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GORGIAS' ARGUMENT DOES NOT INCLUDE ACTUAL CONDITIONALS

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Abstract. *It can be thought that Gorgias' argument on the non-existence consists of three sentences, the first one being an asseveration and the other two being conditionals. However, this paper is intended to show that there is no conditional in the argument, and that the second and third sentences only appear to be so. To do that, a methodology drawn from the framework of the mental models theory is used, which seems to lead to the true logical forms of these last sentences as well.*

Keywords: *conditional, Gorgias, logical form, mental models theory, non-existence*

Some translations of the argument on the non-existence by Gorgias appear to indicate that it has three sentences and that two of them (the second and the third) are conditionals. Probably, this is so because those two sentences contain, in the original Greek version given by Sextus Empiricus, the word *εἰ* ("if"). However, this paper has two main goals. On the one hand, it will be argued, with the help of the mental models theory (e.g., Hinterecker et al. 2016; Johnson-Laird 2010, 2012, 2015; Johnson-Laird & Byrne 2002; Orenes & Johnson-Laird 2012; Quelhas & Johnson-Laird 2017; Quelhas et al. 2017; Ragni et al. 2016), that the two aforementioned sentences are not really conditionals (at least if it is assumed that the suitable interpretation of the conditional is the material one). On the other hand, it will be attempted to show, with the help of works such as, for example, the one of López-Astorga (2017a), that their real logical forms are obvious and that they can be found from analyses based on the very mental models theory.

Thus, firstly, Gorgias' argument will be addressed. It will be commented on what exactly it provides, and then the problems that must be faced if its two last sentences are interpreted as conditionals from the point of view of classical logic will be described. As explained below, those problems are related to certain unwanted consequences of understanding such sentences as material conditionals.

Secondly, the theses and the most important resources of the mental models theory that are needed to make it evident that the two mentioned sentences are not truly (material) conditionals will be presented. And, of course, after that, the way this theory can reveal that that is really so will be pointed out.

Finally, after a brief description of the methodology used in works such as, for example, that of López-Astorga (2017a), it will be shown how that very methodology, which is also based on the mental models theory, can lead one to the actual logical forms of the second and third sentences in Gorgias' argument. Hence, the next section deals with the original fragment in which this last argument is expressed.

Gorgias' Argument on the Non-Existence

As it is well known and said, the argument has three sentences. One of them is undoubtedly an assertion. However, the other two a priori seem to be conditionals. This can be seen in a clearer way if the original text is taken into account.

It is really, as indicated, a fragment given by Sextus Empiricus. It is exactly in *Adversus Mathematicos* VII, 65, and is to be found, for example, as Fragment 3 in Tapia Zúñiga (1980). The Greek original version is as follows:

> …ἓν μὲν καὶ πρῶτον ὅτι οὐδὲν ἔστιν, δεύτερον ὅτι εἰ καὶ ἔστιν, ἀκατάληπτον ἀνθρώπῳ, τρίτον ὅτι εἰ καὶ καταληπτόν, ἀλλὰ τοί γε ἀνέξοιστον καὶ ἀνερμήνευτον τῷ πέλας.
>
> ("…firstly, nothing is; second, if something were, it could not be understood by a human being; thirdly, if it could be understood, of course, it could be neither transmitted nor accounted for to a near person".)

Obviously, the presence of *εἰ* leads one to think about conditionality in the second and the third points. It is true that, given that *εἰ* adjoins *καὶ* in those two cases, other translations can be thought, for example, "even if" instead of just "if". Nevertheless, there is no doubt that, in many of those possible alternative translations, in principle, the conditional sense of the sentences would be preserved.

But, as claimed, this can be a problem, at least if the conditional is understood as in classical logic, that is, in a material way. And this can be noted in an evident manner if we consider possible logical forms for the previous sentences, since they can allow deriving conclusions that appear to be, if not inconsistent, at least different from what Gorgias seems to mean. Clearly, the first sentence causes no difficulty, as its form is simple. Let it be, for example, "““ the universal quanfier, "¬" the negation, and "B" a predicate denoting "to be". Then, the logical form of the first sentence can be, in first-order predicate logic, this one:

(1) $\forall x\ (\neg Bx)$

The problems, as pointed out, begin with the second sentence. If we assume now that "→“ represents conditional relationship and that "*U*" refers to "to be able to be understood by a human being", this formula could be attributed to it:

(2) $\forall x\ (Bx \rightarrow \neg Ux)$

Nonetheless, if this is so, it is clear that, in first-order predicate logic, (2) in turn enables to derive a formula such as the following (where "*a*" is a constant):

(3) $Ua \rightarrow \neg Ba$

And (3) is undoubtedly controversial, since it provides that, if *a* (which stands for any being here) can be understood by a human being, then it is not (or, if preferred, it does not exist). Thus, beyond the fact that this idea does not appear to make sense, it is obvious that it does not describe what Gorgias really wanted to say either. Hence a formula such as (2) is not the best logical form that can be given for the second sentence in this argument.

Indeed, the conditional as understood in classical logic allows drawing, in propositional calculus, formulae such as $\neg q \rightarrow \neg p$ from formulae such as $p \quad q$, which in turn means that, in first-order predicate calculus, it is absolutely correct to conclude (3) from (2) (to check the reasons why, based on works such as those of Gentzen (1934, 1935), the general structure of the derivations in propositional logic are also correct in first-order predicate logic (see, e.g., Deaño 1999)). However, this problem is compounded if we pay attention to the third sentence, as exactly the same difficulty can be observed in it. To note that, only three additional assumptions are necessary: that "$\wedge$" is conjunction, that "T" represents "to be able to be transmitted", and that "A" denotes "to be able to be accounted for to a near person". In this way, the logical form of the third sentence can be:

(4) $\forall x\, (Ux \rightarrow (\neg Tx \quad \neg Ax)$

Nevertheless, again, in the same manner as (3) can be derived from (2), it is now possible to draw this formula from (4) (where "$\wedge$" refers to disjunction):

(5) $(Ta \vee Aa) \rightarrow \neg Ua$

The latter expresses that, if *a* can be transmitted or *a* can be accounted for to a near person, then a human being cannot understand it. So, apart from the fact that (5) is also hard to understand, since it is difficult to accept that something that is transmitted or accounted for to other person cannot be understood by a human being, it presents, as in the case of (3), an idea that does not seem to correspond to the real Gorgias" thought.

But maybe the key is that (2) and (4) are not truly suitable logical forms of the second and third sentences in Gorgias' argument on the non-existence. A possibility can be that there really is no conditional relationship in them and that, therefore, a symbol such as "$\rightarrow$" should not be used to formally represent them. The mental models theory allows exploring this last possibility and, for that reason, the next section is devoted to what that theory claims about the sentences with "if".

The Mental Models Theory and the Conditional

The mental models theory is a psychological approach trying to show, among other important aspects, the way people tend to think when faced with connectives such as those of classical logic. Nonetheless, as far as the aims of this paper are concerned, it is necessary to consider only its account of the conditional. According to it, when an individual thinks about a conditional such as "if A then B", he usually focuses on possibilities that are consistent with it. If all of them are identified, such possibilities are the following (see, e.g., Quelhas et al. 2017: 1004):

(6) A & B
Not-A & B
Not-A & Not-B

These are three possible scenarios: (i) A and B happen, (ii) only B happens, and (iii) neither A nor B happen. However, although this can remind a truth table of classical logic, these scenarios are iconic, and to support the definition of this last concept, the proponents of the theory often resort to accounts such as that of Charles Sanders

Peirce (1931-1958). Johnson-Laird (2012), for example, does that. Nevertheless, what is important in this way is that, because they are iconic, the possibilities in (6) describe complete alternative scenarios with coherence and consistency, which means that, in the real cases, some of them can disappear, be rejected, or be changed. A clear example in this regard can be the following, which is taken from Johnson-Laird and Byrne (2002: 663; see also, e.g., López-Astorga 2016: 291, 2017b: 107):

> "If you are interested in seeing Vertigo, then it is on TV tonight".

This sentence does not admit the three possibilities in (6), but only

(7) You are interested in seeing Vertigo & It is on TV tonight
You are not interested in seeing Vertigo & It is on TV tonight

The third possibility (Not-A & Not-B) cannot be taken into account, because, from what is said by the speaker, it is absolutely clear that Vertigo is on TV tonight, and that fact cannot be denied (we cannot think about a case of Not-B). Thus, given that the cases in which a conditional is true in a truth table of classical logic are those included in (6), which, as it is known, correspond to those of the material interpretation of the conditional, and in (7) the last one is missing, it is evident that the previous example coming from Johnson-Laird and Byrne (2002) is not actually a conditional, even though the word "if" appears in it, at least, if we understand the conditional just in a material way.

The name that Johnson-Laird and Byrne (2002) give to that kind of sentence is "relevance", and it has already been used to analyze theses proposed by ancient philosophers. For example, López-Astorga (2017b) argues that the thesis by Thales of Miletus related to the fact that all of beings have souls, including the beings that are usually thought not to have a soul, can be better interpreted, if it is linked to a set of possibilities such as that of (7), that is, to a set of possibilities with just these two elements: (A & B) and (Not-A & B). Nonetheless, the point here is that it can be useful to better understand Gorgias' argument as well, in particular, obviously, its second and third sentences. If we consider the second one firstly, we can note that, certainly, it does not refer to possibilities such as those of (6), but to a set with a structure similar to the one of (7), that is, to this set:

(8) Something is & It cannot be understood by a human being
Something is not & It cannot be understood by a human being

The third possibility of (6) cannot be accepted here, because, as pointed out, it does not seem possible that something does not exist and, however, it can be understood, and this is so independently of the fact that, as also indicated, this last idea does not appear to be a thesis truly held by Gorgias.

Furthermore, something similar happens with the third sentence in his argument. (6) is not the most suitable set for it but, again, another one akin (at least in its structure) to that of (7):

(9) Something can be understood & It can be neither transmitted nor accounted for to a near person
Something cannot be understood & It can be neither transmitted nor accounted for to a near person

Clearly, the third combination in (6) must be rejected here too, and the reason for that has also been mentioned above. It is very difficult to think about a scenario in which something cannot be understood and, at the same time, it can be transmitted or accounted for to a near person, and it should be added to this, as claimed too, that that does not seem to be an idea actually supported by Gorgias either.

Therefore, it appears that the second and third sentences included in the argument by Gorgias that is being considered in this paper are not real material conditionals, even if they have the word *εἰ* ("if"). But, if they are not material conditionals, which logical forms could we attribute to them in classical logic? Perhaps the mental models theory can help us in this regard as well.

The Logical Forms of the Second and Third Sentences in Gorgias' Argument on the Non-Existence

The way to find the actual logical forms of such sentences can be clear if works such as that of López-Astorga (2017a) are taken into account. Although the proponents of the mental models theory often ignore logical forms and state that human reasoning is made without resorting to them (see, e.g., Johnson-Laird 2010 or Orenes and Johnson-Laird 2012), the main idea in these works is that the iconic possibilities to which sentences refer can be deemed as combinations in a truth table of classical logic in which the formula that is being looked for is true. Thus, from such possibilities, we can come to well-formed formulae in this last logic. However, maybe an example can help to better understand this method.

Let us think about (6). Its three possibilities are linked by means of "&", which suggests that it can make sense to consider them to be logical conjunctions. In fact, although they do not use the word "logical", the adherents of the mental models theory explicitly state that the iconic possibilities represent conjunctions (e.g., Quelhas et al. 2017: 1004). But, if we assume that those possibilities are logical conjunctions, we immediately obtain three formulae:

(10) $p \wedge q$

(11) $\neg p \wedge q$

(12) $\neg p \wedge \neg q$

Thus, given that they are possibilities or alternative scenarios, the next step is to join them by means of disjunctions, which leads us to this new formula:

(13) $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

And finally it is only necessary to build the truth table for (13) in order to find the combinations in which it is true and it is false. If that is done, we can see that, as it is obvious and reminded by López-Astorga (2017a: 36), (13) is true when either p is false or q is true, that is, in exactly the same cases as

(14) $p \rightarrow q$

Hence it can be thought that (14) is the suitable logical form here. Of course, other formulae are possible, since, for example, the combinations in which (13) and (14) are true are also, as it is known, the same in which a formula such as this one is true:

(15) $\neg p \vee q$

However, in any case, it is clear that this can be a method to find underlying logical forms, and, for this reason, it appears that it is worth using it to detect the real form of the *relevance* type sentences. If, as said, the possibilities corresponding to this last kind of sentence are (A & B) and (Not-A & B), based on what has been explained, it is evident that they allow constructing this formula:

(16) $(p \wedge q) \vee (\neg p \wedge q)$

That is, a formula that is always true when q is true, and that is only false when q is so. Therefore, it can be thought that its actual logical form is as follows:

(17) $(p \vee \neg p) \rightarrow q$

(Although without resorting to this method based on the truth tables of classical logic, a similar logical form is assigned to *relevance* in, e.g., López-Astorga 2016).

Certainly, (16) and (17) are true and false in the very same cases, and the latter seems to express the sense of *relevance* better than (14), since what it states is that q is always true, no matter what happens with p, that is, whether or not p is true.

Of course, more logical forms can be indicated for *relevance*, for example,

(18) $(p \vee \neg p) \wedge q$

(19) q

However, given that these four last formulae – (16), (17), (18), and (19) – are true if and only if q is so, it can be thought that all of the forms that can be proposed for *relevance*, the mentioned ones being just some possible examples, emphasize that, as said, what is truly important is that q obtains, and that the truth-value of p is irrelevant.

But, on the other hand, based on works such as the one of Deaño (1999), which, as pointed out, describes the links between propositional calculus and first-order predicate calculus, versions of (17), (18), and (19) in first-order predicate logic can be given. Such versions can be, respectively, the following:

(20) $\forall x\ ((Px \vee \neg Px) \rightarrow Qx)$

(21) $\forall x\ ((Px \vee \neg Px) \wedge Qx)$

(22) $\forall x\ (Qx)$

And, because, as shown with (8) and (9), the second and third sentences in Gorgias' argument on the non-existence are instances of *relevance*, and it appears that such sentences can be better expressed in the language of first-order predicate logic, at this point, it can be stated that possible logical forms of them have been found. Indeed, (20), (21), and (22) can provide possible formal structures for those sentences more suitable than (2) and (4). In particular, keeping the meanings of "*B*" and "*U*" above, the logical forms for the second sentence could be these ones:

(23) $\forall x\ ((Bx \vee \neg Bx) \rightarrow \neg Ux)$

(24) $\forall x\ ((Bx \vee \neg Bx) \wedge \neg Ux)$

(25) $\forall x\ (\neg Ux)$

Any of these logical forms eliminates the problems described for (2), and clearly reflects what Gorgias wanted to say with his second sentence: whether or not something is, it cannot be understood by a human being. From this point of view, it is even more evident that that sentence does not indicate a material conditional relationship between being and being able to be understood by a human being. The relationship expressed by the sentence is obviously different from that one.

As far as the third sentence is concerned, something similar can be argued. On the one hand, keeping the meanings of "*T*" and "*A*" too, its logical forms could be the following:

(26) $\forall x\ ((Ux \vee \neg Ux) \rightarrow (\neg Tx \wedge \neg Ax))$

(27) $\forall x\ ((Ux \vee \neg Ux) \wedge (\neg Tx \wedge \neg Ax))$

(28) $\forall x\ (\neg Tx \wedge \neg Ax)$

(26), (27), and (28) also remove the difficulties of (4) mentioned above, and appear to refer to Gorgias' thought in a clearer way. The point is now that, whether or not something is understood by a human being, what is indisputable is that it cannot be transmitted and it cannot be accounted for to a near person. So, in the same way as the case of the second sentence, this offers even further support to the idea that there is no material conditional relationship here either. Certainly, at a minimum, there is no relationship of that kind between the antecedent and the consequent that the sentence has in natural language (or, if preferred, in his Greek original version), that is, between the fact of being able to be understood and the fact of not being able to be transmitted and accounted for.

Therefore, Gorgias' argument gives stronger evidence for the thesis supported in papers such as that of Johnson-Laird and Byrne (2002), that is, that the fact that a sentence in natural language includes the word "if" does not necessarily imply that that sentence expresses a material conditional relationship between its clauses. Of course, that thesis is not new, but it seems that the mental models theory shows why other interpretations can be correct and how we can come to them. In this way, it can be very interesting to note, for example, that, while the word *εἰ*, that is, the Greek word equivalent to the English word "if", appears in the second and third sentences of the fragment presenting Gorgias' argument on the non-existence authored by Sextus Empiricus and cited above, that word is translated into Spanish by Tapia Zúñiga (1980: 1) as "*aunque*" ("although") in both cases. Certainly, it is not incorrect to translate *εἰ* as "although" in certain contexts or in the case of certain grammatical constructions, but there is no doubt that it is clearly relevant that Tapia Zúñiga uses the Spanish word equivalent to "although" in his translation.

Conclusions

As explained, for example, in López-Astorga (2017a), regardless of the real intentions and goals of the mental models theory, its analysis methodology is useful to identify the actual logical forms of sentences. That has been checked here by means of an argument by the sophist Gorgias. However, it is evident that that methodology can help us find the underlying logical form of any sentence in any language and in any context. And this is so because it takes into account not only syntactic factors (the way the words are combined in sentences), but also semantic and pragmatic factors (it pays attention to the iconic possibilities that can be really accepted).

On the other hand, it can help understand and interpret what thinkers in ancient times meant too. As said, this is not the first paper in that direction. In, for example, López-Astorga (2017b), the mental models theory is taken as a methodological resource as well, the aim being to deal with the fragment by Diogenes Laërtius in which it is stated that Thales of Miletus assigned a soul even to beings without a soul. Coincidentally, as also pointed out, López-Astorga comes to the conclusion that the sentence expressing this last idea is also a *relevance* type sentence. However, maybe the most important point is that analyses akin to the one offered here have already been carried out, and that hence they can make sense.

So, it seems that there are two relevant lines of study that are opening up. The first one is related to the search for an algorithm of any kind to detect the true logical forms corresponding to sentences. The second one is connected to the development of an interpretative methodology to come to a better understanding of ancient texts. The mental models theory can be the key framework in both lines, which are equally interesting and deserve further exploration to the same extent. Precisely, this paper appears to have shown this last point, since it has revealed that it is possible to discover the actual logical form of sentences as puzzling as those of Gorgias' argument on the non-existence, and it has explained what exactly this thinker wanted to express in a very clear way.

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GORGIJAUS SAMPROTAVIME NĖRA MATERIALIOSIOS IMPLIKACIJOS

Miguel López-Astorga

Santrauka. Iš pirmo žvilgsnio atrodo, kad Gorgijaus samprotavime apie neegzistavimą esama trijų sakinių, iš kurių pirmasis – kategoriškas tvirtinimas, o likę du – sąlyginiai teiginiai. Šio straipsnio tikslas yra parodyti, kad du pastarieji iš tikrųjų nėra materialiosios implikacijos sakiniai, jie tokie tik atrodo. Kad tą įrodyčiau, pasitelkiu mentalinių modelių teoriją, kuri leidžia išryškinti tikrąją dviejų minėtų sakinių loginę formą.

Pagrindiniai žodžiai: materiali implikacija, Gorgijus, loginė forma, mentalinių modelių teorija, neegzistavimas

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