ξ_2.

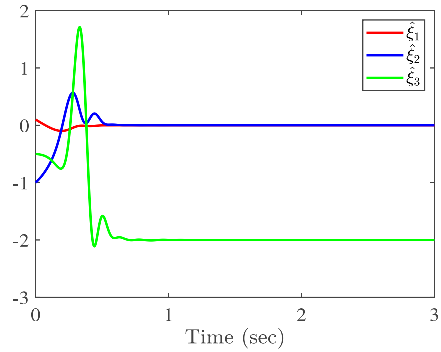

Figure 8. The trajectories of states $\hat{\xi}_1, \hat{\xi}_2$ and $\hat{\xi}_3$.

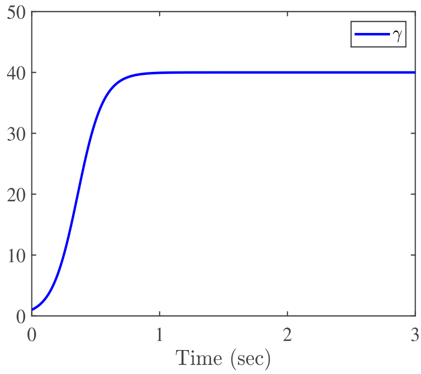

Figure 9. The trajectory of gain γ.

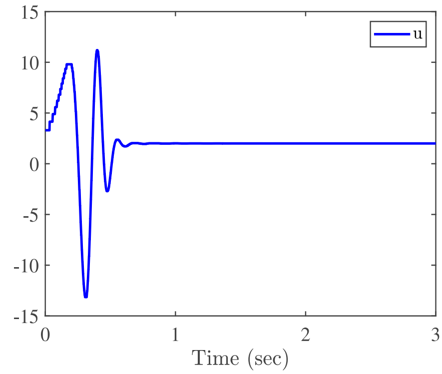

Figure 10. The trajectory of u.

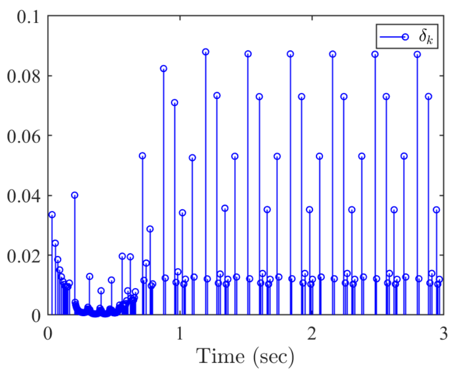

Figure 11. The inter-execution interval δ_k.

and choose the initial conditions as $\xi(0) = [-0.1, -0.3]^{\mathrm{T}}$ and $\hat{\xi}(0) = [0.1, -1, -0.5]^{\mathrm{T}}$. The simulation results are shown in Figs. 7–11. Figures 7–10 demonstrate the effectiveness of the control scheme. The inter-execution interval δ_k is provided in Fig. 11, and consequently, Zeno phenomenon can be excluded. It should be particularly noted that due to the presence of input-matching uncertainties, the event-triggered control schemes proposed in existing work [17] are no longer applicable.

6 Concluding remarks

This article focuses on developing an adaptive event-triggered output-feedback scheme to achieve global stabilization for a class of nonlinear systems characterized by function control coefficients, input matching uncertainty, and an unknown polynomial-function growth rate. To address this challenging problem, we introduce a dynamic gain to manage the additional nonlinearities arising from the function control coefficients and the polynomial-of-output growth rate. Furthermore, the input matching uncertainty is asymptotically estimated using an extended state observer. Notably, we design an event-triggering mechanism featuring a time-varying threshold, which remains strictly positive and gradually decays over time. Additionally, we provide a rigorous proof demonstrating that the resulting closed-loop system is stabilized and that Zeno behavior is effectively precluded. But it is worth pointing out that the system control coefficients we designed are known, and the method mentioned in this article is not applicable when the control coefficients or control directions of the system are unknown.

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