

Apparent L-Falsity and Actual Logical Structures

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Abstract. In 2012, Orenes and Johnson-Laird found interesting results from the cognitive point of view but problematic if analyzed under methods such as the semantic one of extension and intension presented by Carnap. The main difficulty in this way is that Orenes and Johnson-Laird showed that people tend to accept, in the case of certain inferences, conclusions that, by themselves, are, according to the aforementioned semantic method, false in absolutely all of the state-descriptions that can be imagined. However, in this paper, a way to overcome that difficulty is proposed. That way is based upon the idea that the real logical forms of the conclusions accepted by the participants in Orenes and Johnson-Laird's experiments were not the apparent ones, but they corresponded to other very different structures that can be true in some state-descriptions.

Keywords: L-falsity, logical form, semantics, state-description, truth table

Tariamas L-klaidingumas ir tikrosios loginės struktūros

Santrauka. 2012 metais atliktų tyrimų Orenes ir Johnsonas-Lairdas pasiekė tokių rezultatų, kurie buvo įdomūs žvelgiant iš kognityvinės perspektyvos, tačiau problemiški analizuojant tokiais metodais kaip Carnapo ekstensijos ir intensijos semantinis metodas. Esminė kliūtis kyla iš to, kad, anot Orenes ir Johnsono-Lairdo, žmonės yra linkę tam tikrais atvejais priimti tokias išvadas, kurios yra klaidingos esant bet kokiam įmanomam dalykų padėties aprašui. Straipsnyje siūlomas būdas įveikti šią kliūtį. Šis būdas remiasi mintimi, kad Orenes ir Johnsono-Lairdo eksperimento dalyvių priimtų išvadų tikroji loginė forma buvo ne tokia, kaip atrodė iš pirmo žvilgsnio, o atitiko kitokią, labai skirtingą struktūrą, kuri gali būti teisinga pagal kai kuriuos dalykų padėties aprašus.

Pagrindiniai žodžiai: L-klaidingumas, loginė forma, semantika, dalykų padėties aprašas, teisingumo lentelė

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Introduction

As it is well known, Carnap (1947) offered a method intended to analyze and explain how human language truly is. From this perspective, one might hope that a method of that kind is able to account for all of the linguistic phenomena that can happen. But what this idea really achieve is to bring important challenges to that method, since the contemporary literature on human cognition seems to show several situations or facts that appear to be incompatible with the general principles or some essential notions of it.

In this paper, one of such facts will be addressed. It refers to results such as those found in papers such as the one of Orenes and Johnson-Laird (2012). Those results are relevant because they seem to prove that individuals use and accept conditionals sentences that should be totally rejected from the framework raised by Carnap (1947). Nevertheless, what will be argued here is that Orenes and Johnson-Laird's (2012) results are not actually a proof that Carnap's method is incorrect or wrong, and that there are, at least, one way to explain them from that method without requiring its modification.

Thus, to argue in that direction, the first section will explain in detail the particular fact discovered by Orenes and Johnson-Laird and that appears to be incoherent with Carnap's (1947) method. Then the aspects of that method that, in principle, seem to conflict with Orenes and Johnson-Laird's (2012) findings will be presented, as well as an account of why that appears to be so. Third, it will be shown that the problem addressed in this paper is part of a greater issue related to the possible links between logic and human cognition. Finally, the way to solve the problem will be indicated following works such as López-Astorga (2017), which, as it will be described, propose manners to identify the real logical forms of sentences.

Orenes and Johnson-Laird (2012) and their Results Related to Conditionals

Really, Orenes and Johnson-Laird (2012) assume the mental models theory (see, e.g., for its first versions, Oakhill & Garham 1996, and, for more recent versions, Khemlani et al. 2018), and hence their study has the aim to support that theory. Nonetheless, this paper will present neither a comprehensive structure of their research nor a detailed analysis of the way they designed and proposed their experimental tasks. The main important point here is just a partial result that, although they do not mention it (since their interests have a different goal), seems to be undoubtedly inconsistent with the method proposed by Carnap (1947). That result, which, by the way, is clearly coherent with the basic ideas of the mental models theory and, indeed, appears to give evidence in favor of it (obviously, further explanation about this point will not be offered here because it would be to go beyond the aims indicated in the previous section), is that people tend to accept and use conditionals sentences such as these ones:

[I] "...if Maria got the furniture then she didn't get the table..." (Orenes & Johnson-Laird 2012: 374).

[II] "...if Paco bought meat then he didn't buy the chicken..." (Orenes & Johnson-Laird 2012: 374).

Truly, much more examples are to be found in the paper by Orenes and Johnson-Laird (2012). However, [I] and [II] are illustrative enough to show what this paper is intended to make it explicit. [I] and [II] share the fact that one of the combinations of possibilities for which the conditionals are true in classical logic is impossible in their cases. As it is well known, in this last logic, it is absolutely correct to state that

[III] A conditional such as (if P then not-Q) is true if, and only if, either (p and not-Q) is true, or (not-P and not-Q) is true, or (not-P and Q) is true.

Nevertheless, as it can be checked, the last disjunct in [III] is possible neither in [I] nor in [II]. Certainly, in [I], it is not possible not to get the furniture (not-P) and get the table (Q) because a table is furniture. Likewise, in [II], is not possible not to buy meat (not-P) and to buy the chicken (Q) because a chicken is meat (of course, this beyond the fact that pragmatics can reveal in particular occasions that 'furniture' does not include 'table' or that 'meat' does not include 'chicken' in the sentences).

This is not a problem from the framework of the mental models theory at all. In fact, this last theory, which is, as said, the one supported by Orenes and Johnson-Laird, considers many phenomena similar to the one described as the reason why people sometimes come to conclusions contrary to logic and, ultimately, that human reasoning is not necessarily in accordance with logic (this point is also developed and further accounted for in papers such as, e.g., Quelhas & Johnson-Laird 2017, or Quelhas et al. 2010). Nonetheless, it does seem to be clearly a problem for Carnap's (1947) method. The next section tries to explain why.

Do people accept and use L-false sentences?

Given that Carnap's semantic method of extension and intension is complex and includes subtle details, it is only interesting here the particular aspect of it that conflicts with sentences such as [I] and [II]. That aspect is linked to the Carnap's (1947) idea about L-concepts. According to him, there are concepts that are true or false because the language used provides that, irrespective of what occurs in reality. Those are L-concepts, which depend upon an essential notion in modal logic: state-description.

As Carnap (1947) says, a state-description is akin to what in modal logic is deemed as a possible world. Thus, a concept is L-true if, and only if, it "is true in all of the state-descriptions that can be thought" (López-Astorga 2018: 406). Indeed, that is basically what, for example, Carnap (1947) claims in his Definition 2.2. However, another important point in this regard is that, obviously, if a sentence is L-true, its negation has to be L-false,

that is, false in all of the state-descriptions that can be thought (see, e.g., Definition 2-3-a in Carnap 1947).

So, it is clear, and Carnap (1947) also explicitly refers to that fact, that L-true concepts are evidently very similar to the Kantian analytic truths, and that, therefore, tautologies such as [not-(P and not-P)] are L-true, as well as contradictions such as (P and not-P) are L-false.

But, as indicated, what gives their nature to L-concepts is really language, and that means that, beyond logical truths and falsities, a particular sentence can be L-true or L-false because of exclusively linguistic criteria. That is the case of, for example, definitions. Certainly, if a definition is correct in the language, it can be said that it is true in all of the state-descriptions. In this way, there is no doubt that that is the case for a definition such as, for example, this one:

[IV] Dogs are mammals.

Given that [IV] is a correct definition in the language in which it is expressed (in this case, English), it is, following the semantic method of extension and intension, L-true too. And here is exactly where the problem with sentences such as [I] and [II] seems to be. As it is well known and it has been pointed out above, by virtue of the definitions of ‘table’ and ‘chicken’, it is possible and correct to state in English sentences such as these ones:

[V] A table is a kind of furniture.

[VI] Chicken is a kind of meat.

Hence, by virtue of those very definitions, it can be assumed that [V] and [VI] are L-true as well. And all of this is a problem because, as it is also well known, classical logic provides this equivalence:

[VII] (if P then not-Q) if, and only if, (if Q then not-P).

Clearly, [VII] allows transforming [I] into:

[VIII] If Maria got the table, then she did not get the furniture.

And [II] into:

[IX] If Paco bought the chicken, then he did not buy meat.

And transformations such as these ones cause an important difficulty: neither there is any state-description in which a table can be got without getting at once furniture nor there is any state-description in which chicken can be bought without buying at once meat. So, one might think that [VIII] and [IX], by virtue of [V] and [VI], are L-false. However, in spite of this, as shown by Orenes and Johnson-Laird (2012), people usually accept and use [I] and [II], which, as explained, logically implies to accept and use [VIII] and [IX].

A quick and intuitive way to try to solve this problem is to claim that, from a strictly logical point of view, [VIII] and [IX] are not L-false at all. Following [III], there are state-descriptions in which they can be true, and, accordingly, they are not false in all of them. In particular, [VIII] can be true in the state-descriptions in which Maria does not get the table, and [IX] can be so too in the state-descriptions in which Paco does not buy chicken, since [III] clearly makes it explicit that any conditional is true whenever its antecedent is false (as provided by the material interpretation of the conditional).

Nevertheless, although even it could be also argued that the acceptance of sentences such as [I] and [II] (and, therefore, logically, such as [VIII] and [IX]) by the participants in the experiments carried out by Orenes and Johnson-Laird (2012) happens in very particular circumstances created by certain experimental conditions, which, evidently, have an influence on their choices, the main point here is that, as said, L-concepts are so because of linguistic criteria. And, obviously, purely semantic or linguistic criteria can lead to the negations of [VIII] and [IX], that is, to:

[X] not-(if Maria got the table, then she did not get the furniture).

[XI] not-(if Paco bought the chicken, then he did not buy meat).

And the reason is also clear: as pointed out, [V] and [VI] reveal that it is possible neither to get a table without getting furniture nor to buy chicken without buying meat. Hence, it can be claimed that semantics, by virtue of the meanings of ‘table’, ‘furniture’, ‘chicken’, and ‘meat’, leads to necessary assume that [X] and [XI] are L-true. Nonetheless, as accounted for too, the negation of a L-true sentence is L-false under the framework of the semantic method of extension and intension, which means that, if [X] and [XI] are L-true, then [VIII] and [IX] are L-false under that very method, and that, therefore, the problem remains.

Of course, another possible way to try to remove these difficulties can be to raise the idea that all of this is caused by the particular characteristics of the language being used, that is, English, and that, for example, other languages do not admit conditionals of the type of [I] and [II]. However, this is undoubtedly incorrect, since the following translations into the languages indicated between brackets make sense to the native speakers of such languages, who, indeed, often use and accept sentences such as [I] and [II] as well:

[I.1] (Spanish) si María compró mobiliario, entonces ella no compró la mesa.

[I.2] (Portuguese) se Maria pegasse a mobília, então ela não pegaria a mesa.

[I.3] (French) si Maria a eu les meubles, alors elle n’a pas eu la table.

[II.1] (Spanish) si Paco compró carne, entonces él no compró el pollo.

[II.2] (Portuguese) se Paco comprasse carne, então ele não compraria o frango.

[II.3] (French) si Paco a acheté de la viande, alors il n’a pas acheté le poulet.

In fact, in their paper, Orenes and Johnson-Laird (2012) indicate that the original tasks presented in their experiments were in Spanish (the participants were Spanish speakers). And even more can be said. Several studies suggest that, in the case of languages so close as romance languages such as, for example, Spanish, Portuguese, and French, a native speaker of one of them can interpret, in certain contexts, what is transmitted in other of those languages with a minimal effort and study (e.g., Bonvino et al. 2015). So, one might think that any of the sentences [I.1], [I.2], [I.3], [II.1], [II.2], and [II.3] not only can be perfectly understood by native speakers of the language corresponding to it, but also, in certain circumstances, even by a native speaker of other of the languages pointed out between brackets (and, as indicated, with a minimal effort or study).

Hence, sentences such as [I] and [II] seem to be an important challenge to the semantic method of extension and intension by Carnap (1947). However, there is, at least, one possible way to overcome that challenge. It is presented below. But, before that, it is shown that, actually, all of this is a part of a more general important problem.

Logic and Human Cognition

Indeed, the problem is much wider. Carnap's (1947) framework is logical and perhaps the first question to ask, even before studies such as this one are carried out, is whether or not the human way to think and process information is ruled by logic.

The truth is that to answer that question is not easy, since experimental evidence tends to make it explicit that people do not always follow logical rules. That is the case of, for example, logical schemata such as these ones:

[XII] P. So, either P or Q.

[XIII] not-Q. So, if P then not-Q.

[XIV] not-P. So, if P then Q.

As it is well known, [XII], [XIII], and [XIV] are absolutely correct logical inferences. Nevertheless, as the experimental results reported by Orenes and Johnson-Laird (2012) show, people do not always consider them to be valid.

One might think that this is exactly the opposite problem to the one that is being addressed here, since [XII], [XIII], and [XIV] are L-true, and this paper is about L-false sentences. However, the manner to face both of these problems (that people may not accept certain L-true sentences and that they may accept certain L-false sentences) can be very similar.

In López Astorga (2017) and works such as those cited there are the keys to solve both problems. Nonetheless, the next section applies such keys just to the one of L-false sentences, since the case of L-true sentences has been the mainly studied one in that very literature.

Mental Models, Truth Tables, and Logical Forms

The way to solve the problem of L-false sentences such as [I] and [II] is, as said, based upon the framework proposed in works such as, for example, López-Astorga (2017). That approach comes from an old idea: there is no an exact or clear correspondence between the words in natural language and the connectives in classical logic (see, e.g., Deaño 1999). Thus, what López-Astorga really proposes is a method to identify the real logical forms that can be attributed to sentences in natural language.

This, in the case of the problem dealt with in this paper, means that the key of the difficulties can be the fact that, although they include the words ‘if’ and ‘then’, [I] and [II] might not be actual conditional sentences (or at least they might not be that in the logical sense of the conditional relationship). To check doubts such as that one, López-Astorga’s (2017) framework begins by considering important theses of the mental models theory, as, for example, that sentences often refer to sets of possibilities (see, for this particular point in the mental models theory, e.g., Khemlani et al. 2017). In this way, following López-Astorga, if such possibilities are deemed as rows in truth tables, it is easy to identify the real logical forms of the sentences.

Of course, Carnap’s (1947) method has already been related to the mental models theory in different manners (see, e.g., López-Astorga 2018). Nevertheless, the use of the mental models theory here appears to be clearly different, since, beyond the fact that this section resorts to it only in an indirect way (what mainly is interesting here is, as said, López-Astorga’s, 2017, approach, which in turn is based on some theses of the mental models theory), the following argumentation is not mainly intended to find links between the mental models theory and the semantic method of extension and intension, which is what is usually made in the literature. The aim here is, as indicated, just to solve a problem that this last method appears to have, and that can be done, as stated, with the help of López-Astorga’s framework.

Indeed, what López-Astorga (2017) exactly proposes is that the possibilities that can be assigned to a sentence are considered as the rows in which that sentence is true in its truth table. Thus, this, in the cases of [I] and [II], has a clear consequence. As indicated, the last disjunct in [III] is impossible for them. So, their only possibilities are the other two disjuncts included in the second clause of [III], that is, (P and not-Q) and (not-P and not-Q). But, if these two last possibilities are deemed as the only rows in a truth table in which sentences such as [I] and [II] can be true, it is evident that the logical form of these last sentences cannot be the conditional one.

What really the two possibilities linked to them, that is, (P and not-Q) and (not-P and not-Q), express is that, whatever happens, (Q) is not true. In other words, those possibilities show that the antecedent in [I] and [II] is irrelevant, and that (not-Q) is the case and occurs in any event, no matters which the truth value of the antecedent is.

However, if this is so, it is evident that the conclusion is that [I] does not intend to provide a conditional relationship between ‘furniture’ and ‘table’, but only to state that Maria does not get the table, whether or not she gets furniture. Likewise, it is obvious that

[II] does not intend to provide a conditional relationship between ‘meat’ and ‘chicken’ either, but only to state that Paco does not buy the chicken, whether or not he buys meat.

And all of this leads to think that what is truly important in [I] is just its apparent consequent, that is, that Maria does not get the table, and that what is truly important in [II] is also just its apparent consequent, that is, that Paco does not buy the chicken. Hence it seems to be justified to assume that, as it can be derived from works by López-Astorga such as, for example, López-Astorga (2017), the most appropriate logical form for [I] and [II] is not (if P then not-Q), but just the negation of a fact, which, in the case of [I], is that Maria gets the table and, in the case of [II], is that Paco buys the chicken.

But, if, in their deep logical structure, [I] and [II] are simply negations of facts, it is clear that the problem described above disappears. Negations of facts such as those pointed out can be neither L-false nor L-true. Accordingly, from this perspective, it cannot be said that the participants in Orenes and Johnson-Laird’s (2012) study tended to admit L-false sentences. They only accepted negations of certain facts for which one might think that there is always at least one state-description in which they occur.

Conclusions

Therefore, phenomena such as those described by Orenes and Johnson-Laird (2012) are not a problem at all for the semantic method of extension and intension provided by Carnap (1947). As shown, there is at least one possible solution for their difficulties.

Basically, as also accounted for, that solution consists of considering the logical form of the sentences indicated by Orenes and Johnson-Laird to be very different from the conditional logical form, and this even though such sentences include the words ‘if’ and ‘then’, and hence the conditional one a priori seems to be their clear form. This idea, as argued too, allows removing the contradictions that can be associated to those sentences. And its formal process is easy to describe: given a sentence, the real possibilities to which it refers are identified. Then, it is assumed that such possibilities are the rows in which the sentence is true in a truth table. Finally, that truth table allows recovering the real logical form of the sentence.

Of course, one might think that perhaps more solutions are possible. Nevertheless, in any case, the one described above can be deemed as sufficient to support the thesis that to resort to Carnap’s (1947) method keeps being useful to make certain linguistic and philosophical analysis. Several examples can be found in this way, one of them being the paper by López-Astorga (2018). In that paper, an essential axiom that is assumed in the system Prior uses to demonstrate the famous ‘Hintikka’s theorem’ (Prior, 2012; see also Øhrstrøm et al. 2012) is reviewed by means of the method of extension and intension as well, and the conclusion is that it can be said that the axiom is undoubtedly justified from the general framework of the method, which is interpreted as additional evidence in favor of both the aforementioned axiom and the system assuming it.

So, it is obvious that the method can continue to be an aid in several areas and to achieve different purposes. There is no doubt that more problems such as the one analyzed in this paper can keep arising. Nonetheless, the possibility to face them in a similar way as it has been done here always exists. Furthermore, such new difficulties and their attempts to resolve them can also be really useful. The processes to eliminate those problems can lead, as their results, to improvements or nuances of the method, which in turn can even give it a wider scope and a stronger potential. Hence, in principle, it can be claimed that the fact that the semantic method of extension and intension sometimes has to consider challenges coming from the specialized literature is not a great inconvenience for it. Apart from the fact that its basic and general framework might not be influenced by those challenges, trying to overcome them can be truly positive for its development.

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