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Locating Logical Necessity in Models or the Nature of Situations

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Abstract. Logical necessity is a pivotal notion in accounting for logical consequence, while the source of logical necessity is not widely discussed. In this article I bring to attention one peculiar phenomenon that not only the model-theoretic tradition, but also the essentialist account regarding logical necessity could locate the source of it in logical constants or logical notions. I argue that, under the former tradition, logical constants could be assimilated to restrictions of admissible models, and it is more plausible to locate the source of logical necessity in models; while, under the latter account, and borrowing ideas from the former, we should locate the source of logical necessity in the nature of what models represent, i.e., situations.
Keywords. Logical Consequence; Logical Necessity; Model-theoretic Account; Essentialism; Logical Constants

Loginio būtinumo kilmės alternatyvos: modeliai ar situacijų prigimtis

Santrauka. Loginis būtinumas yra kertinė sąvoka interpretuojant loginį sekimą, tačiau diskusijos apie loginio būtinumo kilmę yra retos. Šiame straipsnyje atkreipiu dėmesį į vieną ypatingą reiškinį, kuris implikuoja, kad ne tik modelio teorijos tradicija, tačiau ir esencialistinis požiūris į loginį būtinumą gali jo kilmę sieti su loginėmis konstantomis ar loginėmis sąvokomis. Aš teigiu, kad, laikantis ankstesnės tradicijos, loginės konstantos galėtų būti asimiliuotos su priimtinų modelių apribojimais ir kad loginio būtinumo šaltinį yra labiau tikėtina aptikti modeliuose; laikantis antrojo požiūrio bei skolinantis idėjų iš pirmojo, loginio būtinumo kilmė turėtų būti aptikta prigimtyje to, kam modeliai atstovauja, t. y. situacijose.
Pagrindiniai žodžiai: loginis sekmuo, loginis būtinumas, modelių teorija, esencializmas, loginės konstantos

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Introduction

Logical consequence is concerned with one specific form of truth-preserving, which involves an absolutely strong modal notion – the notion of logical necessity. The idea is that what distinguishes logical consequence from other kinds of inferences is, among others, the absolute guarantee or 'mustness' that links the conclusion to the premises. This paper explores one specific aspect of logical necessity, that is the source of it, and argues against the accounts which try to locate the source in logical constants or logical entities.

Regarding the importance of logical necessity, some cursory survey of history would suffice to show the predominance that this notion enjoys. For instance, in ancient times, when the subject of logic began to shape, Aristotle viewed a syllogism as a "discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so" (Aristotle 24b, 18). Coming to the pre-modern era, the last universal genius Leibniz held that "true reasoning depends on necessary or eternal truths, such as those of logic, numbers, and geometry, which bring about an indubitable connection of ideas and infallible consequences" (Leibniz 1989: 209). Meanwhile, modern text emphasizes the significance of this notion no less than ancient times, as Bell and Machover nicely gloss in a text on mathematical logic: "What makes [the conclusion] a logical consequence of [the premises] is the fact that if [the premises] are true then [the conclusion] must be true as well" (Bell et al. 1977: 5).

Nowadays, any philosophical account of logical consequence cannot circumvent Tarski's model-theoretic account of logical consequence, since this account is deemed by many as the one true definition of logical consequence, and thus enjoys truism to some extent. What is less known and discussed is the fact that it is his claim that the account should satisfy the so-called adequacy requirement.

> It seems to me that everyone who understands the content of [my] definition must admit that it agrees quite well with ordinary usage. This becomes still clearer from its various consequences. In particular, it can be proved, on the basis of this definition, that every consequence of true sentences must be true, and also that the consequence relation ... is completely independent of the sense of the extralogical constants which occur in these sentences. (Tarski 1956: 417)

The idea is that there are two most important features that any account of logical consequence should meet – necessity and formality. While we can read the requirement of necessity from the 'must' in the text, formality is here tantamount to the requirement that logical consequence should be completely independent of the sense of the extralogical constants. However, while the model-theoretic account satisfies the formality requirement straightforwardly (since in models we can assign any set-theoretic structure to extralogical constants agreeing with their grammatic category), it is far less clear that it can also easily account for the necessity requirement, let alone what exactly the proof that Tarski alluded to is.

Regarding this adequacy (or inadequacy) of accounting for necessity, lots of discussions spawn from Etchemendy's influential 1990 book *The Concept of Logical Consequence*.

This book launches a series of attacks against Tarski's account, which culminates with the conclusion that Tarski's "logical consequence does not capture, or even come close to capturing, any pre-theoretic conception of the logical properties" (Etchemendy 1990: 6). One of the major attacks of Etchemendy is to claim that Tarski committed a fallacy, that is, that Tarski aimed his account to capture the necessity of logical consequence, but his proof could not legitimately establish the goal (See discussions in Sher 1996; Gómez-Torrente 1998; Ray 1996).

One aspect which contributes to the intricacy of the problem is Etchemendy's distinction between interpretational semantics and representational semantics: the former semantics regards models as interpretations of non-logical expressions in our actual world, while the latter regards models as depicting possible worlds[1]. And, since this distinction came out, philosophers have discussed many implications of it (See discussions in MacFarlane 2000; Sher 1996; Shapiro 1998).

Much of the above discussions will be relevant for this paper, but an adjudication between opposite sides of them is not the destination. While this paper also focuses on the problem of accounting for logical necessity, it pulls this question back into a more general background and inquiries into a specific question – what is the source of logical necessity. To wit, this paper questions the possibility and plausibility of locating logical necessity in logical constants or logical entities. What makes this worth discussing is not only that not much literature has explicitly discussed the source of logical necessity, but also that, as we shall see, the already existing discussions have more or less endorsed a common route.

Under the model-theoretic tradition, not many people have patently explained logical necessity in terms of logical constants, but since it is in the spirit of this tradition to put much focus on logical constants, I deem this thesis as a natural extension of the model-theoretic account of logical consequence, and it is most prominently exemplified in Sher (2021). Differently from the model-theoretic tradition, in the current literature, one can also notice the growing popularity of accounting for logical necessity in essentialist terms (Correia 2012; Hale 2013; Keefe et al. 2018; Leech 2021). Specifically, proponents of this route often jettison the talk of the model-theoretic tradition altogether and appeal to the nature or essence of things to account for necessity. One typical conclusion resulting from this strand is the thought that the source of logical necessity should be located in the nature or essence of logical entities, thus, in explaining some φ to be logically necessary, we should appeal to logical entities that φ incorporates, specifically, the essence or nature of them.

Thus, the main phenomenon that I will expose in this paper is that although these two philosophical strands differ, they can nevertheless converge as certain prevalent versions of them choose to locate the source of logical necessity in logical constants or logical entities. But I will argue that, eventually, these attempts on both sides will not work for

[1] In this paper, I shall endorse the latter view throughout. This should not be a limitation of my discussion since models seem to be flexible enough to accommodate both readings (that is exactly why there is the debate about choosing between these two distinctions).

similar reasons, and my positive claim is that we should instead locate logical necessity in models or the nature of situations, that is, what models represent.

Here is the plan for this paper: in Section 1), I expound upon the opposition between essentialism and anti-essentialism when it comes to the explanation of necessity, and how they eventually converge with regard to locating the source of logical necessity, i.e., locating it in logical notions; in Section 2), I examine a specific account under the anti-essentialist strand – Gila Sher's account of logical consequence, logicality and logical necessity – and I raise two objections questioning the plausibility of locating the source of logical necessity in logical constants. Moreover, I argue that we should invoke models instead to account for logical necessity; in Section 3), I argue that a similar kind of reasoning applies to the essentialist strand, thus we need to jettison the essentialist thought of locating the source of logical necessity in logical entities, and instead it should be located in the essence of things that are represented by models – the situations; in Section 4), I consider one potential problem for locating logical necessity in models, and I shall come to the conclusion that my positive thesis is defendable nonetheless.

1. Essentialism and Anti-essentialism about Logical Necessity

If we understand essentialism as the thesis that objects have some of their properties essentially, and that anti-essentialism argues otherwise, then, the opposition between them is too broad a gap to facilitate our discussion. Since our discussion regards the source of logical necessity, I will pin down where exactly the opposition lies by firstly specifying the explanatory relationship between essentialism and modality. Now, a coarse glimpse into history would suffice to notice a change: the explanatory relation between essence and necessity – mostly metaphysical necessity – was reversed in the late 20th century. Before Fine's seminal paper *Essence and Modality*, it was somewhat standard to hold that we should explain essence in terms of necessity, thus the thesis *a property P's being essential for something a* should be analyzed as it is necessary that when *a* exists, then it possesses the property *P*. As it is well-known, Fine's essentialist theory arises from his dissatisfaction that, under this explanation, certain asymmetries of *de re* necessity involving two entities cannot be displayed, with the principal example being it is the essence of a singleton that it contains its sole element, but not the other way around.

Fine's remedy for this problem is simple, as he contends that it should rather be the opposite way that we approach these notions. That is, metaphysical necessities should arise from the essence or nature of all entities, be they objects, concepts or whatever, and it is the essence of things that functions as a more fundamental notion than metaphysical necessities. Therefore, it is necessary for Socrates to be a human being is to be explained by it is Socrates' essence to be a human. And, to extend the thesis, logical necessity should be explained in terms of the essence of what can be loosely called 'logical entities', and it is now just one species of metaphysical necessity since logical entities are just one species among the myriad of entities in the world.

Understood this way, anti-essentialism regarding logical necessity can be deemed as the theoretic strand that does not invoke the essence of logical entities to explain the notion of logical necessity[2]. This is, of course, an umbrella term which encompasses many distinct accounts of logical necessity, the options ranging from invoking the notion of possible worlds to reducing it to the analytic truth, etc. What is especially noteworthy is that the development of mathematic logic in the last 150 years has certainly shed new light on the issue of accounting for logical consequence, which has in its progress rendered the notion of logical necessity more comprehensible. Under the now widely recognized and accepted framework developed in Tarski's *On the Concept of Truth in Formal Languages* and the later-developed model-theoretic tools, we can define that: φ is logically necessary if and only if φ is a logical truth if and only if φ is true in all models. This is already deviant from the essentialist view which explains logical necessity in terms of essence. What is more, under this account we can even define metaphysical necessities in terms of logical necessity by combing metaphysical laws with logical consequence, and logical necessity is thus elevated as the strongest modal notion. Therefore, the contrast between essentialism and anti-essentialism is not only concerned with whether essence is invoked for explanation, but also the theoretic priority between logical and metaphysical necessity.

Then, after spelling out the opposition, the question is how could these two accounts converge with such a substantive gap between them? Notice that in order for the model-theoretic account to work at all, we have to specify a set of logical constants, keep them fixed, and then use models to assign semantic value to all the non-logical terms in a grammatically reasonable way. Also, under the essentialist account, logical entities in arguments are singled out and assigned with an essential role for explaining logical necessity. Therefore, it is this pivotal role of logical constants and logical entities which gives rise to the possible intersection of essentialism and anti-essentialism.

Let us see this in more details. Under the camp of the model-theoretic account of logical consequence we look specifically into Gila Sher's account of logicality as well as logical necessity. Her account has two separable components, the first being the Tarski-Sher thesis, i.e., utilizing the notion of invariance under permutations to demarcate the line between logical and non-logical constants. As explicated by Sher (2021), there are properties which do not care about differences in certain groups of individuals, and logical properties could withstand the most extensive changes of individuals, or, in another word, all the permutations of individuals. Notice that this criterion of logicality alone cannot guarantee us an account of logical necessity, and here the second component comes into the picture as Sher maintains that the notion of logical necessity is within reach once we have attained logicality of logical constants. For instance, the permutation invariance of the property 'identity' gives reason for Sher to attribute logical necessity to the following principle describing this property:

(Id) Every individual is-identical-to itself

[2] Here I intend this strand to include accounts which deny the existence of essence generally and accounts which remain neutral on this issue, for instance, Sher's account.

On the essentialist side, the thought we are going to focus on is a descendent of Finean essentialism, and it reaches a full-blood explication in Hale's work (1996, 2013). Though some recent works (Correia 2012; Leech 2021) have spotted problems in this line of thought and tried to remedy it, they largely remain Finean in spirit nevertheless. The core idea of this account is that logical truth or logical consequence is logically necessary in virtue of the nature or essence of logical entities, while the nature or essence of a thing, according to Hale, is just what it is to be that thing or what distinguishes that thing from every other thing or what is given by its definition (Hale 2013). For instance, Hale gives the following explanation for the necessary truth involving conjunction:

> □ (A conjunction of two propositions A and B is true only if A is true and B is true) because conjunction just is that binary function of propositions the value of which is a true proposition iff both its arguments are true propositions. (Hale 2013: Ch.5)

To take stack, notice that although the two above-presented accounts are essentially different in their theoretic apparatus and reasoning process, they agree on their conclusion that we can attain logical necessity through logical constants or logical entities alone. While, on the latter strand, it is the essence, nature or real definition of logical entity that accounts for logical necessity, in the former case the permutation-invariance of logical constants gives rise to it. In my explication, I have postponed the details concerning these two accounts; instead, what I am eager to bring to attention is the 'coincidence' that, despite the irreconcilable differences between the two philosophical strands, they nonetheless both appeal to logical notions when accounting for logical necessity. And it is this paper's goal to argue against this thought.

2. Sher's Account and the Two Objections

In this section, I shall give a more detailed exposition of Sher's account and consider two objections: while the first objection remains more or less at the intuitive level, the second objection is much more theory-laden as it appeals to Sagi's account of semantic constraints.

Let us start with a closer look into the so-called Tarski-Sher thesis. Though Tarski's attitude regarding the issue of demarcating logical constants has gone through some changes, he has always attached acute significance to the notion of a logical constant:

> Underlying our whole construction is the division of all terms of the language discussed into logical and extra-logical. This division is certainly not quite arbitrary. If, for example, we were to include among the extra-logical signs the implication sign, or the universal quantifier, then our definition of the concept of consequence would lead to results which obviously contradict ordinary usage. On the other hand, no objective grounds are known to me which permit us to draw a sharp boundary between the two groups of terms. (Tarski 1956: 418)

In the concluding remarks of the same paper, Tarski explicitly expresses his pessimism regarding justifying the traditional boundary between logical and extra-logical expressions by addressing that – maybe – no further positive results could be brought out in this inves-

tigation. It turns out that Tarski came up with a criterion in *What are logical notions?*, a paper published posthumously in 1986, whose content is based on two lectures given by Tarski in 1966 and 1973. And the criterion given looks concise and perspicuous enough: "that we call a notion 'logical' if it is invariant under all possible one-one transformations of the world onto itself." (Tarski 1986: 149)

The way Sher (see Sher 1991) approaches this criterion is to view logical notions as operators which are characteristic functions representing them. And, following the convention traced to Lindström's (1966), her version replaces and refines (without changing the basic spirit of Tarski's account) 'permutation' with 'isomorphism' or 'bijection', and 'world' with 'structure'. And the following is one way that we can formulate her criterion:

> **Invariance under Isomorphism**: An operator *O* is logical iff it is invariant under all isomorphisms of its argument-structure.

Without the need for delving into details, we can appreciate the merit of this *Invariance Under Isomorphism* criterion. That is, it provides a mathematically precise and tractable way for us to determine logical operators' logicality. And it is no surprise that all the standard logical operators come out logical according to this criterion. What is more, this criterion provides a basis for deciding many indecisive cases, for instance, quantifiers like 'there are exactly *n* things such that' and 'is a well-ordering' and 'most' could be logical when they represent definite operators.

If this *Invariance Under Isomorphism* criterion really provides us with a satisfactory account of logicality, then it has several significant implications: 1) The most obvious and immediate implication is that this criterion would put an end to the dispute to the ongoing debate surrounding demarcating logical constants. Importantly, this account is not an *ad-hoc* account which merely enumerates the list of logical constants; rather, it provides a precise and plausible standard for incorporating or excluding certain terms or operators as logical constants[3]; 2) secondly, this account could provide us with resources to ascribe certain features to logical consequence (top-neutrality, generality etc.). Without a criterion for logicality, our understanding regarding some or most of these features only remains at a pre-theoretic level, but now we can give grounds for these features.

It is the second implication that I shall bring into discussion. In Sher's most recent paper on the significance of permutation invariance (i.e., Sher 2021), she utilizes this criterion to explain the notion of logical necessity. More specifically, Sher proposes four theses aiming to establish the connection between logical constants' logicality and logical necessity:

> Thesis 1. Every property is invariant under some 1–1 and onto replacement(s) of individuals.
>
> Thesis 2. Some properties have a higher degree of invariance than others; some, but not all, properties have maximal invariance.

[3] Here I shall not address various objections towards the sufficiency and necessity of this criterion of demarcating logical constants as they are largely independent of my own objection.

So far, this is really just an old articulation of Sher's invariance criterion. In requiring 1–1 and onto replacement(s) of individuals, we are just investigating isomorphic structures. This means that when some properties are invariant under all the 1–1 and onto replacement(s), thus maximal invariant, then it is invariant under all the isomorphic structures. However, what is new is the following two theses:

> Thesis 3. The higher the degree of invariance of a given property, the greater the degree of necessity of the laws/principles governing/describing it.
>
> Thesis 4. The higher the degree of invariance of a given field of knowledge, the greater the degree of necessity of, or available to, its laws/principles.

Therefore, the general idea is that, after recognizing logical constants as formal operators possessing the essential feature of being invariance under isomorphism, Sher further specifies a set of formal laws that these logical constants feature in, which helps her attain logical necessity. To give a concrete example, it is easily provable that the property 'is identical to' is invariant under any permutation of individuals; then, the principles governing or describing this property certainly cannot distinguish between any individuals either. And, since the principle governing or describing this property is just

(Id) Every individual is-identical-to itself,

we have reasons to think that (Id) is a law possessing the highest level of modal force (thus, it qualifies as logically necessary), because no matter what kind of actual or counter-actual individual we put into the schema, this principle still holds.

Here I shall bring up my first objection and use the familiar 'is-identical-to' case to illustrate my point. Under a somewhat standard analysis of our understanding of logical necessity, what we want for this notion is for some laws to hold in all logically possible worlds, i.e., the proposition that an individual is-identical-to itself should hold in all logically possible worlds. And the problem is that this critical part of our criterion for modality is largely missing in Sher's account. To be more precise, what can be derived from principle (Id) by the process of replacement of any actual-counterfactual individuals is nothing more than a set of instantiations of the alleged law, for instance:

(a) Object *a* is identical to itself; Object *b* is identical to itself; Object *c* is identical to itself…

But what we really want for a logically necessary law *qua* law is that it holds in all actual and counter-actual situations or possible worlds, for instance:

(b) Object *a* is identical to itself in possible *World One*; Object *a* is identical to itself in possible *World Two*; Object *a* is identical to itself in possible *World Three*…

But it is also unfair to say that Sher's account incorporates no modality. For Sher emphatically stresses that when we consider the group of objects that we should replace

in order to test invariance, we not only have to consider all the actual objects, but also counterfactual objects as well. Now, the question is that Sher does not provide any explicit account of the notion of counterfactual objects, rather, she settles with our common understanding of this notion which is "completely-intuitive and pre-theoretical," and which is open to further diverse precisifications. But even if we admit the legitimacy of invoking counterfactual objects, this kind of modality barely suffices for our purpose, for incorporating counterfactual objects only amounts to attaching additional instantiations to the ever-lasting list (a), what is still wanting is some particular instantiation holding in all possible scenarios like (b).

At this point, one may wonder what better choice we may possess. And here a natural suggestion is that we should appeal to models. For since logical constants are not able to fully incorporate what model-theoretic apparatus could offer us, we should turn to models instead to account for what logical constants cannot by themselves. Next, I shall borrow from Gil Sagi's idea of semantic constraints to explicate this suggestion (Sagi 2014). While Sagi's idea is a revision of the common conception of the logical form, I argue that this revision can be extended to Sher's explanation of the logical necessity.

Sagi starts her account with the thesis which represents the centrality of logical terms:

1) The logical validity of an argument is determined by its form.
2) The form of an argument is determined by the logical vocabulary and the arrangement of all terms in the argument.

Now, while Sagi acknowledges the plausibility of 1), she demurs from 2) and proposes that the form of an argument is rather determined by what is held fixed in the argument under all interpretations. One might wonder at this point what is the difference between these two theses: Is it not that to determine the logical vocabulary just the same process as determining what is held fixed in the argument? It turns out that the answer to this question is not that obvious, and is maybe contrary to intuition. For Sher, these two theses indeed collapse into one, as Sher clearly holds the following: a) logical vocabularies are just those terms denoting notions which are maximally permutation-invariant, and b) logical vocabularies are held fixed in the argument under all interpretations. What is lacking is, of course, a reason for us to move from a) to b), that is, why maximally permutation-invariance gives us reason to hold them fixed? And Sher would most probably bridge these two by invoking the aforementioned thesis that this route is going to help us explain the logical necessity (and other salient qualities of logic) without difficulties.

Now, although Sagi also agrees with the importance of holding some terms fixed in the argument (that is what makes the model-theoretic account possible), she rejects that we should find some independent justification for choosing or demarcating these terms (permutation invariance, for instance). Rather, if we come back to the very purpose of fixing terms in the first place, we should notice that when demarcating logical terms and giving interpretations for them, we essentially deal with the whole set of models and determine what general features these models have. And, after specifying these terms,

we can then assign specific models to non-logical expressions of a language. Thus, fixing the meaning of some terms in the argument can be viewed as restricting the admissible models for the language.

For instance, consider the meta-semantic clause we give for conjunction:

I ($\varphi \wedge \psi$) is true if and only if I (φ) is true and I (ψ) is true

This is, of course, a prominent example of how to spell out semantics for the conjunction symbol. Viewed traditionally, this kind of clause should be given at the outset, and what we do essentially with this clause is giving an interpretation of the conjunction symbol itself. But, as Sagi argues, this traditional understanding can also be somewhat misleading, for they tend to suggest that logical terms are somewhat alienable from the whole semantic system, or being significant on their own. This kind of impression is nicely reflected by Sher's paragraph:

> The meaning of a logical constant is not given by the definitions of particular models but is part of the same metatheoretical machinery used to define the entire network of models.... The meaning of logical constants is given by rules external to the system. (Sher 1991: 49)

In contrast, viewed in the newly proposed way, this clause can be read as the statement that the only models that are admissible are those which abide by this restriction, or, in another word, we exclude those models which are deviant. This reading can readily be applied to all the other meta-semantic clauses that we specify under model-theoretic settings. So, contra Sher, there is no need for some extra rules external to the semantic system which deal with models not individually but in an all-encompassing fashion, rather, the semantic system is just a set of admissible models, and each logical term contributes to restricting them. Because these kinds of constraints are directly related to giving interpretations to languages, Sagi dubs them *semantic-constraints*. We can, as Sagi does, use the symbol Δ to specify the set of all the semantic constraints; then, Δ-models would be those which are admissible under these constraints.

It is noteworthy that Sagi's account stands not in complete opposition with the traditional term-based account in determining the form of an argument; rather, it can be deemed as a more flexible and general extension. What is in stark opposition with Sher's route, as I shall argue, is rather our explanation of logical necessity.

Now, within the setting of semantic constraints, it is clear that φ is logically true if and only if φ is true in all Δ-models, with Δ being the set of semantic constraints that are linked to certain terms in φ. Compared with the more traditional definition of logical truth, it makes explicit the role of logical constants in contributing to logical truth, i.e., restricting models. And now, if we go on asking which intuitive idea is captured by this definition, it is the following: we usually understand logical necessity as true in all logically possible worlds, and we deem each Δ-model as representing a logically possible world. Thus φ's being true in all Δ-models would represent that φ's are true in all the logically possible worlds, and this in turn amounts to that φ is logically necessary. In short, under this ac-

count, for φ to be logically necessary is just for it to be true in each and every Δ-model. Notice that this is a reductive explanation of logical necessity, and logical constants play, compared with Sher's account, at most a negative role in this explanation, i.e., the role of excluding inadmissible models. And if we come back to the problem of the two lists posted earlier, we should come to the conclusion that it is only through this invoking of models that could help us obtain list (b) which exhibits logical necessity.

To take stack, logical constants are indeed significant, but they specify only what all the admissible models have in common, and it is ultimately admissible models which do the explanatory work. Thus, Sher's reliance on the permutation-invariance to account for logical necessity should give way to admissible models. This concludes our discussion of the model-theoretic route of accounting for logical necessity.

3. Essentialist Account of Logical Necessity

In the last section, I have argued for the thesis that under the model-theoretic strand, models, compared with logical constants, are better suited to account for logical necessity. This section extends the argument to the essentialist strand.

As mentioned in Section 1), it is not any type of essentialist account that is under investigation. Rather, the essentialist account at issue specifies the essence as the real definition of things, thus, *definitional essentialism* is a suitable name of it. For instance, to account for metaphysical necessity and reverse the traditional explanatory priority, Fine not only takes essence as primitive, but also proposes that essence can be illuminated by the notion of real definition, while *x*'s real definition consists of all the propositions that are true in virtue of the nature (or identity) of *x*. And, according to Hale, we can "think of the nature or identity of a thing as what is given by its definition – that is, the definition of the thing, and not that of some word for the thing or concept of the thing" (Hale 2013: 132–3). Similarly, Lowe proposes that "the essence of something, X, is what X is, or what it is to be X" (Lowe 2008: 35).

Although this account is mainly concerned with metaphysical necessity, we can further obtain more refined kinds of necessity just by restricting the class of objects that are invoked in explanation. Therefore, conceptual necessities would be those propositions necessarily true in virtue of the nature of concepts, and logical necessities can be explained by the nature of what can be loosely called the 'logical entities'.

Lots of people regard Fine's paper to be one of the most important papers on essentialism of the last decades, and some of them try to elaborate the idea further to make it more convincing. For instance, Correia (2012) argues that Fine's account commits one to a distinction between brute or basic and derivative essentialists facts, and then presents a conception of derivation in terms of logical consequence[4]. Revisions as such are certainly important steps to make an essentialist account of logical necessity more plausible, but they nevertheless remain in the same vein. Now, if we dwell on our discussions of the model-theoretic account

[4] Correia dubs his account as 'the rule-based' account, see an extended defence of it in Correia (2020).

in the last section, we can smell that maybe something is off when we put so much weight on logical entities. For, under the model-theoretic tradition, logical constants are by themselves incapable to secure logical necessity and they can be assimilated to something else, and the lesson should apply to the essentialist account of logical necessity. To introduce what exactly my proposal is, I borrow one critical passage in Shalkowski (2004):

> Essentialists have at least two ways of explaining logical necessity. One is to explain logical necessities as those true in virtue of the natures of logical items, perhaps propositions and their constituent concepts or else other truth bearers and their constituents. Alternatively, logical necessities might be explained as those propositions true in virtue of the natures of every situation or every object and property, thus preserving the idea that logic is the most general science. (Shalkowski 2004: 79)

Shalkowski seems to suggest in the passage that there are two equally good and mutually compatible routes for an essentialist account of logical necessity: The first of those is instantiated by Fine's and Hale's account which explains logical necessity in terms of the nature of logical notions; while the second route is still somewhat unexplored by essentialists, that is, explaining logical necessity in the nature of every situation of the world. But notice that these two routes do not look similar at all, for logical items are abstract functions whereas situations involve concrete objects and properties (under a common understanding). So, the question is: *Are they really equally plausible routes?* Next, I shall argue that the answer is no, and it is the second route that is more plausible.

Remember that, in the last section, instead of giving logical terms their own interpretations, we view relevant clauses as spelling out semantic constraints for admissible models. What this implies for the essentialist camp is that, instead of viewing logical entities as those whose nature is truth-value function and which exist like a kind of abstract but still objective entities, we should regard them as more closely related to or even embedded in the things, properties and situations of the world. How do we establish this relation? The natural thought is that we use models as the bridge. That is, since logical constants are constraints of admissible models and admissible models represent possible situations, then logical constants represent constraints of possible situations. Now, the problem is to cash this out in the terms of essentialism.

Let us use the familiar case to illustrate my idea:

I ($\varphi \wedge \psi$) is true if and only if I (φ) is true and I (ψ) is true

I have already argued that, from the model-theoretic perspective, this clause can be read as the statement that the models that are admissible are those which abide by this restriction. Now, since models just represent possible situations, then what this clause states is a restriction of possible situations that for any complex situation in the form 'φ and ψ' to obtain, φ must obtain and ψ must obtain. In another word, any violation of this restriction would incorrectly state what situations are possible. What this tells us about the nature of logical notions is that logical notions play merely a role of excluding impossible situations, or, situations that are, in essentialist terms, precluded by the nature of situations.

What exactly is the nature of situations? Following the route of equating the nature or the essence of things with the real definition, we can say that this nature is manyfold: 1) In every situation, there are objects; 2) In every situation, objects have properties and hang in various relationships, etc. Thus, we can also see that what makes admissible models represent possible situations is that models just specify the nature of situations: 1) They are specified by a model domain containing individual objects; and 2) They are specified by a model's interpretation function which gives extensions to predicates.

This gives us sufficient reason for choosing Shalkowski's second route over the first, and this reason is parallel to that we appealed to in the last section. We stick to the thought that logical necessity comes from being true in all logically possible worlds, or being true no matter what the situations would be like. Under the model-theoretic account, since models just represent logically possible worlds, logical necessity is attained by being true in all models. And, because logical constants merely exclude inadmissible models, we should invoke models to account for logical necessity. Now, similarly, under the essentialist account, what a possible situation could be like is determined by the nature of situations, or, in another word, the possibility of a situation is determined exactly by its being compatible with the nature of situations. Meanwhile, the nature of logical notions is that they merely serve to exclude impossible situations, or situations that are precluded by the nature of situations since they are incompatible with it. We should therefore conclude that 'logically necessary' is obtained in virtue of the nature of situations.

Note that this paper does not aim to adjudicate between the essentialist and the anti-essentialist account. Actually, as we have seen, both sides have something to say about accounting for logical necessity. Rather, the aim is merely to point out that, on both accounts, we have reasons not to choose logical constants or the nature of logical constants to account for logical necessity. That is, they play merely a heuristic role relative to models or the nature of situations specified by models[5].

4. The Scope of Models and Situations

In this section, I shall address a critical problem for the model-theoretic account. Notice that our reason for choosing models to account for logical necessity relies on the thesis that models represent possible worlds, or that there is a one-one correspondence between them. But does this thesis really stand?

This poses an acute problem considering that model domains are essentially just sets, and sets have their limitations. Specifically, there are certain paradoxical problem in sets' foundation such as Russell's Paradox, which could result in possible situations which seemingly do not correspond to any model. For instance, presumably, there is a possible world where everything is self-identical, which is just our actual world, but there is simply no model to correspond to, at least for models of first-order languages. The reason is that in order for a first-order model to correspond to such a possible world, the set assigns to

[5] My thanks go to an anonymous reviewer for pointing this out.

the predicate should be everything in the world, which makes it no longer a set, but rather a proper class. However, given the constraint that the domain of a first-order model should be a set, no model could assign this proper class to the predicate.

An attempt to remedy the situation is alluded to by Etchemendy (1990), and Sher (1996)[6], and it is currently, to the best of my knowledge, the most plausible attempt. As observed by Sher (1996), the pivotal thesis in establishing logical necessity under the model-theoretic account is that there is always a Tarskian model which could represent any situation there could be. And Sher argues that, at least under the realm of first-order logic and with the help of the well-established properties of the first-order logic, we can indeed attain a proof. More specifically, the focal idea of this proof is to appeal to the first-order deductive system and the Completeness Theorem. A simplified version of the proof is as follows:

P1. Axioms express necessary truths.
P2. Rules of inference preserve necessary truths.
P3. (From P1 and P2) Any φ that is provable expresses necessary truth.
P4. According to completeness theorem of first-order logic, any φ that is model-theoretically truth is also provable.
Conc. (From P3 and P4) Any φ that is model-theoretically truth expresses necessary truth.

This sounds like a neat and compelling argument, at first sight, the problem with this argument lies within the first premise of this argument. And it is concerned with how we should understand the notion of the necessary truth under a proof system. One initial thought is that the axioms do not express necessary truth through which our inference can start from absolutely sound foundations, rather, they are relativized to specific purposes and systems, and thus they always carry a sense of arbitrariness and relativity. More significantly, the notion of truth here is deviant from that in the model-theoretic account, i.e., for some proposition to be true is for it to be proved, or for us to have evidence for it. Based on this conception, one could argue that logical necessity under the proof system is to be understood as necessity of thought (see Prawitz 2005). But notice that this is a long way from our conception of what logical necessity is under the model-theoretic account. Therefore, this argument cannot really give us the thesis that we need.

Based on our discussions, I would like to point out a tentative solution to the problem. Note that, up until now, we have taken the following route: to determine the extension of admissible models, we should firstly determine the scope of situations, and then, since models represent situations, we come derivatively to the set of admissible models. The idea underlying this route is that we seem to be far more familiar with situations than with models since we have a direct and causal contact with the former, while models are merely abstract representational devices. Thus, prioritizing situations would establish models on an ontologically and epistemologically good footing. The drawback of this

[6] It is essentially borrowed from Georg Kreisel's 'squeezing argument' in Kreisel (1967).

route is that once we have come to the realm of possible situations, our perceptions lose their validity, and people's intuitions hardly agree with each other.

The tentative solution here consists of a reversing of the priority and letting models decide which possible situations there are. The thought is that since it is the nature of situations which decide which situations are possible, and models just represent situations by specifying the nature of situations, then, whatever is constructable, using models would not be in contradiction with or precluded by the nature of situations, and thus being possible. This constitutes a far more substantive understanding and usage of models since they are now constructive rather than descriptive. Moreover, we are led to the conclusion that what can be constructed model-theoretically would be logically possible. This thesis could be controversial since people have intuitions about what really is logically possible and can thus make a comparison. For instance, when asking what all the possible variations in the world vis-à-vis Cerberus (Sher's dog) are, Sher finds herself muddled in a series of thorny problems regarding "recalcitrant questions of identity, essential properties, moral and rational agency, meaning, etc." (Sher 1996: 661), and there is simply no easy answer for these questions.

How plausible this tentative solution is remains to be decided, but it needs not to be done here. An acute problem as it may be, this should not affect the conclusion we have drawn from our discussions, that is, models or the nature of situations specified by models are better suited to account for logical necessity compared to logical constants.

Conclusion

In this paper, I try to bring to attention one intriguing phenomenon regarding the explanation of logical necessity. That is, both essentialism and anti-essentialism could locate the source of logical necessity in logical notions. I argue that, on the anti-essentialist side, and, more specifically, the model-theoretic account, logical constants are not fitted for the job because they only specify what inadmissible models there are, and we should invoke admissible models to account for logical necessity; similarly, on the essentialist account, the nature of logical notions is that they merely serve to exclude impossible situations, while it is the nature of situations that decide which situations are possible, and thus could lead us to logical necessity. Therefore, on both accounts, we have better options to locate the source of logical necessity – in models or in the nature of situations that models represent.

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