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Prescribed-time leader-following scaled consensus for nonlinear multiagent systems*

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Abstract. In this paper, the leader-following (L-F) scaled consensus (SC) for first-order multiagent systems (MASs) is considered within a prescribed time (PT). First, a distributed control protocol is proposed by incorporating a time-varying gain function and scaled factors. This protocol is designed to drive the states of the leader and followers to a predefined scaled relationship within the PT. It is worth noting that, by appropriately selecting the scaled factor, the protocol can also achieve complete consensus, bipartite consensus, or cluster consensus among the agents. To further enhance the control performance and reduce communication frequency, an improved control protocol is proposed by incorporating an event-triggered mechanism in which the event-triggering function is constructed using time-varying variables. Moreover, based on the PT stability theory, sufficient conditions are derived to ensure that SC is achieved within the PT. Finally, the effectiveness of the proposed control protocol is verified through two numerical simulation examples.

Keywords: prescribed time, scaled consensus, nonlinear MASs, event-triggered protocol, time-varying scaling function.

1 Introduction

In the development of intelligent systems, computer science and complex system theory, MASs have been widely and deeply applied in the fields of aircraft formation, cooperative equipping of multiple robotic arms, cooperative control of multiple robots, and network resource allocation [2, 25]. The MAS is composed of numerous autonomous agents that engage in both collaboration and competition with each other. Through agent interaction

and collaborative efforts, these systems are capable of addressing complex problems. Compared with traditional single systems, MASs demonstrate remarkable flexibility and adaptability, particularly in scenarios involving dynamic changes and uncertainties. They have the ability to promptly adjust their strategies in reaction to emerging challenges, thereby ensuring robust performance and strong resilience. Taking UAV formation missions as an example. MASs can realize highly cooperative flight through the implementation of distributed control. This ensures that the UAVs can effectively avoid collision and autonomously adjust their flight paths according to the environmental changes when executing the mission. This type of cooperation not only enhances work efficiency but also minimizes the necessity for manual intervention. Consequently, distributed collaborative control has become a pivotal research frontier in the control field, offering an effective methodology for addressing complex problems.

In the cooperative control of MASs, all agents need to reach consensus on shared objectives through local interactions, thereby achieving consensus behavioral states. Therefore, the consensus problem is regarded as the core issue of cooperative control and has been widely applied in fields such as formation control [7] and swarm coordination [9]. This significant research direction has attracted considerable attention from scholars and has yielded abundant research outcomes. For example, the consensus for integrator-type (IT) MASs was investigated via sampled-date control in [11]. By incorporating a time-base generator, a practical fixed-time consensus protocol is proposed in [18] for IT MASs. The output consensus for heterogeneous MASs with dynamic event-triggered (ET) mechanism was studied in [19]. The L-F consensus for linear MASs with transient performance via reset control approach was studied in [10]. It is evident that the aforementioned research works [10, 11, 18, 19] can be divided into two categories based on the presence or absence of a leader: leaderless consensus [11, 18] and L-F consensus [10, 19]. In leaderless consensus control, all agents achieve consensus through independent interactions without relying on any external reference signal. In contrast, under L-F consensus control, each follower agent need to track the leader's state through local information interactions.

From a dynamical perspective, the above works mainly considered the IT MASs [11, 18] and linear MASs [10, 19]. Nevertheless, nonlinear systems are widely utilized across various fields such as economics, engineering, and physics. Therefore, investigating such systems not only deepens our understanding of the fundamental principles of complex systems, but also lays a crucial foundation for developing more efficient and innovative solutions. Compared to linear systems, the study of nonlinear systems faces considerably greater challenges. These challenges primarily stem from the complex interactions among internal variables and the resulting intricate dynamic behaviors. Currently, there are several research findings pertaining to the consensus of nonlinear systems. For example, the consensus tracking control problem for a class of nonlinear multiinput-multioutput heterogeneous MASs was discussed in [16]. The fixed-time containment control for second-order nonlinear MASs with switching topologies was studied in [15].

In the study concerning consensus of MASs, the core objective is to design a distributed control protocol. This protocol should enable all agents to achieve consensus in their states or behaviors through local information exchange. However, achieving state

consensus among agents is challenging due to uneven resource allocation, varying task complexity, and diverse task requirements. To overcome the limitations of traditional consensus concepts, researchers have proposed a broader and more universal concept – SC [21]. The SC refers that each agent in the network flexibly adjusts its own state to achieve a predefined scaled relationship, rather than simply pursuing state consensus.

Similar to the consensus concept in MASs, SC has also attracted considerable research interest due to its enhanced adaptability and broad applicability. This mechanism demonstrates significant potential in various scenarios, including efficient resource allocation, cooperative robot formation, and numerous tasks requiring multiagent coordination. With the continuous advancement of research, the application domain of SC has expanded from initial first- and second-order systems to more complex high-order systems. This expansion has yielded plentiful and fruitful research outcomes. The SC of MASs with time delays was investigated in [1, 30, 31]. The SC with switching networks was studied in [5, 17]. Meanwhile, the SC with ET control was discussed in [28].

Although many scholars have conducted in-depth discussions on the issue of SC in MASs, there is relatively little research on achieving SC in nonlinear systems. The proposal of this situation not only reflects the flexibility and efficiency of SC in practical applications, but also provides important theoretical support and practical directions for further research on SC.

Convergence time is a crucial index that need to be considering during the design of control protocols. Under the control, the consensus can be categorized into asymptotical consensus [13, 23] and time-limited consensus [8, 14, 27]. Asymptotic consensus refers to the property that as time approaches infinity, the states of all agents asymptotically converge to a common value or a small bounded region. In contrast, the time-limited consensus emphasizes that the consensus can be achieved within a limited time and it has fast convergence and high accuracy. The time-limited consensus includes finite-time consensus [27], fixed-time consensus [14], and PT consensus [8]. In both finite-time and fixed-time consensus, the convergence time of the system generally depends on either the initial state or the control parameters. This dependency significantly influences the convergence performance of the system. Therefore, such parameters must be carefully designed rather than arbitrarily selected.

In practical scenarios, it is essential to accomplish relevant tasks within the PT. For example, in scenarios such as power grid scheduling and UAV flight, the system is required to be able to achieve the desired behavior within the PT. Therefore, the PT consensus framework was proposed. In the current research on cooperative control of MASs, the problem of PT consensus has become a cutting-edge hotspot, and scholars have achieved remarkable progress in distributed control protocol design, convergence rate optimization, robustness analysis, and multiple related dimensions. For instance, the PT consensus for input-constrained linear MASs was considered in [29]. The adaptive PT consensus for a class of nonlinear networked MASs was studied in [24].

Based on the above discussion, although substantial research outcomes exist in the field of SC, most existing studies are established under the frameworks of linear systems with fixed-time or finite-time convergence. To better support engineering applications such as smart grids and multirobot collaboration, this paper introduces a PT conver-

gence mechanism and integrates it with the SC framework. The proposed method enables scaled coordination of agent states within the PT, thereby significantly improving the system response speed. The main innovations can be categorized into three distinct components:

(i) This paper devises a novel distributed control protocol by utilizing scaled factors and incorporating a time-varying scaling function. This protocol guarantees that first-order nonlinear MASs attain SC within the PT frame.

(ii) Compared with consensus, SC exhibits greater flexibility and broader adaptability. Papers [22] and [3] investigated asymptotic SC and fixed-time SC, respectively. The present paper further explores the PT SC problem for nonlinear MASs. In contrast to existing methods, the PT convergence mechanism offers a significant advantage: the upper bound of the convergence time can be predefined directly, independent of the system's initial states and control parameters. Meanwhile, the proposed control strategy significantly reduces the time required to achieve consensus. This characteristic makes SC highly suitable and effective in practical applications, further enhancing its practical value.

(iii) To reduce communication frequency and minimize control expenses, a novel ET mechanism is devised along with an enhanced control protocol that incorporates a power law function. This control protocol ensures SC of MASs within a PT, while also ensuring the absence of Zeno behavior throughout the control process.

The remaining sections are organised as follows. First, some preliminaries and the problem statement are given in Section 2. The PT SC for nonlinear MASs problem is studied in Section 3. The effectiveness of the proposed method is demonstrated through numerical examples in Section 4. Finally, Section 5 presents the conclusions and future research directions.

Notations. Define the set of real numbers, the set of nonnegative real numbers, and the set of positive real numbers as $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, and $\mathbb{R}_{> 0}$. For any real number $a \in \mathbb{R}$, its absolute value is defined as $|a|$. $\|u\|_1$ and $\|u\|_\infty$ represent the 1-norm and the infinite norm of the vector u, respectively. For an arbitrary symmetric matrix Q, Q^T, $\lambda_{\min}(Q)$, and $\lambda_{\max}(Q)$ denote the transpose, the minimum, and maximum eigenvalues of Q, respectively. Let $[Q]_{ij}$ denote the (i, j)th entry of matrix Q. The matrix $Q > 0$ denotes that the matrix Q is positive definite. Given a vector $h = (h_1, h_2, \ldots, h_n)^{\mathrm{T}} \in \mathbb{R}^n$, we define $\mathrm{sign}(h) = (\mathrm{sign}(h_1), \mathrm{sign}(h_2), \ldots, \mathrm{sign}(h_n))^{\mathrm{T}}$ by applying the sign function $\mathrm{sign}(\cdot)$, i.e.,

$$\mathrm{sign}(h) = \begin{cases} 1, & h > 0, \\ 0, & h = 0, \\ -1, & h < 0. \end{cases}$$

2 Preliminaries

This section presents graph theory concepts, relevant lemmas, and the problem formulation.

2.1 Algebraic graph theory

Consider a MAS comprising a single leader (agent 0) and N followers indexed by $i = 1, 2, \ldots, N$. Follower interactions are modeled via communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. Therein, $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is a set of nodes representing N agents, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set describing the interaction among agents, i.e., $(i, j) \in \mathcal{E}$ indicates direct state information flow from agent j to agent i. In this framework, agent j belongs to the neighbor set $\mathcal{N}_i$ of agent i, where $\mathcal{N}_i = \{j \mid (i, j) \in \mathcal{E}\}$ formally defines all topological neighbors. The interconnection topology is characterized by weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$, where $a_{ij} > 0$ if the agent i can receive information from the agent j, otherwise, $a_{ij} = 0$. This paper assumes a loop-free graph structure, where no vertex connects to itself so that $a_{ii} = 0$. The degree matrix denoted as $D = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$, where $d_i = \sum_{j=1}^{N} a_{ij}$ for $i = 1, \ldots, N$. The Laplacian matrix is defined as $L = [l_{ij}] = D - \mathcal{A}$, i.e., $[L]_{ij} = -a_{ij}$ for $i \neq j$, and $[L]_{ii} = \sum_{j \in \mathcal{N}_i} a_{ij}$. The graph $\mathcal{G}$ is said to be connected if any two distinct vertices are joined by a path. Let $B = \mathrm{diag}\{b_1, b_2, \ldots, b_N\}$ represent the information transmission between the leader and follower. $b_i > 0$ means that follower i can obtain information from the leader. Otherwise, $b_i = 0$. To facilitate later derivation, the matrix H is taken to be $H = L + B$. Given a connected undirected graph $\mathcal{G}$ with at least one leader-accessible follower, the matrix H is positive definite.

2.2 Some useful definitions and lemmas

Consider the following system:

$$\dot{s}(t) = g\big(s(t)\big) \quad \forall t \geqslant 0, \qquad s(0) = s_0, \tag{1}$$

where $s(t) \in \mathbb{R}$ denotes the state variable, and $g(s(t)) : \mathbb{R} \to \mathbb{R}$ represents a continuous nonlinear function satisfying condition $g(0) = 0$.

Definition 1. (See [4].) The global PT stability is guaranteed for the origin of system (1) if $\lim_{t \to T} s(t) = 0$ and $s(t) \equiv 0$ for $t \in [T, +\infty)$, where T is an arbitrarily chosen positive constant, called the settling time, and $s_0 \in \mathbb{R}$.

Lemma 1. (See [32].) For $x_i \in \mathbb{R}$, $i = 1, \ldots, n$,

$$\left(\sum_{i=1}^{N} |x_i| \right)^p \leqslant \sum_{i=1}^{N} |x_i|^p \leqslant N^{1-p} \left(\sum_{i=1}^{N} |x_i| \right)^p \quad \text{if } 0 < p \leqslant 1,$$

$$N^{1-p} \left(\sum_{i=1}^{N} |x_i| \right)^p \leqslant \sum_{i=1}^{N} |x_i|^p \leqslant \left(\sum_{i=1}^{N} |x_i| \right)^p \quad \text{if } 1 < p < +\infty.$$

Introduce a family of time-varying functions

$$\vartheta(t) = \begin{cases} \frac{T_\rho^l}{(T_1 - t)^l}, & t \in [t_0, T_1), \\ 1, & t \in [T_1, +\infty), \end{cases} \tag{2}$$

where the settling time $T_\rho > 0$ is a user-defined parameter that can be prespecified according to performance requirements. t_0 is the initial time, $l > 2$ is a positive constant, and $T_1 = t_0 + T_\rho$.

For the sake of simplicity, we let $t_0 = 0$, then the following lemma is given.

Lemma 2. (*See* [26].) *Let $V(s(t))$ be a positive and continuously differentiable function. If there exist two positive real constants c_1 and c_2 such that*

$$\dot{V}\big(s(t)\big) \leqslant \begin{cases} -c_1 V(s(t)) - c_2 \frac{\dot{\vartheta}(t)}{\vartheta(t)} V(s(t)), & t \in [0, T_\rho), \\ 0, & t \in [T_\rho, +\infty), \end{cases}$$

then the origin of system (1) *is globally PT stable with the PT T_ρ. Furthermore, the solution can be derived as follows*:

$$V\big(s(t)\big) \leqslant \vartheta^{-c_2}(t)\exp(-c_1 t)V(0), \quad t \in [0, T_\rho),$$
$$V\big(s(t)\big) \equiv 0, \quad t \in [T_\rho, +\infty).$$

2.3   Problem formulation

Consider a MAS consisting of $N + 1$ agents, where the dynamics of the agent i and the leader are described as follows:

$$\begin{aligned} \dot{s}_i(t) &= g\big(s_i(t)\big) + u_i(t), \quad i = 1, \dots, N, \\ \dot{s}_0(t) &= g\big(s_0(t)\big), \end{aligned} \tag{3}$$

where $s_i(t) \in \mathbb{R}$ represents the state variable of the ith follower, and $s_0(t) \in \mathbb{R}$ is the state variable of leader. $g(\cdot)$ is a bounded nonlinear function that satisfies condition $\|g(s_i(t))\|_\infty \leqslant \kappa$, where κ is a positive design parameter.

Definition 2. (See [20].) Consider the MAS (3), the PT SC is solved if the following equations are satisfied:

$$\lim_{t \to T_\rho^-} \big|\omega_i s_i(t) - s_0(t)\big| = 0,$$
$$\big|\omega_i s_i(t) - s_0(t)\big| \equiv 0 \quad \forall t \in [T_\rho, +\infty),$$

where the scaled factor $\omega_i \in \mathbb{R} \setminus \{0\}$ is a constant for the ith follower agent, and T_ρ represents the PT.

The main task of this paper is to propose some feasible control protocols such that all agents in the MAS (3) can achieve PT SC.

Remark 1. Following the SC characterization established in Definition 2, SC is essentially a generalized form of consensus. By flexibly adjusting the scaled factor parameters, MASs can achieve different forms of consensus states. Specifically, when all agents adopt the same scaled factor, the system achieve complete consensus. When the scaled

factor satisfies $\omega_i \in \{-1, 1\}$, the system exhibits bipartite consensus. Moreover, through deliberate design of the scaled factor, the system can attain more intricate behavioral states. These theoretical findings demonstrate both the universality of the studied SC problem and the broad application prospects of the proposed framework in distributed collaborative control.

Assumption 1. The topology graph $\mathcal{G}$ between followers is an undirected connected graph, and at least one follower can receive the information from the leader.

3 Main results

To achieve L-F SC within the PT, this paper proposes the following control protocol:

$$u_i(t) = -\frac{1}{\omega_i}\left[\left(c_1 + c_2\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)\psi_i(t) + \gamma_1 \operatorname{sign}\big(\psi_i(t)\big)\right], \quad i = 1, 2, \ldots, N, \quad (4)$$

where c_1, c_2, and γ_1 are positive numbers, $\vartheta(t)$ is given by (2), and

$$\psi_i(t) = \sum_{j=1}^{N} a_{ij}\big(\omega_i s_i(t) - \omega_j s_j(t)\big) + b_i\big(\omega_i s_i(t) - s_0(t)\big).$$

Define the SC error between the ith follower and the leader as $\tilde{s}_i(t) = \omega_i s_i(t) - s_0(t)$, $i = 1, 2, \ldots, N$. As a result, the following result is obtained:

$$\begin{aligned}
\dot{\tilde{s}}_i(t) &= \omega_i g(s_i(t)) - \left(c_1 + c_2\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)\psi_i(t) - \gamma_1 \operatorname{sign}\big(\psi_i(t)\big) - g(s_0(t)) \\
&= G(\omega_i) - \left(c_1 + c_2\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)\psi_i(t) - \gamma_1 \operatorname{sign}\big(\psi_i(t)\big),
\end{aligned} \quad (5)$$

where $G(\omega_i) = \omega_i g(s_i(t)) - g(s_0(t))$.

Define $\tilde{s}(t) = (\tilde{s}_1(t), \tilde{s}_2(t), \ldots, \tilde{s}_N(t))^{\mathrm{T}}$ and $G(\breve{\omega}) = (G(\omega_1), G(\omega_2), \ldots, G(\omega_N))^{\mathrm{T}}$, then (5) can be rewritten as

$$\dot{\tilde{s}}(t) = G(\breve{\omega}) - \left(c_1 + c_2\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)H\tilde{s}(t) - \gamma_1 \operatorname{sign}\big(H\tilde{s}(t)\big),$$

where $H = L + B$ and $\operatorname{sign}(H\tilde{s}(t)) = (\operatorname{sign}(\psi_1(t)), \operatorname{sign}(\psi_2(t)), \ldots, \operatorname{sign}(\psi_N(t)))^{\mathrm{T}}$.

Remark 2. Unlike prior methods in [14, 27], this paper introduces scaled factors and time-varying functions into the controller design. This approach significantly enhances the flexibility and robustness of the controller structure. Based on this design, the MAS can adapt to different environmental requirements and rapidly achieve various forms of state convergence, including complete consensus, bipartite consensus, and cluster consensus.

Theorem 1. *Under Assumption* 1, *if the control parameters c_2 and γ_1 satisfy*

$$c_2 \geqslant \frac{1}{l\lambda_{\min}(H)}, \qquad \gamma_1 \geqslant \kappa \max_{1\leqslant i\leqslant N} \{1+|\omega_i|\}, \qquad (6)$$

then the MAS (3) *with control protocol* (4) *achieves L-F SC within the PT T_ρ.*

Proof. Choose the Lyapunov function as

$$V(t) = \tilde{s}^{\mathrm{T}}(t) H \tilde{s}(t).$$

The derivative of $V(t)$ with respect to time and using condition (6) is computed as

$$
\begin{aligned}
\dot{V}(t) &= 2\tilde{s}^{\mathrm{T}}(t) H \dot{\tilde{s}}(t) \\
&\leqslant 2\tilde{s}^{\mathrm{T}}(t) H \left[G(\breve{\omega}) - \left(c_1 + c_2 \frac{\dot{\vartheta}(t)}{\vartheta(t)} \right) H\tilde{s}(t) - \gamma_1 \operatorname{sign}(H\tilde{s}(t)) \right] \\
&\leqslant 2\big\| H\tilde{s}(t) \big\|_1 \big\| G(\breve{\omega}) \big\|_\infty - 2\lambda_{\min}(H) \left(c_1 + c_2 \frac{\dot{\vartheta}(t)}{\vartheta(t)} \right) V(t) - 2\gamma_1 \big\| H\tilde{s}(t) \big\|_1 \\
&\leqslant 2\big\| H\tilde{s}(t) \big\|_1 \left[\kappa \max_{1\leqslant i\leqslant N} (|\omega_i|+1) - \gamma_1 \right] - 2\lambda_{\min}(H) \left(c_1 + c_2 \frac{\dot{\vartheta}(t)}{\vartheta(t)} \right) V(t) \\
&\leqslant -2c_1 \lambda_{\min}(H) V(t) - 2c_2 \lambda_{\min}(H) \frac{\dot{\vartheta}(t)}{\vartheta(t)} V(t).
\end{aligned}
$$

Therefore, one can obtain that

$$
\dot{V}(t) \leqslant \begin{cases} -2c_1\lambda_{\min}(H)V(t) - 2c_2\lambda_{\min}(H)\frac{\dot{\vartheta}(t)}{\vartheta(t)}V(t), & t \in [0, T_\rho), \\ -2c_1\lambda_{\min}(H)V(t), & t \in [T_\rho, +\infty). \end{cases}
$$

By recalling Lemma 2, it follows that $V(t) \equiv 0$, and thus $\tilde{s}(t) \equiv 0_N$ for $t \in [T_\rho, +\infty)$, which indicates that the L-F SC can be achieved within the PT T_ρ.

Next, it is necessary to ensure the boundedness of the control input $u_i(t)$, $i = 1, 2, \ldots, N$.

$$
\begin{aligned}
|u_i(t)| &= \left| -\frac{1}{\omega_i} \left[\left(c_1 + c_2 \frac{\dot{\vartheta}(t)}{\vartheta(t)} \right) \psi_i(t) + \gamma_1 \operatorname{sign}(\psi_i(t)) \right] \right| \\
&\leqslant \frac{1}{|\omega_i|} \left[\left(c_1 + c_2 \frac{\dot{\vartheta}(t)}{\vartheta(t)} \right) |\psi_i(t)| + \gamma_1 \right] \\
&\leqslant \frac{1}{|\omega_i|} \left[\left(c_1 + \frac{lc_2}{T_\rho} \right) \vartheta^\tau(t) \sqrt{N\lambda_{\max}(H)V(0)} + \gamma_1 \right] \\
&\leqslant \frac{1}{|\omega_i|} \left[\left(c_1 + \frac{lc_2}{T_\rho} \right) \sqrt{N\lambda_{\max}(H)V(0)} + \gamma_1 \right],
\end{aligned}
$$

where $\tau = 1/l - c_2\lambda_{\min}(H)$.

Based on the preceding analysis, the following result is derived:

$$\left|u_i(t)\right| \leqslant \kappa_{1i} V^{1/2}(0) + \kappa_{2i}, \quad t \in [0,\, T_\rho),$$

and

$$\left|u_i(t)\right| = 0, \quad t \in [T_\rho,\, +\infty),$$

where

$$\kappa_{1i} = \frac{1}{|\omega_i|}\left(c_1 + \frac{lc_2}{T_\rho}\right)\sqrt{N\lambda_{\max}(H)} \quad \text{and} \quad \kappa_{2i} = \frac{\gamma_1}{|\omega_i|}.$$

Consequently, the boundedness of the control input is established, which establishes the proof of Theorem 1. □

Remark 3. This paper investigates the SC of MASs based on undirected graph topology. To design an effective control protocol, a sign function is introduced to eliminate the significance of nonlinear terms. It is particularly noteworthy that due to the asymmetric of the Laplacian matrix of general directed graph, it is difficult to directly generalize this protocol to directed networks. But, it can be extended to strongly connected and detail-balanced graph. Under this framework, the local neighborhood error

$$\sum_{j=1}^{N} \theta_i a_{ij}\bigl(\omega_i s_i(t) - \omega_j s_j(t)\bigr) + b_i\bigl(\omega_i s_i(t) - s_0(t)\bigr)$$

is changed by introducing the balance factor θ_i. In terms of constructing the Lyapunov function, we need to replace the matrix H by θH, where $\theta = \mathrm{diag}\{\theta_1, \theta_2, \ldots, \theta_N\}$, which can effectively extend the relevant conclusions to the case of strongly connected and detail-balanced networks.

The ET control provides an efficient approach to decreasing communication frequency, while conserving system resources. Hence, the control protocol (4) is improved, and the following ET control protocol is proposed:

$$u_i(t) = -\frac{1}{\omega_i}\left[\left(c_3 + c_4\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)\psi_i\bigl(t_k^i\bigr) + \gamma_2\,\mathrm{sign}\bigl(\psi_i\bigl(t_k^i\bigr)\bigr)\right], \quad t \in \bigl[t_k^i, t_{k+1}^i\bigr), \quad (7)$$

where t_k^i is the kth ET instant of agent i with $k = 0, 1, \ldots$, and $t_0^i = 0$,

$$\psi_i(t) = \sum_{j=1}^{N} a_{ij}\bigl(\omega_i s_i(t) - \omega_j s_j(t)\bigr) + b_i\bigl(\omega_i s_i(t) - s_0(t)\bigr),$$

and the definitions of other parameters are the same as the ones in (4).

The measurement error of the ith agent is given by the following expression:

$$
\begin{aligned}
e_i(t) = {}& -\frac{1}{\omega_i}\left[\left(c_3 + c_4\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)\psi_i\bigl(t_k^i\bigr) + \gamma_2\,\mathrm{sign}\bigl(\psi_i\bigl(t_k^i\bigr)\bigr)\right] \\
& + \frac{1}{\omega_i}\left[\left(c_3 + c_4\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)\psi_i(t) + \gamma_2\,\mathrm{sign}\bigl(\psi_i(t)\bigr)\right].
\end{aligned}
$$

For the MAS (3) to attain L-F SC, the proposed ET function is given by

$$\phi_i(t) = \big|e_i(t)\big| - \frac{\eta_1}{|\omega_i|}\big|\psi_i(t)\big| - \frac{\eta_2\varrho(t)}{|\omega_i|}, \quad i = 1, 2, \ldots, N, \tag{8}$$

where η_1 and η_2 are positive constants. $\varrho(t)$ is characterized by

$$\varrho(t) = \begin{cases} \vartheta^{-m}(t), & t \in [0, T_\rho), \\ 0, & t \in [T_\rho, +\infty), \end{cases}$$

where m is a positive constant.

For agent i, the $(k+1)$th triggering instant is denoted by t_{k+1}^i, which is determined by

$$t_{k+1}^i = \inf\big\{t > t_k^i\colon \phi_i(t) \geqslant 0\big\}.$$

Theorem 2. *Under Assumption* 1, *if the control parameters* c_3, c_4, *and* γ_2 *satisfy*

$$c_3 > \eta_1, \qquad c_4 \geqslant \frac{1}{l\lambda_{\min}(H)}, \qquad \gamma_2 \geqslant \kappa \max_{1 \leqslant i \leqslant N}\big\{|\omega_i| + 1\big\} + \eta_2,$$

then the MAS (3) *with control protocol* (7) *achieves L-F SC within the PT* T_ρ.

Proof. Take the Lyapunov function to be

$$V(t) = \frac{1}{2}\tilde{s}^{\mathrm{T}}(t)H\tilde{s}(t),$$

where $\tilde{s}(t) = (\tilde{s}_1(t), \tilde{s}_2(t), \ldots, \tilde{s}_N(t))^{\mathrm{T}}$.

Differentiating $V(t)$ with respect to time thus yields

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} \psi_i(t)\big(\omega_i g\big(s_i(t)\big) - g\big(s_0(t)\big)\big) \\
&\quad + \sum_{i=1}^{N} \omega_i \psi_i(t)\left[e_i(t) - \frac{1}{\omega_i}\left(c_3 + c_4\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)\psi_i(t) - \frac{\gamma_2}{\omega_i}\operatorname{sign}\big(\psi_i(t)\big)\right] \\
&\leqslant \kappa \max_{1 \leqslant i \leqslant N}\big\{|\omega_i| + 1\big\} \sum_{i=1}^{N} \big|\psi_i(t)\big| + \eta_1 \sum_{i=1}^{N} \psi_i^2(t) \\
&\quad - \left(c_3 + c_4\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right) \sum_{i=1}^{N} \psi_i^2(t) - \gamma_2 \sum_{i=1}^{N} \big|\psi_i(t)\big| + \eta_2\varrho(t) \sum_{i=1}^{N} \big|\psi_i(t)\big| \\
&\leqslant -\left(c_3 - \eta_1 + c_4\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right) \sum_{j=1}^{N} \psi_i^2(t).
\end{aligned}$$

In addition,

$$\sum_{i=1}^{N}\left(\sum_{j=0}^{N}\big(\tilde{s}_i(t) - \tilde{s}_j(t)\big)\right)^2 \geqslant \lambda_{\min}(H)\tilde{s}^{\mathrm{T}}(t)H\tilde{s}(t) = 2\lambda_{\min}(H)V(t).$$

Therefore, one can obtain that

$$\dot{V}(t) \leqslant \begin{cases} -2(c_3 - \eta_1)\lambda_{\min}(H)V(t) - 2\lambda_{\min}(H)c_4\frac{\dot{\vartheta}(t)}{\vartheta(t)}V(t), & t \in [0, T_\rho), \\ -2(c_3 - \eta_1)\lambda_{\min}(H)V(t), & t \in [T_\rho, +\infty). \end{cases}$$

It follows from Lemma 2 that $V(t) \equiv 0$, and thus $\tilde{s}(t) \equiv 0_N$ for $t \in [T_\rho, +\infty)$. Therefore, the L-F SC can be attained within the PT T_ρ, that is, $\omega_i s_i(t) = s_0(t)$, $i = 1, 2, \ldots, N$, for $t \geqslant T_\rho$.

The finite range of the control input $u_i(t)$, $i = 1, 2, \ldots, N$, is rigorously demonstrated following the analytical framework of Theorem 1:

$$\begin{aligned} |u_i(t)| &= \left| -\frac{1}{\omega_i}\left[\left(c_3 + c_4\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)\psi_i(t_k^i) + \gamma_2 \operatorname{sign}\left(\psi_i(t_k^i)\right)\right]\right| \\ &\leqslant |e_i(t)| + \frac{1}{|\omega_i|}\left[\left(c_3 + c_4\frac{\dot{\vartheta}(t)}{\vartheta(t)}\right)|\psi_i(t)| + \gamma_2\right] \\ &\leqslant \frac{1}{|\omega_i|}\left(c_3 + \eta_1 + \frac{lc_4}{T_\rho}\right)\vartheta^{1/l}(t)|\psi_i(t)| + \frac{\eta_2\varrho(t)}{|\omega_i|} + \frac{\gamma_2}{|\omega_i|} \\ &\leqslant \frac{1}{|\omega_i|}\left(c_3 + \eta_1 + \frac{lc_4}{T_\rho}\right)\sqrt{N\lambda_{\max}(H)V(0)} + \frac{\eta_2\varrho(t)}{|\omega_i|} + \frac{\gamma_2}{|\omega_i|}. \end{aligned}$$

Therefore, the controller is bounded.

In the light of

$$\phi_i(t) = |e_i(t)| - \frac{\eta_1}{|\omega_i|}|\psi_i(t)| - \frac{\eta_2\varrho(t)}{|\omega_i|},$$

we have

$$D^+|e_i(t)| \leqslant |\dot{e}_i(t)| \leqslant \frac{1}{|\varrho_i|}\frac{lc_4}{(T_\rho - t)^2}|\psi_i(t)| + \frac{1}{|\varrho_i|}\left(c_3 + \frac{lc_4}{T_\rho - t}\right)|\dot{\psi}_i(t)|.$$

In addition,

$$|\psi_i(t)| \leqslant \sum_{i=1}^{N}\left(\psi_i^2(t)\right)^{1/2} \leqslant N^{1/2}\left(\sum_{i=1}^{N}\psi_i^2(t)\right)^{1/2} \leqslant \sqrt{2N\lambda_{\max}(H)V(0)},$$

$$\begin{aligned} |\dot{\psi}_i(t)| &= \left|\sum_{j=1}^{N}a_{ij}\left(\omega_i\dot{s}_i(t) - \omega_j\dot{s}_j(t)\right) + b_i\left(\omega_i\dot{s}_i(t) - \dot{s}_0(t)\right)\right| \\ &\quad + b_i\left|\omega_i\left(g\left(s_i(t)\right) + u_i(t)\right) - g\left(s_0(t)\right)\right| \\ &\leqslant \sum_{j=1}^{N}|a_{ij}|\left(|\omega_i||u_i(t)| + |\omega_j||u_j(t)|\right) + \sum_{j=1}^{N}|a_{ij}|\left(|\omega_i||g\left(s_i(t)\right)| + |\omega_j||g\left(s_j(t)\right)|\right) \\ &\quad + b_i\left(|\omega_i|\left(|g\left(s_i(t)\right)| + |u_i(t)|\right) + |g(s_0)(t)|\right) \\ &\leqslant 2N\bar{\omega}\bar{a}\chi + 2N\bar{\omega}\bar{a}\kappa + \bar{b}\left((1 + \bar{\omega})\kappa + \bar{\omega}\chi\right) \leqslant \chi_1, \end{aligned}$$

where $\bar{\omega} = \max_{i=1,2,\dots,N}\{|\omega_i|\}$, $\bar{a} = \max\{|a_{ij}|\}$, $\bar{b} = \max_{i=1,2,\dots,N}\{b_i\}$,

$$\chi = \frac{1}{|\omega_i|}\left(c_3 + \eta_1 + \frac{lc_4}{T_\rho}\right)\sqrt{N\lambda_{\max}(H)V(0)} + \frac{\eta_2 \varrho(t)}{|\omega_i|} + \frac{\gamma_2}{|\omega_i|},$$

and

$$\chi_1 = (2N\bar{\omega}\bar{a} + \bar{b} + \bar{\omega}\bar{b})(\kappa + \chi).$$

Given $e_i(t_k^i) = 0$, we conclude that

$$
\begin{aligned}
\big|e_i(t)\big| &\leqslant \int_{t_k^i}^{t} \big|\dot{e}_i(z)\big|\,\mathrm{d}z \\
&\leqslant \frac{1}{|\omega_i|}\int_{t_k^i}^{t}\left[\frac{lc_4}{(T_\rho - z)^2}\big|\psi_i(z)\big| + \left(c_3 + \frac{lc_4}{T_\rho - z}\right)\big|\dot{\psi}_i(z)\big|\right]\mathrm{d}z \\
&\leqslant \frac{1}{|\omega_i|}\int_{t_k^i}^{t}\left(\frac{lc_4\sqrt{2N\lambda_{\max}(H)V(0)}}{(T_\rho - z)^2} + \left(c_3 + \frac{lc_4}{T_\rho - z}\right)\chi_1\right)\mathrm{d}z. \quad (9)
\end{aligned}
$$

In view of (9) and the ET function (8), we have

$$
\begin{aligned}
\big|e_i(t_{k+1}^i)\big| &= \frac{\eta_1}{|\omega_i|}\big|\psi_i(t_{k+1}^i)\big| + \frac{\eta_2}{|\omega_i|}\varrho(t_{k+1}^i) \\
&\leqslant \frac{1}{|\omega_i|}\int_{t_k^i}^{t_{k+1}^i}\left(\frac{lc_4\sqrt{2N\lambda_{\max}(H)V(0)}}{(T_\rho - z)^2} + \left(c_3 + \frac{lc_4}{T_\rho - z}\right)\chi_1\right)\mathrm{d}z \\
&\leqslant \big(t_{k+1}^i - t_k^i\big)\chi_2,
\end{aligned}
$$

where

$$\chi_2 = \frac{1}{|\omega_i|}\left(\frac{lc_4\sqrt{2N\lambda_{\max}(H)V(0)}}{(T_\rho - t_{k+1}^i)^2} + \left(c_3 + \frac{lc_4}{T_\rho - t_{k+1}^i}\right)\chi_1\right).$$

Since $T > t_{k+1}^i$, then $\chi_2 > 0$. This means

$$t_{k+1}^i - t_k^i \geqslant \frac{\eta_2\varrho(t_{k+1}^i)}{(\chi_2|\omega_i|)} > 0,$$

which implies that Zeno behavior does not exist. Therefore, Theorem 2 is proved. $\square$

Remark 4. In the design of ET control protocol (7), the function $\varrho(t)$ is meticulously introduced. It can both ensure that the system achieves SC within a PT and effectively eliminate Zeno behavior. Notably, the selection of $\varrho(t)$ is not exclusive. Any function that converges to zero within a PT can serve as a viable alternative.

Remark 5. In [12] and [6], although the authors proposed some control protocols to study the consensus of MASs within a PT, all agents only can achieve complete consensus or bipartite consensus. This concept is further refined through the introduction of scaled factors and a time-varying scaling function, leading to the design of an advanced SC control protocol. The proposed protocol offers considerable flexibility and versatility. By appropriately selecting the scaling factors, it can be specialized to achieve various consensus forms, including complete consensus, bipartite consensus, and cluster consensus. This makes the proposed method not only applicable to traditional consensus problems but also capable of solving more complex and diverse MAS problems.

4 Simulation example

This section validates the effectiveness of the proposed distributed PT control protocol through two simulation examples. Different communication topologies, initial states, and PT parameters are configured in these two cases to thoroughly demonstrate the adaptability and robustness of the proposed control strategy under various conditions.

Example 1. Consider a MAS composed of five agents, which includes one leader and four followers labeled as 0 and $1-4$, respectively. The information transmission topology within the system is illustrated in Fig. 1. The dynamics of the followers and the leader are described by

$$\dot{s}_i(t) = 0.02 \sin\big(s_i(t)\big) + 0.02 + u_i(t), \quad i = 1, \ldots, N,$$
$$\dot{s}_0(t) = 0.02 \sin\big(s_0(t)\big) + 0.02.$$

According to Theorems 1 and 2, the parameters are selected as $c_1 = 3.68$, $c_2 = 3.6$, $l = 2.001$, $t_0 = 0s$, $T_\rho = 0.4s$, $\gamma_1 = 0.08$, $c_3 = 1.001$, $c_4 = 1.49$, and $\gamma_2 = 0.09$. The scaled factors are $\omega_1 = 0.1$, $\omega_2 = 0.2$, $\omega_3 = -0.1$, and $\omega_4 = -0.2$. All conditions of Theorems 1 and 2 are satisfied. The initial state is designed as $s_0(0) = -2.1$, $s_1(0) = -5.23$, $s_2(0) = 1.1$, $s_3(0) = 3.2$, and $s_4(0) = -3.9$.

Under protocol (4), the control input and the states of all agents are shown in Fig. 2. It can be verified that all agents achieve SC. Furthermore, we choose the parameter $\omega_i = 1$ for $i = 1, \ldots, 4$, then the simulation results are shown in Fig. 3, which demonstrate that all agents achieve complete consensus within the prescribed time. Additionally, when the parameters $\omega_1 = \omega_2 = 1$ and $\omega_3 = \omega_4 = -0.6$ are chosen, the corresponding control
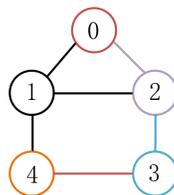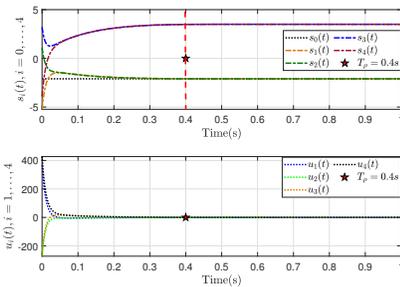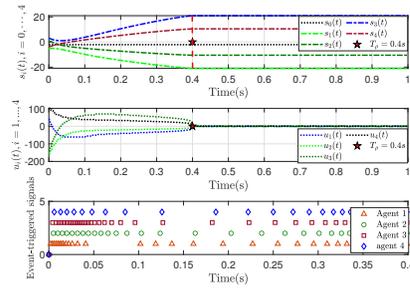

Figure 1. The communication topology.
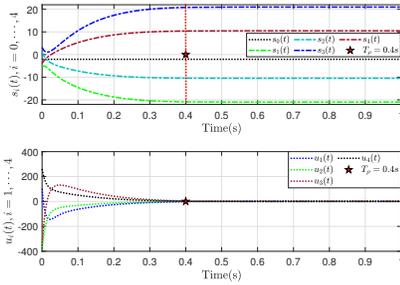
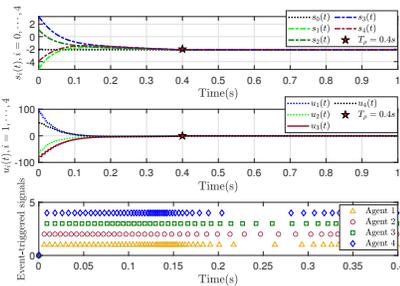**Figure 2.** The trajectories of $s_i(t)$ and $u_i(t)$ with $\omega_1 = -\omega_3 = 0.1, \omega_2 = -\omega_4 = 0.2$.
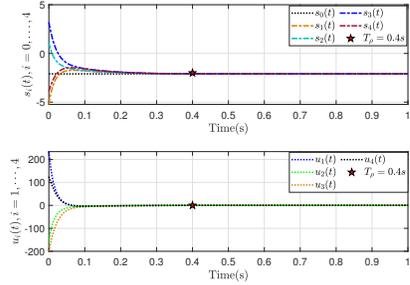
Figure 3. The trajectories of $s_i(t)$ and $u_i(t)$ with $\omega_i = 1, i = 1, \ldots, 4$.
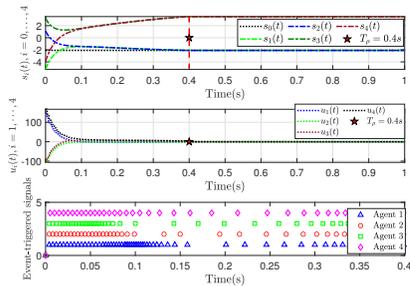

Figure 4. The trajectories of $s_i(t)$ and $u_i(t)$ with $\omega_1 = \omega_2 = 1, \omega_3 = \omega_4 = -0.6$.

Figure 5. $s_i(t)$, $u_i(t)$, and event-triggered instants with $\omega_1 = -\omega_3 = 0.1, \omega_2 = -\omega_4 = 0.2$.

Figure 6. $s_i(t)$, $u_i(t)$, and event-triggered instants with $\omega_i = 1, i = 1, \ldots, 4$.

Figure 7. $s_i(t)$, $u_i(t)$, and event-triggered instants with $\omega_1 = \omega_2 = 1, \omega_3 = \omega_4 = -0.6$.

input and state trajectories of the agents are depicted in Fig. 4. Analysis shows that under the PT condition, all agents achieve cluster consensus. In summary, the results clearly illustrate the influence of scaled factors on the system behavior and effectively validate the feasibility and effectiveness of the control protocol.

Under the control protocol (7), the trajectory obtained from numerical simulations demonstrate that the MAS achieves SC, complete consensus, and cluster consensus within

the PT (see Figs. 5–7). Moreover, the three figures show that the discontinuous nature of the controller updates reduces control costs.

Example 2. Consider a MAS with one leader and four followers labeled as 0 and $1-4$, respectively. The communication topology is shown in Fig. 8. The dynamics of both the followers and the leader are modeled by

$$\dot{s}_i(t) = 0.02 \sin\big(s_i(t)\big) + 0.02 + u_i(t), \quad i = 1, \ldots, 4,$$
$$\dot{s}_0(t) = 0.02 \sin\big(s_0(t)\big) + 0.02.$$

The initial conditions for both the leader and the four followers are set to $s_0(0) = -2$, $s_1(0) = 3.6$, $s_2(0) = 1.6$, $s_3(0) = -4$, and $s_4(0) = -6$. Based on the parameter selection rules outlined in the stability analysis, the numerical value of the design parameters are meticulously chosen as $c_1 = 1.68$, $c_2 = 4.12$, $l_1 = 2.01$, $l_2 = 2.001$, $t_0 = 0s$, $T_\rho = 0.6s$, $\gamma_1 = 0.12$, $c_3 = 1.001$, $c_4 = 0.798$, and $\gamma_2 = 0.13$. Here l_1 and l_2 represent the values of l in Theorems 1 and 2, correspondingly.

The simulation results are shown in Figs. 9–11 and 12–14, corresponding to the PT continuous distributed control and the discontinuous distributed control strategies, respectively. The results demonstrate that under both control schemes, the states of the leader and followers achieve SC, complete consensus, and cluster consensus within the PT. Moreover, the simulations under discontinuous distributed control reveal that the controller exhibits discontinuous triggering behavior.
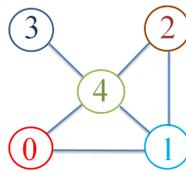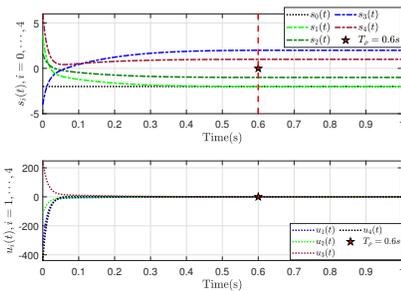

Figure 8. The communication topology.

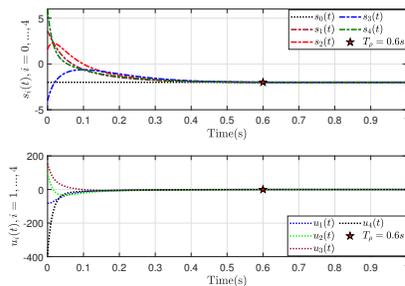**Figure 9.** The trajectories of $s_i(t)$ and $u_i(t)$ with $\omega_1 = 1$, $\omega_2 = 2$, $\omega_3 = -1$, $\omega_4 = -2$.
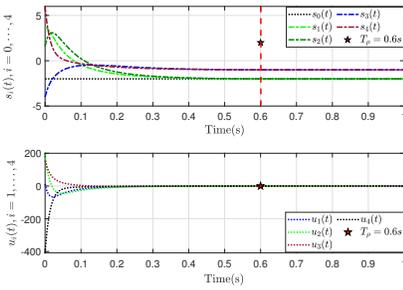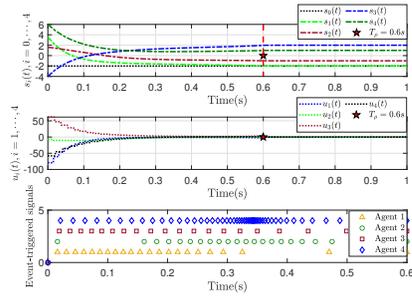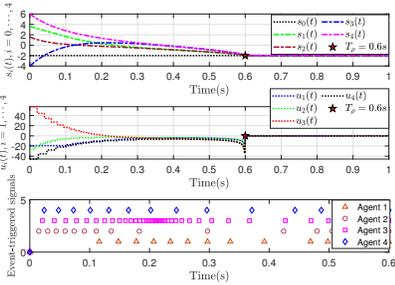

Figure 10. The trajectories of $s_i(t)$ and $u_i(t)$ with $\omega_i = 1$, $i = 1, \ldots, 4$.
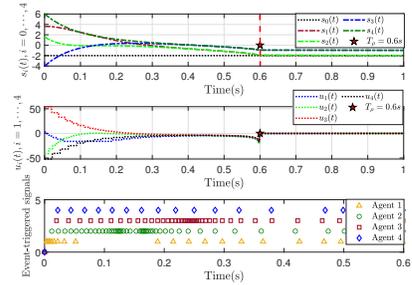
Figure 11. The trajectories of $s_i(t)$ and $u_i(t)$ with $\omega_1 = \omega_2 = 1$, $\omega_3 = \omega_4 = 2$.

Figure 12. $s_i(t)$, $u_i(t)$, and event-triggered instants with $\omega_1 = 1$, $\omega_2 = 2$, $\omega_3 = -1$, $\omega_4 = -2$.

Figure 13. $s_i(t)$, $u_i(t)$ and event-triggered instants with $\omega_i = 1$, $i = 1, \ldots, 4$.

Figure 14. $s_i(t)$, $u_i(t)$, and event-triggered instants with $\omega_1 = \omega_2 = 1$, $\omega_3 = \omega_4 = 2$.

5 Conclusion

This paper investigated the PT SC for first-order nonlinear MASs. First, a continuous control protocol was proposed by introducing the time-varying parameters and normalization coefficients. By leveraging PT stability theory, some sufficient conditions were derived to guarantee L-F SC within the PT. Moreover, to minimize communication demands and resource consumption, an enhanced ET protocol was developed by carefully designing the trigger function. In addition, we also obtained some conditions for achieving L-F SC within the PT interval. To empirically verify the efficacy of the protocols that have been developed, a series of numerical simulations were conducted and presented. In the future work, we will extend our research results to high-order MASs and propose some more feasible control protocols to more complex dynamics.

Conflicts of interest. The authors declare no conflicts of interest.

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