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Progressive Confirmation of Two Mental Systems

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Abstract. From current cognitive science, some theories propose that the human mind includes two systems: a system leading quick intuitions and a system ruling slow logical reasoning. Given that the systems are in the mind, one might think that their existence is difficult to check. This paper presents a way to gradually confirm the hypothesis about the two systems. It adopts the conception of the two systems as presented by the theory of mental models. Moreover, following Carnap's proposal of reduction, the paper describes two procedures to incrementally confirm the hypothesis. One of them investigates human performance at inferences in the form of Modus Tollendo Tollens. The other one investigates inferences from an exclusive disjunction having three disjuncts.
Keywords: confirmation; exclusive disjunction; mental model; Modus Tollendo Tollens; reduction

Laipsniškas dviejų mentalinių sistemų patvirtinimas

Santrauka. Kai kurios šiuolaikinio kognityvinio mokslo teorijos teigia, kad žmogaus prote veikia dvi sistemos: sistema, vykdanti greitą intuityvų mąstymą, bei sistema, vadovaujanti lėtam logiškam mąstymui. Būtų galima manyti, kad šių sistemų egzistavimą patikrinti sudėtinga. Šiame straipsnyje pateikiamas būdas palaipsniui patvirtinti šių dviejų sistemų egzistavimą. Pasitelkiamas dviejų sistemų, pasireiškiančių per mentalinių modelių teoriją, principas. Be to, laikantis Carnapo redukcijos idėjos, straipsnyje aprašomos dvi procedūros, kuriomis hipotezė patvirtinama palaipsniui. Viena iš jų tyrinėja, kaip žmogaus protas nagrinėja žmogaus proto veiklą darant išvedimus pagal *modus tollendo tollens*. Kita hipotezė tyrinėja išvedimus iš griežtosios disjunkcijos su trimis disjunktais.
Pagrindiniai žodžiai: patvirtinimas, griežtoji disjunkcija, mentalinis modelis, *modus tollendo tollens*, redukcija

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Introduction

From different approaches, it has been claimed that dual processes work in the human mind (e.g., Evans 2008, 2009). Thus, it has been said that the processes belong to two systems: System 1 and System 2 (e.g., Johnson-Laird et al. 2015). System 1 has to do with heuristic and intuitive mental actions. System 2 is in general said to be linked to logic and rational processes. Hence, it is usually deemed as a slower system. This general framework seems to be a solution for many problems cognitive science has at present (see also, e.g., Stanovich 2012). For example, this difference between two systems allows explaining why physicians tend to only pay attention to the probability of coronary disease, while ignoring that of heart attack. The reason for this is that coronary disease and heart attack are often linked. So, one might think that System 1 leads physicians to review just the possibility of coronary disease (Reyna 2004) Likewise, de Groot's (1965) findings about chess masters can be interpreted in this way too. If chess masters only perceive what is essential for each movement and each piece, that is because of System 1 (Inglis and Simpson 2006).

It is true that, by means of functional magnetic resonances, each of the systems has been attributed to a different part of the brain (Goal and Dolan 2003). However, the main difficulty of the general framework claiming the presence of two systems in the mind may be its empirical confirmation. It tries to describe what happens inside the human mind. Hence, the verification of the framework is hard.

The present paper is intended to show that the hypothesis about System 1 and System 2 can be confirmed at least in a gradual way. On the one hand, the paper assumes the theory of the mental models, which is a dual-process theory (e.g., Khemlani et al. 2018). On the other hand, it also takes the idea of reduction that Carnap (1936, 1937) gives to progressively confirm relations between predicates. Thus, the final aim is to present two bilateral reduction sentences based on the framework of the theory of mental models to increasingly confirm the existence of the two systems indicated. The advantages of doing this are at least two. It reveals that there is clear empirical support for essential theses of the theory of mental models. In addition, it allows arguing that some components of Carnap's philosophy keep being suitable nowadays.

The first sections will address the way the theory of mental models explains the workings of System 1 and System 2 (of course, that way is not the only possible one; there are different dual-process theories, and not all of them assign the same characteristics to the two systems). They will resort to two examples of logical tasks people often perform badly. One of them is *Modus Tollendo Tollens*. In the other one, participants have to respond whether or not, given an exclusive disjunction including three disjuncts, the three disjuncts can be true at the same time. The first section will deal with *Modus Tollendo Tollens*. The second section will be devoted to the task with the exclusive disjunction and its three disjuncts.

The third section will follow Carnap's (1936, 1937) general framework. Because, according to that framework, total verification cannot be achieved, but only phased confirmation, a bilateral reduction sentence linked to *Modus Tollendo Tollens* will be offered. The last section will present something similar regarding the example related to the disjunction having three disjuncts: it will present a bilateral reduction sentence corresponding to that example. Thereby, the third and fourth sections will propose two possible manners to gradually confirm the central idea of dual-process theories. This is because Carnap (1936) understands bilateral reduction sentences as formulae to successively confirm connections between predicates. While Carnap's approach is about experimental research, as shown below, the difficulties people have with *Modus Tollendo Tollens* and exclusive disjunctions with three disjuncts are reported in the literature from experimental results.

System 1 and *Modus Tollendo Tollens*

The basic idea of the theory of mental models is that, by reasoning, people derive 'conjunctions of possibilities' from sentences (e.g., Johnson-Laird and Ragni 2019). The problem is that only System 2 can detect all the possibilities in a particular conjunction of possibilities (see also, e.g., Byrne and Johnson-Laird 2020).

Given (1),

(1) If A then B

Its possibilities would be, in principle, those in (2).

(2) Π(A & B) & Π(not-A & B) & Π(not-A & not-B)

Where 'Π' indicates possibility.

One way to express how the theory links (1) to (2) can be (3).

(3) {If A then B} ⇒ {Π(A & B) & Π(not-A & B) & Π(not-A & not-B)}

Where '⇒' means that the right possibilities in curly brackets are the possibilities corresponding to the left sentence in curly brackets.

This does not mean that the theory of mental models is a modal logic. This is because the logical form is not relevant within it (e.g., Johnson-Laird 2010). The meanings play a more important role in reasoning, which is not about formal deductions or derivations. Thus, the semantic content of the clauses signified by A and B in (1), (2), and (3) can change the possibilities (see also, e.g., Quelhas et al. 2010). Take the following sentence as an example:

If they eat fruit, then they eat oranges

If it is assumed that A is 'they eat fruit' and B corresponds to 'they eat oranges', the relation would not be (3) but (4).

(4) {If A then B} ⇒ {Π(A & B) & Π(A & not-B) & Π(not-A & not-B)}

In Relation (3), the conditional appears to be material, since it is compatible with Π(not-A & B) but not with Π(A & not-B). The case of (4) is the opposite: it is compatible with Π(A & not-B) but not with Π(not-A & B). This is because oranges are fruits. So, while the case of (3) seems to show that the theory of mental models is compatible with classical logic, the case of (4) appears to reveal the opposite. In classical propositional logic, a conditional such as 'if A then B' cannot be true in a scenario in which A is true but B is not. In fact, the latter is the only combination forbidden for a conditional in classical logic.

Besides, the fact that both (3) and (4) are linked to Π(A & B) also reveals that the theory of mental models is different from the habitual normal modal logics (in the sense they have in works such as that of Kripke 1963). Symbol '&' can work as conjunction in logic, but 'Π' cannot work as the operator of possibility in the habitual normal modal logics. (1) leads to Π(A & B) and, therefore, to Π(A). The habitual normal modal logics do not accept that. They do not enable to infer that clause A is possible from the fact that a sentence such as 'if A then B' is true (see also, e.g., Espino et al. 2020). In the material interpretation of the conditional characteristic of classical logic, the conditional 'if A then B' can be true even if Clause A is impossible. As said, that conditional could only be false in a circumstance in which A were the case and B were false (see also, e.g., Jeffrey 1981).

The most interesting point for the present paper is that Relations such as (3) and (4) are only possible with System 2. Following the theory, System 1 tends not to represent negations. Thus, all of the possibilities in (3) and (4) cannot be considered with System 1. In both cases, the relation is reduced to (5) (see also, e.g., López-Astorga et al. 2022).

(5) $\{\text{If A then B}\} \Rightarrow_{S1} \{\Pi(A \,\&\, B)\}$

The difference between '$\Rightarrow$' and '$\Rightarrow_{S1}$' is just that the former represents the relation between the sentence and the possibilities in the case of System 2. On the other hand, the latter expresses that very relation in System 1.

If the possibilities with negations in (3) are removed, the result is (5). The same occurs with (4). If the possibilities with negated clauses are eliminated, only the possibility in (5), that is, Π(A & B), remains.

All of this is important for the case of *Modus Tollendo Tollens*. Its logical structure in the classical propositional calculus is that in (6).

(6) $[(p \rightarrow q) \wedge \neg q] \therefore \neg p$

Where '$\rightarrow$' is the material conditional, '$\wedge$' stands for conjunction, '$\neg$' indicates negation, and '$\therefore$' represents logical deduction.

If, with thematic content, the conditional in (6) is regular and not modulated, that is, it is not a conditional in which the content modifies the possibilities (as it happens, e.g., in (4) and similar examples), the mental process following the theory of mental models would be akin to that described in (7) (see also, e.g., Byrne and Johnson-Laird 2009).

(7) Π(p & q) & Π(not-p & q) & Π(not-p & not-q)
Not-q
―――――――――――――――――――――――
Therefore, not-p

According to the theory of mental models, (7) is correct. The second premise, that is, not-q, shows that the conjuncts Π(p & q) & Π(not-p & q) in the first premise are not the case. This is because in both Π(p & q) and Π(not-p & q), q is true. Hence, they are incompatible with not-q. The only possibility is Π(not-p & not-q) and p is not true in that possibility.

This process cannot be developed with System 1. System 1 leads to (5). So, its mental process would not be that shown in (7) but that in (8) (e.g., Byrne and Johnson-Laird 2009).

(8) Π(p & q)
Not-q
――――――――――――――――
Therefore, no inference possible

Inference (8) does not contain any possibility including not-q. Accordingly, it cannot be inferred what happens with p in a scenario in which q does not occur.

This is the account the theory of mental models gives to explain the difficulties that, as shown in the literature (see also, e.g., Cramer et al. 2021), people usually have with *Modus Tollendo Tollens*. There are many experimental reports in this way. Just two examples of experiments in which people have a poor performance in *Modus Tollendo Tollens* tasks are "*if there is the letter d on one side of the card, then there is the number 3 on the other side* and *there is the number 7 on the other side*" (Cramer et al. 2021: 2337, italics in text; coming from Wason 1968) and "*if Mary is in Dublin, then Joe is in Limerick* and *Joe is in Cambridge*" (Cramer et al. 2021: 2337, italics in text; coming from Johnson-Laird, Byrne and Schaeken 1992). The explanation of cases such as these is easy from the theory of mental models. In the first example, the participants do not note that it is possible that the letter d is not on one side and the number 3 is not on the other side. Therefore, they cannot conclude that, when the number 7 appears, the letter d cannot be on the card. In the second one, the participants do not identify, as a possible scenario, the circumstance in which Mary is not in Dublin and Joe is not in Limerick. Hence, they cannot infer that, when Joe is in Cambridge, Mary is not in Dublin. The problem is that System 1 does not allow them to pay attention to the key possibilities. A careful consideration and, accordingly, System 2, would enable to take the latter possibilities into account.

However, individuals have problems not only with *Modus Tollendo Tollens*, but also with other logical structures. Another case is that of the inferences from a disjunction with two characteristics: it is exclusive, and it has three disjuncts.

System 1 and the disjunctions that are exclusive and have three disjuncts

The theory of mental models also links disjunction to conjunctions of possibilities. If exclusive, the possibilities that can be derived are those in (9) (see also, e.g., Quelhas et al. 2019).

(9) {Either A or B, but not both of them} $\Rightarrow$ {Π(A & not-B) & Π(not-A & B)}

Here, the meaning can also have an influence, that is, the possibilities can be modulated (see also, e.g., Quelhas and Johnson-Laird 2017). An example can be this sentence:

Either they eat fruit or they eat oranges

If, again, A represents that 'they eat fruit' and B stands for that 'they eat oranges', the relation is not (9) but (10).

(10) {Either A or B} $\Rightarrow$ {Π(A & B) & Π(A & not-B)}

The content makes the sentence inclusive: 'but not both of them' is removed and Π(A & B) is included. In addition, given that it is not possible eating oranges without eating fruit, Π(not-A & B) is not in (10).

The case of disjunction allows checking that the theory of mental models is not a habitual normal modal logic, either. By virtue of both (9) and (10), it is possible to deduce Π(A) from 'either A or B', and that is not correct in the habitual normal modal logics (see also, e.g., Khemlani et al. 2017). (9) leads to Π(A & not-B), and, therefore, to Π(A). (10) is related to Π(A & B), and, therefore, to Π(A), and to Π(A & not-B), and, therefore, to Π(A).

The action of the two systems is present in disjunction as well. People can be aware of the negative clauses only by means of System 2. With System 1, just the positive clauses can be identified. Thus, in the case of a regular exclusive disjunction, that is, a non-modulated exclusive disjunction, the relation is not that in (9) but that in (11) (see also, e.g., Johnson-Laird et al. 2021).

(11) {Either A or B, but not both of them} $\Rightarrow_{S1}$ {Π(A) & Π(B)}

All of this enables to see what happens when the particular exclusive disjunction does not have two disjuncts, but three. If the mental system is System 2, the correspondence is described in (12) (see also, e.g., Khemlani and Johnson-Laird 2009).

(12) {Either (either A or B, but not both of them) or C, but not both of them} $\Rightarrow$ {Π(A & B & C) & Π(A & not-B & not-C) & Π(not-A & B & not-C) & Π(not-A & not-B & C)}

Relation (12) allows checking that, given a disjunction such as the left one in curly brackets in (12), if the question is whether or not the three disjuncts can be true at once, the answer has to be positive. The first possibility in (12), that is, Π(A & B & C), shows that.

The problem arises when the system working is System 1. This system does not take negative clauses into account. Hence, the relation System 1 establishes is not (12) but (13) (e.g., Khemlani and Johnson-Laird 2009).

(13) {Either (either A or B, but not both of them) or C, but not both of them} $\Rightarrow_{S1}$ {Π(A) & Π(B) & Π(C)}

The reason why (13) is the relation corresponding to System 1 is the following. The case of A & B implies that the first clause of the disjunction (i.e., 'either A or B, but not both of them') is negated. This is because 'A and B' means that 'either A or B, but not both of them' is not the case. Hence, A & B cannot be considered, and Π(A & B & C) is transformed into Π(C). The elimination of the negated clauses transforms in turn Π(A & not-B & not-C) into Π(A), Π(not-A & B & not-C) into Π(B), and Π(not-A & not-B & C) into Π(C). Since the theory of mental models refers to possibilities, it is not necessary to include Π(C) in the conjunction of possibilities twice, the result being (13).

This is the account the theory of mental models offers to explain the logically incorrect answer individuals often give in tasks with disjunctions including three disjuncts. If, in tasks with thematic content, it is stated that 'either (either A or B, but not both of them) or C, but not both of them' is true, and it is asked whether or not A, B, and C can be true at the same time, the response is usually negative. According to the theory of mental models, the reason is that the system people resort to is System 1: they reflect assuming (13). And in (13) the possibilities can be understood as alternative possibilities (e.g., Khemlani and Johnson-Laird 2009).

An example of an experimental task in the literature supporting this is as follows: "Suppose that only one of the following assertions is true: (1) You have the mints. (2) You have the gumballs or the lollipops, but not both. Also, suppose you have the mints. What, if anything follows? Is it possible that you also have either the gumballs or the lollipops? Could you have both?" (Khemlani and Johnson-Laird 2009: 618). Although, according to classical logic, it is possible to have the mints, the gumballs, and the lollipops at once, the participants tended to respond negatively. Propositional logic allows having the mints, the gumballs, and the lollipops because, if you have the mints, the second alternative has to be false. But, if you have both the gumballs and the lollipops, the second alternative is false. Nevertheless, if System 1 guides the participants' answer, they can only think about three scenarios. In one of them, you have the mints. In the second one, you have the gumballs. In the last one, you have the lollipops. To note that it is possible to have all the candy (mints, gumballs, and lollipops) at the same time, it is required the use of System 2.

From this framework, it is possible to provide processes to progressively confirm the existence of System 1 and System 2. Although they are mental systems that cannot be empirically perceived, their confirmation is not impracticable. It is enough to consider Carnap's (1936, 1937) thesis about reduction. That will not lead to a definitive verification. Nevertheless, Carnap (1936, 1937) does not support the idea of coming to definitive verifications. From Carnap's point of view, definitive verification is possible for no hypothesis or theory. Scientists can only gradually confirm their proposals.

Modus Tollendo Tollens and its bilateral reduction sentence

Carnap (1936) proposes reduction sentences, reduction pairs, and bilateral reduction sentences as essential conditional (or biconditional) structures to check whether there is any relation between properties (or predicates). Carnap appears to think about the relations that scientific tests and their results can have to certain properties (or predicates). The idea is to use reduction for progressive confirmation of the relations between the properties (or predicates). This is relevant not only from the point of view of philosophy of science. It is also within cognitive science. The literature in the latter subject area has shown that, given a new hypothesis, the first action people tend to do is to confirm it (e.g., Dunbar and Klahr 2012; Wason 1968). So, the importance of Carnap's framework might be not only that it is useful to confirm hypotheses. It might reproduce the natural way individuals deal with hypotheses too.

To build bilateral reduction sentences in the sense Carnap (1936) indicates, it is required to assume first-order predicate calculus. For this reason, the formulae corresponding to this and the next section will be well-formed formulae in that calculus. Likewise, both in this section and in the next one, previous works in the literature constructing sentences related to reduction will be followed (e.g., López-Astorga 2022; where reduction is used to confirm other theses of the theory of mental models).

Starting with the case of *Modus Tollendo Tollens*, it is necessary to define three predicates. The first one would be M.

$M =_{df}$ to perform a task such as (14).

(14) If A then B
It is not the case that B
―――――――――――
Therefore?

Although the clauses in (14) should have thematic content, the conditional in the first premise ('if A then B') needs to be a regular conditional, that is, a non-modulated conditional.

The second predicate would be N.

$N =_{df}$ to respond something different to 'it is not the case that A' to (14).

And the last predicate would be S^1.

$S^1 =_{df}$ to reason using System 1.

This enables to build this well-formed formula in first-order predicate logic:

(15) $\forall x\,[Mx \rightarrow (Nx \rightarrow S^1x)]$

Where '∀' stands for the universal quantifier.

Within Carnap's (1936, 1937) framework, (15) would be a reduction sentence for S^1 if (16) holds.

(16) $\exists x\,(Mx \wedge Nx)$

Where '∃' represents the existential quantifier.

Given that experiments in which people perform tasks such as (14) are possible, and the literature reveals that there are occasions in which the answer is not that expected in *Modus Tollendo Tollens* (e.g., Cramer et al. 2021), (15) can be accepted as a reduction sentence for S^1. This already allows processes of onward confirmation of the thesis about the existence of System 1 and System 2. Nevertheless, an even stronger sentence can be built:

(17) $\forall x\ [Mx \rightarrow (Nx \leftrightarrow S^1x)]$

Where '↔' indicates biconditional relation.

Formula (17) keeps providing that, when in a task such as (14), the answer is not the negation of the consequent, the system is System 1. It is stronger because it adds something else: if *Modus Tollendo Tollens* is performed by using System 1, the conclusion cannot be the negation of the consequent.

Following Carnap (1936), (17) could be a bilateral reduction sentence for S^1, which would confirm to a larger extent than (15) the main idea of the dual-process theories, if (18) is true.

(18) $\exists x\ Mx$

As said, experiments using (14) are possible. So, it can be claimed that it is possible to find pieces of evidence in favor of the thesis about the two mental systems by means of *Modus Tollendo Tollens*.

Tasks with exclusive disjunctions presenting three disjuncts and their bilateral reduction sentences

It is also possible to construct bilateral reduction sentences corresponding to tasks such as (19).

(19) One of these two sentences is true, but not both of them
-Either A or B, but not both of them
-C

Therefore, A, B, and C are possible at once?

Task (19) has to have thematic content too. Besides, its exclusive disjunctions have to be regular, that is, not modulated.

To build a reduction sentence related to (19), two new predicates are necessary. One of them is D.

D $=_{df}$ to perform a task such as (19).

And the second one is I.

I $=_{df}$ to give a negative response to (19).

The reduction sentence would be (20).

(20) $\forall x\ [Dx \rightarrow (Ix \rightarrow S^1x)]$

As in (15), Carnap's (1936) framework requires (20) to meet a condition. If not, (20) cannot be a reduction sentence for S^1. That condition is (21).

(21) $\exists x\ (Dx \wedge Ix)$

Requirement (21) is not a problem either. The literature also reports the application of tasks akin to (19) in which the participants' answers tend to be compatible with the definition of I (e.g., Khemlani and Johnson-Laird 2009).

Formula (20) would already be a means of getting evidence in favor of dual-process theories. But a stronger formula is possible here as well. That sentence is (22).

(22) $\forall x\ [Dx \rightarrow (Ix \leftrightarrow S^1x)]$

Again, (22) expresses the same as (20) adding something else. According to (20), to give a negative response to (19) is a sufficient cause to use System 1. However, following (22), to use System 1 is also a sufficient cause to give a negative response to (19).

To be a bilateral reduction sentence for S^1, (22) only needs (23) to hold.

(23) $\exists x\ Dx$

As mentioned, the literature shows that (23) is the case. Therefore, tasks such as (19) can be pieces of evidence supporting the hypothesis about the two systems too.

Conclusions

A very relevant idea in cognitive science today is that of the two mental systems working in parallel. One of those systems is quick and related to intuitions: System 1. The other system is slow and linked to analytic processes: System 2. *A priori*, one might think that this hypothesis is extremely hard to check. Nonetheless, if a reflection about possibilities such as that proposed by the theory of mental models and Carnap's framework referring to reduction are taken into account, it is possible to think about easy ways to continuously confirm the hypothesis.

Two kinds of intellectual tasks can be useful. The first one is *Modus Tollendo Tollens*. The prediction of the theory of mental models for this task is that it should be difficult to perform. The reason is that System 2 is required to come to the correct conclusion. System 1 does not suffice.

The second task involves proposing an exclusive disjunction including three disjuncts. The task asks whether or not the three disjunctions can happen at the same time. The prediction of the theory of mental models is that the answer should be negative. The explanation of that response is, again, that System 1 is not enough, and System 2 is needed.

As indicated, experimental studies seem to support the predictions of the theory of mental models in both cases. This enables to construct bilateral reduction sentences, in the sense Carnap describes, to increasingly confirm that the two mental systems exist. It is only necessary to take as predicates these actions: to perform the tasks pointed out, to give the responses the theory of mental models predicts, and to reason by using System 1.

Accordingly, although, as Carnap acknowledges for every theory, the absolute confirmation of the general idea of the dual-process theories is not possible, cognitive scientists can incrementally confirm that idea by means of the concept of reduction. This seems to enable us to claim the validity of some of Carnap's theses today. Besides, it has already been proposed in different fields (for the particular case of cognitive science and the theory of mental models, see, e.g., López-Astorga 2022). Thus, it appears to make sense to continue in this direction. Perhaps the next step is to explore possible cases in which the predictions of the theory of mental models do not hold true in tasks such as those considered in the present paper. The aim would be to check whether or not the theory can explain those results. Those cases could be, for example, related to modulation processes. However, if most of the cases could not be explained, it would be necessary to review the support the bilateral reduction sentences above give to the thesis of the existence of the two systems.

References

Byrne, R. M. J., Johnson-Laird, P. N., 2009. *If* and the Problems of Conditional Reasoning. *Trends in Cognitive Science 13*(7): 282–287. Access online: https://doi.org/10.1016/j.tics.2009.04.003

Byrne, R. M. J., Johnson-Laird, P. N., 2020). *If* and *Or*: Real and Counterfactual Possibilities in Their Truth and Probability. *Journal of Experimental Psychology: Learning, Memory, and Cognition 46*(4): 760–780. Access online: https://doi.org/10.1037/xlm0000756

Carnap, R., 1936. Testability and Meaning. *Philosophy of Science 3*(4): 419–471. Access online: https://doi.org/10.1086/286432

Carnap, R., 1937. Testability and Meaning – Continued. *Philosophy of Science 4*(1): 1–40. Access online: https://doi.org/10.1086/286443

Cramer, M., Hölldobler, S., Ragni, M., 2021. When Are Humans Reasoning with Modus Tollens? *Proceedings of the Annual Meeting of the Cognitive Science Society 43*(43): 2337–2343.

Dunbar, K. N., Klahr, D., 2012. Scientific Thinking and Reasoning. In: Holyoak, K. J., Morrison, R. G. (eds.). *The Oxford Handbook of Thinking and Reasoning*, 701–718. New York: Oxford University Press. Access online: https://doi.org/10.1093/oxfordhb/9780199734689.013.0035

Espino, O., Byrne, R. M. J., Johnson-Laird, P. N., 2020. Possibilities and the Parallel Meanings of Factual and Counterfactual Conditionals. *Memory & Cognition 48*: 1263–1280. Access online: https://doi.org/10.3758/s13421-020-01040-6

Evans, J. St. B. T., 2008. Dual-Processing Accounts of Reasoning, Judgment, and Social Cognition. *Annual Review of Psychology 59*(1): 255–278. Access online: https://doi.org/10.1146/annurev.psych.59.103006.093629

Evans, J. St. B. T., 2009. How Many Dual-Process Theories Do We Need? One, Two or Many? In: Evans, J. St. B. T., Frankish, K. (eds.). *In Two Minds: Dual Processes and Beyond*, 33–54. Oxford: Oxford University Press. Access online: http://doi.org/10.1093/acprof:oso/9780199230167.003.0002

Goel, V. & Dolan, R. J. 2003. Explaining modulation of reasoning by belief. *Cognition 87*(1): B11–B22. Access online: https://doi.org/10.1016/s0010-0277(02)00185-3

Groot, A. de, 1965. *Thought and Choice in Chess*. The Hague: Mouton de Gruyter.

Inglis, M., Simpson, A., 2006. Characterising Mathematical Reasoning: Studies with the Wason Selection Task. In: Bosch, M. (ed.). *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education*, 1768–1777. Sant Feliu de Guíxols.

Jeffrey, R., 1981. *Formal Logic: Its Scope and Limits*. New York: McGraw-Hill.

Johnson-Laird, P. N., 2010. Against Logical Form. *Psychologica Belgica 50*(3/4): 193–221. Access online: http://doi.org/10.5334/pb-50-3-4-193

Johnson-Laird, P. N., Byrne, R. M. J., Schaeken, W., 1992. Propositional Reasoning by Model. *Psychological Review, 99*(3), 418–439. Access online: https://doi.org/10.1037/0033-295X.99.3.418

Johnson-Laird, P. N., Khemlani, S., Goodwin, G. P., 2015. Logic, Probability, and Human Reasoning. *Trends in Cognitive Sciences 19*(4), 201–214. Access online: https://doi.org/10.1016/j.tics.2015.02.006

Johnson-Laird, P. N., Quelhas, A. C., Rasga, C., 2021. The Mental Model Theory of Free Choice Permissions and Paradoxical Disjunctive Inferences. *Journal of Cognitive Psychology*. Access online: https://doi.org/10.1080/20445911.2021.1967963

Johnson-Laird, P. N., Ragni, M., 2019. Possibilities as the Foundation of Reasoning. *Cognition* 193. Access online: https://doi.org/10.1016/j.cognition.2019.04.019

Khemlani, S., Byrne, R. M. J., Johnson-Laird, P. N., 2018. Facts and Possibilities: A Model-Based Theory of Sentential Reasoning. *Cognitive Science 42*(6), 1887–1924. Access online: https://doi.org/10.1111/cogs.12634

Khemlani, S., Hinterecker, T., Johnson-Laird, P. N., 2017. The Provenance of Modal Inference. In: Gunzelmann, G., Howes, A., Tenbrink, T., Davelaar, E. J. (eds.). *Computational Foundations of Cognition*, 663–668. Austin: Cognitive Science Society.

Khemlani, S., Johnson-Laird, P. N., 2009. Disjunctive Illusory Inferences and How to Eliminate Them. *Memory and Cognition* 37(5): 615–623. Access online: https://doi.org/10.3758/MC.37.5.615

Kripke, S. A., 1963. Semantical Analysis of Modal Logic I: Normal Modal Propositional Calculi. *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 9: 67–96. Access online: https://doi.org/10.1002/malq.19630090502

López-Astorga, M., 2022. The Definition of Modulation and Its Reduction Sentences. *Open Insight XIII*(27): 106–118. Access online: https://doi.org/10.23924/oi.v12i27.506

López-Astorga, M., Ragni, M., Johnson-Laird, P. N., 2022. The Probability of Conditionals: A Review. *Psychonomic Bulletin and Review 29*: 1–20. Access online: https://doi.org/10.3758/s13423-021-01938-5

Quelhas, A. C., Johnson-Laird, P. N., 2017. The Modulation of Disjunctive Assertions. *The Quarterly Journal of Experimental Psychology* 70(4): 703-717. Access online: https://doi.org/10.1080/17470218.2016.1154079

Quelhas, A. C., Johnson-Laird, P. N., Juhos, C., 2010. The Modulation of Conditional Assertions and Its Effects on Reasoning. *The Quarterly Journal of Experimental Psychology 63*(9): 1716–1739. Access online: https://doi.org/ 10.1080/17470210903536902

Quelhas, A. C., Rasga, C., Johnson-Laird, P. N., 2019. The Analytic Truth and Falsity of Disjunctions. *Cognitive Science 43*(9). Access online: https://doi.org/10.1111/cogs.12739

Reyna, V. F. 2004. How People Make Decisions that Involve Risk: A Dual-Process Approach. *Current Directions in Psychological Science 13*(2): 60–66. Access online: https://doi.org/10.1111/j.0963-7214.2004.00275.x

Stanovich, K., 2012. On the Distinction between Rationality and Intelligence: Implications for Understanding Individual Differences in Reasoning. In: Holyoak, K., Morrison, R. (eds.). *The Oxford Handbook of Thinking and Reasoning*, 343–365. New York: Oxford University Press. Access online: https://doi.org/10.1093/oxfordhb/9780199734689.013.0022

Wason, P. C., 1968. Reasoning about a Rule. *Quarterly Journal of Experimental Psychology 20*(3): 273–281. Access online: https://doi.org/10.1080/14640746808400161