

Reflections on Williamson on Logic and Validity

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1. Introduction

Over many years now, Tim Williamson and I have engaged in debates over fundamental issues concerning logic. On some things, we have disagreed strongly; on others, we have agreed.

Recently, Williamson has published a paper which bears on these matters, ‘Is Logic about Validity?’¹. The general thrust of the paper is to the following effect: There is a view abroad that logic is ‘centrally about validity’; this view is mistaken; logic tells us something fundamental and important about ‘the world’; moreover, this misunderstanding has pernicious effects on debates about which logic is correct²; in particular, the view distorts debates which appeal to the strength of a logic.

Williamson himself summarises the contents of the paper in its introduction as follows (p. 2, Williamson’s italics):

Section 1 explains how the idea of logic as metaphysics is a natural upshot of Tarski’s famous account of logical consequence and logical truth, when we respect the mundane distinction between mentioning symbols and simply using them. Section 2 explains why the natural methodology for logic as metaphysics is an *abductive* one, like that familiar from theorizing in natural science. It shows how confusion between logic and metalogic has caused misunderstanding of methodological issues, particularly about the role of logical *strength* in evaluating logics.

¹ Williamson (2025). At the time of writing (15 October, 2024), this has not appeared in print. A preprint can be found at <https://www.philosophy.ox.ac.uk/files/logicvaliditypdf>. What follows (including page references) relate to this (accessed 20/09/2024). Whether the published version will be different, I do not know. Williamson gave a version of the paper at the *Second International Conference on Logic and Philosophy*, University of Vilnius, May 2024. My own remarks here are drawn from the paper I gave at the same conference, ‘Überconsistent Logics and Dialetheism: Some Initial Thoughts’. Many thanks go to the audience there for their comments and thoughts, and especially to Tim himself. Thanks also go to Thomas Ferguson, Hartry Field, and Jitka Kadlečiková for comments on a first draft of this paper.

² Williamson is a logical monist. So am I. That is one of our agreements. He holds that the correct logic is classical logic. I hold that it is a paraconsistent logic. That is perhaps the major disagreement. More of this below.

There is much in the paper that is interesting and deserving of comment³; but the aim of this paper is not to discuss all the matters raised. Its aim is much more limited: to discuss those issues which pertain to a continuing discussion of our mutual concerns. As I will explain, there is more agreement on some matters than he takes there to be. (Often, I think, the differences are more matters of emphasis than substance.) On others there certainly is not.

2. Logic and ‘the World’

Let us start with the nature of logic itself. Williamson holds that logic delivers “very general structural truths about the world” (p. 8). What he means by ‘structural’, and why the truths are structural, he does not elucidate. Neither is it clear exactly what he means by *the world*, but I do not think that these matter too much for present purposes. What *is* clear is that logic is not – or anyway, not simply – about language.

He starts the essay by citing me as an advocate of the view he is criticizing. Now, I certainly hold that logic is about validity; validity is about arguments; arguments are phrased in language; so logic certainly tells us something about language. But that does not imply that that is *all* it tells us about, and I have never said that it is. A theory of validity is entangled with many issues about the world. Indeed, Williamson himself articulates his own view by applying the Tarskian account of validity.

I am not sure why Williamson attributes the view of logic he is attacking to me. He refers to Priest (2016). The page reference he cites is clearly a typo. But here is the passage I presume he is referring to:

The central notion of logic is validity, and its behaviour is the main concern of logical theories. Giving an account of validity requires giving accounts of other notions, such as negation and conditionals. Moreover, a decent logical theory is no mere laundry list of which inferences are valid/invalid, but also provides an explanation of these facts. An explanation is liable to bring in other concepts, such as truth and meaning. A fully-fledged logical theory is therefore an ambitious project. Examples of such projects are the Aristotelian theory of the syllogism, augmented by Medieval accounts of truth conditions (supposition theory); Frege’s classical logic, augmented by Tarski’s model theoretic account of validity; intuitionistic logic, augmented by a proof-theoretic account of meaning; and so on. (Priest 2016: 353)

Now, the word ‘logic’ has many meanings. Perhaps the most fundamental distinction to be drawn is that between pure logic and applied logic⁴. In the passage above I am talking about logic in the sense of the canonical application of a pure logic as providing a theory of what follows from what and why⁵. As the quote says, however, that matter sinks deep into many issues other than validity itself.

³ In particular, there is an extended critique of Gillian Russell’s views which bears careful consideration.

⁴ On some other disambiguations, see Priest (2014).

⁵ Obviously, there can be mathematical investigations of pure logics, and it is perfectly natural to refer to these as logic (as Williamson notes, p. 1) – though in a different sense of the word.

In particular, any account of validity tells us that certain things follow from the empty set of premises, and (in most logics) this set is non-empty. This is its set of logical truths. And logical truths are, well, true. They may be true for a very special reason, but they are true. Let us suppose (as does Williamson) that Excluded Middle, $A \vee \neg A$, is one of these. In first-order logic, this is a schema, and the validity of the schema indicates that every sentence of this form is (logically) true. So it is true that Pigs fly or they do not, that Trump is corrupt or he is not, and so on. If one follows Williamson in a preparedness to use second-order logic – which I am happy to do here – all the sentences in this schema can be generalised in the form of a single sentence, $\forall p(p \vee \neg p)$ ⁶.

But, with or without this move, logic tells us that some things are true. Whether one should call these truths structural, I do not know. But they are not just about language. They are certainly expressed in language – as is a theory about anything. But the statements of Excluded Middle contain no reference to linguistic entities of any kind⁷. One of the things, then, that logic, as a theory of validity, delivers is certain truths about the world. What one might mean by *truth* is certainly contestable; but this much, I think, is clear. Melbourne is in the Southern Hemisphere. What makes this statement true? In part, it is so because of the meanings of the words ‘Melbourne’, ‘Southern’, and so on; but this hardly suffices. Geographical facts about the world in which we live are necessary.

So truth is not just about language, and this is so even if we are dealing with logical truths. For example, one may debate the truth of Excluded Middle⁸. Thus, some people hold that the future is ‘open’. Aristotle held such a view. Facts about the past and present are now fixed. Certain (‘contingent’) facts about the future are not yet so – for example (at the time of writing) that the war between Ukraine and Russia will be going on at the end of 2025. Let us call this statement W . W is now neither true nor false. Given this (and assuming that disjunction works in the standard way), $W \vee \neg W$ – and so its propositional quantification, $\forall p(p \vee \neg p)$ – is not true⁹. Another example: As is well known, Brouwer contested Excluded Middle on the ground that mathematical entities are not abstract existents, but are mental constructions. Hence, the truth about such entities can be obtained only in terms of such constructions, and that means that truth becomes identified with proof. Excluded Middle is a casualty¹⁰.

The truth of Excluded Middle, then, shows us that these two views are false. This is certainly something about the ‘nature of the world’¹¹.

⁶ Propositional quantifiers are just zero-adic second-order quantifiers.

⁷ At least, not normally. Some may, e.g.: “Donald Trump is corrupt” is a grammatical sentence of English or it is not. But that is clearly a very special kind of case.

⁸ As Williamson notes, p. 10.

⁹ See Øhrstrøm and Hasle (2024).

¹⁰ See Iemhoff (2019).

¹¹ Further on the bearing of logic on metaphysics and vice versa, see Priest (2019a).

3. Logical Abductivism

Something else that Williamson and I agree on is that when one chooses the correct (or at least currently best) theory of logic, one does so in generally the same way that one chooses the best theory about anything – be it in physics, economics, metaphysics, ethics, or anything else¹². Williamson coined the term ‘logical anti-exceptionalism’ for this view. I find this term rather misleading, and prefer the simpler ‘logical abductivism’. The inference to the best logical theory is an inference to the best explanation, that is, an abductive one.

When one produces a theory, one does so to explain something. What it is that is to be explained provides the data against which we assess how good the theory is. If it explains only a fragment of it, or entails data-points that do not seem to be true, these things speak against it. Of course, as in all theoretical endeavours, the data are fallible, and we may in the end conclude that some of them are wrong – though if this is the case, it is methodologically desirable that we find an independent explanation of why we were mistaken. For example, as observed, Brouwer contested the truth of certain statements of the form $A \vee \neg A$, where A is a mathematical sentence. Whether he was correct to do so is certainly a contentious matter, but the example shows us that the correctness of data is rationally contestable, and so fallible, due to their entanglement with other issues¹³. In particular, as I said in the quotation above, a full-blooded account of what is valid and why is liable to engage with the notion of truth and its domain.

Of course, even when there is no contention about the data, rarely will it be the case that the theory fits the data exactly; and perhaps there are different theories that fit the data equally well. We are therefore driven to apply other criteria. *Exactly* what these are may be somewhat contentious, but it is standard in the philosophy of science to cite in this context: simplicity, strength, unifying power, non-*ad hoc*ness, consistency. I note that some putative criteria may be domain-specific. Thus, in physics, the numerical accuracy of data-points expressed in quantitative terms is a point of merit. This may play no role in logic and some other areas. Indeed, it is not always the case in the natural sciences. Darwin’s theory of evolution, for example, was accepted for largely qualitative reasons. (This is one reason why I do not care for the term ‘antiexceptionalism’, which may suggest – as it has to some critics of the view – that the factors involved in the abductive inference are *exactly* the same in logic and physics.)

The main point here, however, is that we choose the best theory of logic by an abduction to the one which provides that best account overall – on adequacy to the data and the other relevant criteria. In this matter, Williamson and I are in agreement¹⁴.

I will turn to some details concerning the abduction involved in the next part of this paper; but for reasons that will become obvious in due course, I will discuss another matter first: that of the so-called ‘contra-classical logics’.

¹² See, for example, Priest (2016).

¹³ Arguably, Brouwer lost this debate, but matters could clearly have gone the other way.

¹⁴ There is, of course, the question of how best to articulate the somewhat vague phrase ‘best account overall’. For my own suggestion about the matter, see, e.g., Priest (2016); and on one detailed example, see Priest (2019b). As far as I know, there is no disagreement between Williamson and myself on this articulation, either.

4. Interlude: Contra-Classical and Überconsistent Logics

Let \mathbb{V}_L be the set of valid inferences of a logic, L . Let CL be classical logic, and NL a non-classical logic. Under the homophonic translations of their languages, then, for most well-known NL , $\mathbb{V}_{NL} \subsetneq \mathbb{V}_{CL}$. However, this is certainly not true of all non-classical logics. Non-classical logics that do not satisfy this condition have come to be known as ‘contra-classical logics’.

Perhaps the best known such logic is Aristotelian Syllogistic. This concerns statements of four kinds, which are as follows. The first column gives its name; the second is the expression in English; the third is a standard way to express this in contemporary logic:

<i>a</i> form	all <i>As</i> are <i>Bs</i>	$\neg\exists x(Ax \wedge \neg Bx)$
<i>e</i> form	no <i>As</i> are <i>Bs</i>	$\neg\exists x(Ax \wedge Bx)$
<i>i</i> form	some <i>As</i> are <i>Bs</i>	$\exists x(Ax \wedge Bx)$
<i>o</i> form	some <i>As</i> are not <i>Bs</i>	$\exists x(Ax \wedge \neg Bx)$

Now, syllogistic declares valid inferences such as *Darapti*:

All *As* are *Bs*
 All *As* are *Cs*
 So some *Bs* are *Cs*.

Expressed in the notation of modern logic, this is:

$\neg\exists x(Ax \wedge \neg Bx)$
 $\neg\exists x(Ax \wedge \neg Cx)$
 $\exists x(Bx \wedge Cx)$

which is certainly not valid.

Of course, there are well-known suggested repairs, but they all make a mess somewhere else. One such suggestion is to express the *a* form as $\exists xAx \wedge \neg\exists x(Ax \wedge \neg Bx)$. But the *a* form and the *o* form are contradictories for Aristotle. This is not the case if the extra existential clause is added to the *a* form. Some medieval logicians got around this problem by understanding the *o* form as $\neg\exists xAx \vee \exists x(Ax \wedge \neg Bx)$, but this validates the inference:

There are no *As*.
 So some *As* are not *Bs*

which is of course not valid in classical logic¹⁵. Still, perhaps no one takes syllogistic as a serious candidate for a theory of logical validity today.

A contra-classical logic that *is* now being taken very seriously is connexive logic. Connexive logics are logics which validate certain principles concerning the conditional. The principles are venerable (as the names they are now being given suggest); and though

¹⁵ For further discussion of syllogistic, see Priest (2006), 10.8.

now unorthodox, they have been endorsed by many over the history of Western logic. They can be formulated in somewhat different ways, but the following shall suffice for the present purposes:

Aristotle: $\vdash \neg(A \rightarrow \neg A)$

Boethius: $A \rightarrow B \vdash \neg(A \rightarrow \neg B)$

Clearly, these inferences are not classically valid (assuming that one identifies \rightarrow with the material conditional). Indeed, added to classical logic, they make every inference valid¹⁶.

One class of contra-classical logics that are now starting to be investigated systematically is that of logics in which the set of logical truths is itself inconsistent. The logics have as yet no standard name. I will call them *überconsistent logics*. If an überconsistent logic is not to be trivial, the logic must be paraconsistent. But familiar paraconsistent logics, such as *LP*, are not überconsistent. There are now many known überconsistent logics¹⁷; let me just mention two here. Both are extensions of *LP*¹⁸.

The first is (full) second-order *LP*¹⁹. This extends first-order *LP* to second-order, in exactly the same way that classical first-order logic is extended to second-order. To see why it is überconsistent, simply consider monadic second-order quantifiers. In an interpretation, first-order quantifiers range over a non-empty domain D_1 . Second-order quantifiers range over a non-empty domain, D_2 , whose members are of the form $\langle D^+, D^- \rangle$, where $D^+ \cup D^- \subseteq D_1$. (D^+ is the extension of a variable; D^- is its anti-extension.) Whatever D_2 is, $\models \forall X \forall x (Xx \vee \neg Xx)$; but if one takes D_2 , as is standard in the classical case, to be maximal, then $\models \exists X \exists x (Xx \wedge \neg Xx)$. That is, $\models \neg \forall X \forall x (Xx \vee \neg Xx)$.

The second example of an überconsistent logic takes us back to connexive logics. *LP* can be extended with a detachable conditional in many ways. Many of these extensions are not überconsistent, but some natural extensions are.

For example, take *LP* to be formulated as a 3-valued logic, the values of which are $\{1\}$, $\{1, 0\}$, and $\{0\}$ ²⁰. If v is a valuation of formulas, a simple ponible conditional (that is, one that validates *modus ponens*) is given by the following truth/falsity conditions:

- $1 \in v(A \rightarrow B)$ iff $1 \notin v(A)$ or $1 \in v(B)$
- $0 \in v(A \rightarrow B)$ iff $1 \in v(A)$ and $0 \in v(B)$

The truth table these give is as follows:

A \ B	{1}	{1, 0}	{0}
{1}	{1}	{1, 0}	{0}
{1, 0}	{1}	{1, 0}	{0}
{0}	{1}	{1}	{1}

¹⁶ For further discussion of connexive logics, see Wansing (2023).

¹⁷ A (partial) list of such logics can be found in Wansing (202x: 9).

¹⁸ If one takes the truth predicate to be a logical predicate (for which, I think, there are as good reasons as for taking identity to be a logical predicate), adding the *T*-Schema inferences to *LP* produces a third example.

¹⁹ As far as I know, the überconsistency of this was first noted in Priest (2002: 33).

²⁰ See Priest (2002), 4.6.

A possible connexive conditional can be obtained simply by changing the falsity conditions. $A \rightarrow B$ is false just if $A \rightarrow \neg B$ is true²¹. This gives the falsity conditions:

- $0 \in v(A \rightarrow B)$ iff $1 \notin v(A)$ or $0 \in v(B)$

and produces the truth table:

$A \backslash B$	{1}	{1, 0}	{0}
{1}	{1}	{1, 0}	{0}
{1, 0}	{1}	{1, 0}	{0}
{0}	{1, 0}	{1, 0}	{1, 0}

These are very natural falsity conditions. (*If you go to the movies, will you take me with you? No, if I go to the movies I will not take you with me.*) And as may easily be checked, the conditional now verifies **Aristotle** and **Boethius**. As it is also easy to check, given that conjunction works in the usual way, they also verify $(A \wedge \neg A) \rightarrow A$ and $(A \wedge \neg A) \rightarrow \neg A$. Hence they verify $\neg((A \wedge \neg A) \rightarrow A)$, and so the logic is überconsistent.

There is, of course, much more to be said about all these matters²². But this will do for present purposes. Let us now return to Williamson.

5. The Matter of Strength

As I noted, an abductive account of the best theory of logical validity is delivered by performance overall on a raft of criteria. Williams discusses the performance of classical logic versus non-classical logics on one of these, *strength*, and argues that that classical logic out-performs non-classical logics on this virtue. The rest of this essay concerns his remarks on this matter²³. Of course, even if Williamson is correct in this claim, there remains the question of what happens in the over-all evaluation when all the relevant criteria are taken into account. As Williamson says (p. 12), “other things being equal, we prefer scientifically stronger theories to scientifically weaker ones.” The *ceteris paribus* clause it not to be ignored; but that topic is not on the agenda here.

First, as we have already noted, for most non-classical logics, NL , under the homomorphic translations of their languages, it is generally true that $\mathbb{V}_{NL} \subseteq \mathbb{V}_{CL}$. So classical logic is stronger in this sense. But the significance of this fact is less than clear because, under non-homomorphic translations, this need not be the case. For example, there is the Gödel-Gentzen translation that embeds classical logic in intuitionist logic²⁴. Under this

²¹ This is a very general construction, and it can be applied to any conditional where truth and falsity (at a world) are specified independently. See Wansing (2023).

²² For some of it, see Priest (202x).

²³ For another discussion of the issue, see Hjortland (2017), 5.2.

²⁴ See Moschovakis (2022), 4.1.

embedding, every inference valid in classical logic is valid in intuitionist logic²⁵. Much more straightforwardly, as we have seen, this relationship fails for contra-classical logics, including various extensions of *LP*. Williamson notes, in effect that quantified intuitionist logic is a contra-classical logic, since it contains $\neg\forall p(p \vee \neg p)$, but says (p. 12): “the law of excluded middle as a universal generalization is far more informative deductively than its negation.” But the machinery that delivers the überconsistent extensions of *LP* (second-order logic, a connexive conditional) is clearly deductively highly informative – giving universal generalisations, not merely negations thereof²⁶.

But how, if at all, do these matters bear on the notion of strength relevant to the appropriate abduction? Here is what Williamson says (p. 13):

In their own right, scientifically stronger logics answer more of our questions in logic. In their auxiliary role as background logics for theories in other fields, they increase the scientific strength of those non-logical theories, and enhance their explanatory and predictive power, by extracting more relevant consequences from them.

Concerning logics ‘in their own right’, even if $\mathbb{V}_{NL} \subsetneq \mathbb{V}_{CL}$, it is not true that *CL* answers more questions about logic than *NL*. For any inference, \mathcal{I} , any logic will answer the question ‘Is \mathcal{I} valid?’ – though it may obviously disagree with other logics on the answer.

Moreover, it is not the case that one theory is better than another *simply* because it tells you that there are more things of a certain kind. One account of evolution is not better than another simply because it tells you that there are *more* species. This is especially the case when some of the *more* are highly dubious. Leave aside the counter-intuitiveness of Explosion. We have such well known problematic classical validities as $\models A \supset (B \supset A)$ and $\models \neg A \supset (A \supset B)$ ²⁷. These things tell against the classical theory of the conditional. At the very least, they have to be addressed with a bunch of apparently *ad hoc* mechanisms, weakening the logic against other important methodological criteria.

Let us turn to logic in its ‘auxiliary role’, as the underlying logic of theories in other fields. It is certainly the case that if *T* is any theory and $\mathbb{V}_{NL} \subsetneq \mathbb{V}_{CL}$, then $T \vdash_{NL} A$ implies $T \vdash_{CL} A$, though not necessarily the converse. However, it is not the case that $\mathbb{V}_{NL} \subsetneq \mathbb{V}_{CL}$ if *NL* is a contra-classical logic. The consequence of *T* under the logics is incommensurable by subsethood.

If classical logic has an advantage over non-classical logics here, it is not a quantitative one; it must be qualitative. And Williamson holds that classical logic does have such an advantage: it has been the underlying logic of many successful scientific theories. In another paper, *Alternative Logics and Applied Mathematics*, he makes the point as follows:

²⁵ Williamson notes this, but takes it to be irrelevant, since we are dealing with an already interpreted object language (p. 11). I am not so sure. One thing that could be at issue here is exactly what the intended interpretation should amount to. However, I shall not pursue this matter here, since it would take us off into a mare’s nest of issues.

²⁶ And, of course, if the truth predicate and the *T*-Schema inferences are added, much extra deductive power is generated.

²⁷ As well as their knock-on effects for quantification, such as the logical truth of $\exists x(Px \supset \forall yPy)$ and the non-logical-truth of $\neg\exists x(\forall yPy \supset \neg Px)$. See Wansing (2024).

The hardest test of deviant logic is mathematics, which constitutes by far the most sustained and successful deductive enterprise in human history. With only minor exceptions, mathematicians have freely relied on classical logic, including principles such as the law of excluded middle, $A \vee \neg A$. (Williamson 2018: 399)

Now, I do not think that pure mathematics requires classical logic, as is witnessed by the fact that there are perfectly respectable branches of mathematics, such as the theory of smooth infinitesimals, that require intuitionist logic. More generally, if one is a mathematical pluralist, any logic may be the underlying logic of a perfectly good pure mathematical structure²⁸. But let us set this matter aside. As the title of Williamson’s paper indicates, it is not pure mathematics that he has in his sights, but applied mathematics.

Now, scientists in the social sciences *do* use non-classical logics. Here, for example, is the cover information from a recent book on linguistics (Gregory 2015): “This book will take linguistics students beyond the classical logic used in introductory courses into the variety of non-standard logics that are commonly used in research”. Perhaps this is one of the minor exceptions. More likely, Williamson has the natural sciences in his sights.

Yet, even here, Williamson’s claim is suspect. For a start, the claim is anachronistic. Mathematics has been applied for millennia, and classical (aka Frege/Russell) logic was invented only just over 100 years ago. Of course, mathematicians reasoned before that; but the reasoning was informal and did not answer to any formal logic.

What is more, at times, applied mathematicians clearly flouted the principles of classical logic. For example, the reasoning that was actually used for the infinitesimal calculus from its discovery/invention till the 19th century could not have been classical, since it took infinitesimals to behave in a contradictory fashion²⁹.

Of course, we now know how to formalise this bit of mathematics by using first-order classical logic and the theory of limits. Maybe it is even true that all the mathematics that has been applied in the natural sciences can be formalised by using this³⁰; but that in itself tells us little. It can be formalised in other ways.

Thus, for example, an intuitionist holds that finite (or, more generally, decidable) domains satisfy Excluded Middle. Hence, if they are reasoning about such a domain, they can take Excluded Middle to be a ‘contingent’ assumption true of the domain. And, as it is well known, adding Excluded Middle to intuitionist logic delivers classical logic. Hence, an intuitionist can reason classically about such domains.

In a similar way, a paraconsistent logician can reason classically about consistent domains. They just add the rule:

Exp: $(A \wedge \neg A) \vdash \perp$

²⁸ See Priest (2024), Chs. 1 and 2. See Ch. 3 for a discussion of applied mathematics in this context.

²⁹ See Brown and Priest (2004) and Sweeney (2014).

³⁰ Though who knows what will happen in the future, now that we are seeing the development of well-articulated pure mathematical structures based on non-classical logics?

as ‘contingently’ preserving truth in the domain. This is a version of Explosion, and it rules out inconsistency on pain of triviality. (Indeed, the collapse into triviality is the only way that a classical logician themselves can rule out inconsistency.) And, in most paraconsistent logics (certainly in *LP*), the addition of **Exp** delivers classical logic³¹. Naturally, if the logic is überconsistent, one cannot add the rule of inference for all *A*; in *LP*, we can add it for those *A* that do not contain the vocabulary whose addition delivers überconsistency (second order logic, a connexive conditional); and that is sufficient for classical first-order validity, in which standard scientific theories may be thought to be formulated³².

Finally, on the topic of strength, I note that all classical models are (standard) *LP* models, but not vice versa. So any situation about which a classical logician can reason, can be reasoned about by an *LP* logician (by adding **Exp**); but not vice versa. So there is a clear sense in which *LP* is a stronger logic than classical logic.

6. Conclusion

I do not expect what I have said in this essay to resolve the matters over which Williamson and I disagree. But I hope, at least, that they advance our discussions of these matters. I hope, also, that the points where we *do* agree are now clearer, and that this builds a stronger base for continuing discussions.

References

- Brown, B., and Priest, G., 2004. Chunk and Permeate, a Paraconsistent Inference Strategy; Part I, the Infinitesimal Calculus. *Journal of Philosophical Logic* 22: 379–388.
- Gregory, H., 2015. *Language and Logics: An Introduction to the Logical Foundations of Language*, Edinburgh: Edinburgh University Press.
- Hjortland, O., 2017. Anti-Exceptionalism about Logic, *Philosophical Studies* 174: 631–658.
- Iemhoff, R., 2019. Intuitionism in the Philosophy of Mathematics. In: E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, <https://plato.stanford.edu/entries/intuitionism/>.
- Moschovakis, J., 2022. Intuitionistic Logic, in E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, <https://plato.stanford.edu/entries/logicintuitionistic/>.
- Øhrstrøm, P., and Hasle, P., 2024. Future Contingents. In: E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, <https://plato.stanford.edu/entries/future-contingents/>.
- Priest, G., 2002. Paraconsistent Logic. In: D. Gabbay and F. Guenther (eds.), *Handbook of Philosophical Logic*, 2nd ed., Vol. 6, Dordrecht: Kluwer Academic Publishers, 287–393.
- Priest, G., 2006. *Doubt Truth to be a Liar*, Oxford: Oxford University Press.
- Priest, G., 2014. Revising Logic. In: P. Rush (ed.), *The Metaphysics of Logic*, Cambridge: Cambridge University Press, Ch 12.

³¹ In paraconsistent logics without Excluded Middle (such as *FDE*), one has to add this as well.

³² Williamson says (p. 17), “In the case of mathematics, proponents of weak non-classical logics often propose to recover the full strength of classical mathematics by supplementing their logic with versions of classical schemas such as excluded middle restricted to instances in the language of mathematics (Hjortland 2017). That strategy runs into trouble with applications of classical mathematics in the natural and social sciences, because accepting the relevant instances of the classical schemas often conflicts with the proposed rationale for going non-classical in the first place (Williamson 2018).” I am not exactly sure what he is referring to – there are no page references – but as far as I can see, I am not suggesting with the above comments anything of the kind attributed to Hjortland in that paper.

- Priest, G., 2016. Logical Disputes and the a Priori. *Logique et Analyse* 59(236): 347–366.
- Priest, G., 2019a. Metaphysics and Logic: an Observation in Metametaphysics. *Giornale di Metaphysica* 1: 68–77.
- Priest, G., 2019b. Logical Theory-Choice: the Case of Vacuous Counterfactuals. *Australasian Journal of Logic* 16(7): 283–297.
- Priest, G., 2024. *Mathematical Pluralism*. Cambridge: Cambridge University Press.
- Priest, G., (202x). *Überconsistent Logics and Dialetheism. Some Initial Thoughts*, to appear.
- Sweeney, D. J., 2014. Chunk and Permeate: the Infinitesimals of Isaac Newton. *History and Philosophy of Logic* 35(1): 1–23.
- Wansing H., 2023. Connexive Logic. In E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, <https://plato.stanford.edu/entries/logic-connexive/>.
- Wansing, H., 2024. Constructive Logic is Connexive and Contradictory. *Logic and Logical Philosophy* 33, 2 January. DOI: 10.12775/LLP.2024.001.
- Wansing, H., (202x). *Beyond Paraconsistency. A Plea for a Radical Breach with the Aristotelian Orthodoxy in Logic*, to appear.
- Williamson, T., 2018. Alternative Logics and Applied Mathematics. *Philosophical Issues* 28(1): 399–424. DOI: 10.1111/phils.12131
- Williamson, T., 2025. Is Logic about Validity?, to appear in Brendel, E., Carrara, M., Ferrari, F., Hjortland, O., Sagi, G., Sher, G., and Steinberger, F. (eds.), *Oxford Handbook of the Philosophy of Logic*, Oxford: Oxford University Press.