ESTIMATION OF PARAMETERS AND VERIFICATION OF STATISTICAL HYPOTHESES FOR GAUSSIAN MODELS OF STOCK PRICE

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Abstract. We construct models of asset prices on the Ukrainian stock market and analyse their applicability by checking appropriate statistical hypotheses using actual observed data. We also analyse the presence of jumps in the dynamics of different assets and estimate the Hurst coefficient for the logarithm of the price of the asset by two different methods.

Keywords: Ukrainian stock market, Ukrainian Stock Exchange, fractional Brownian motion, estimation of Hurst coefficient.

1. Introduction

The modern economy heavily relies on various statistical methods. Particularly widespread are the methods of statistical analysis of stock prices in the stock markets, to predict the behaviour of stocks in the future. The exchange segment of the Ukrainian stock market is not sufficiently analysed in comparison to the stock markets of developed countries. In Ukraine, there are ten operating stock exchanges, namely:

- PJSC “Ukrainian Stock Exchange” (Kyiv) ;
- PJSC “East European Stock Exchange” (Kyiv);
- PJSC “Perspectiva Stock Exchange” (Dnipropetrovsk);
- PrJSC “Ukrainian Interbank Currency Exchange” (Kyiv);
- PrJSC “Ukrainian International Stock Exchange” (Kyiv);
- PJSC “Kyiv International Stock Exchange” (Kyiv);
- PJSC “PFTS ua-exchange” (Kyiv);
- PrJSC “Prydniprovska Stock Exchange” (Dnipropetrovsk);
- PrJSC “Stock Exchange “INNEX” (Kyiv);
- PrJSC “Ukrainian stock exchange” (Kyiv).

We analyse data from the Ukrainian Stock Exchange, which focuses on the classical model of trading stocks. Since 2009, the Ukrainian Stock Exchange trades on all major types of securities (stocks, bonds, options, futures) and trading volumes have been steadily increasing. In this paper we investigate methods of parameter estimation and verification of statistical hypotheses for a Gaussian model of the Ukrainian Stock Exchange Index, which is calculated as the average price of 10 Ukrainian "blue chip" stocks (stocks of Ukraine's largest companies and leaders in their fields). For technical analysis we use data for received from the website of the Ukrainian Stock Exchange [3] for the period from 31 March 2014 to 28 August 2014 with observations taken every 5 minutes (a total of 8000 observations). A short description of the analysed data is presented in Table 1.

Table 1. Analysed assets

Marking	Frequency of observations	Asset	Period
UX-5m	every 5 minutes	Ukrainian Stock Exchange Index	31/3/2014 – 28/8/2014

The following figure shows dynamics of the analysed asset with respect to its value at the beginning of the period analysed.

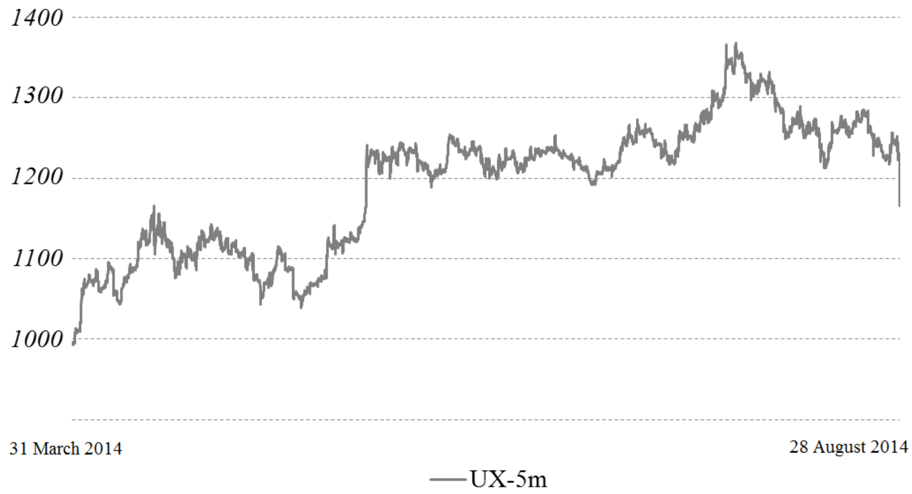

Figure 1. Dynamics of Ukrainian Stock Exchange Index during the analysed period

2. Analysis of the presence of jumps in the dynamics of the Ukrainian Stock Exchange Index

Financial markets sometimes generate a significant number of gaps in the price of certain shares, so-called jumps. Many practical and theoretical studies prove the existence of jumps and show their significant impact on financial management (in particular, risk-management and hedging of the portfolios of securities). The problem of jump identification is quite complex, because only discrete data are available to researchers. Recently, a variety of techniques and statistical tests, which allow the determination of jumps in the dynamics of stock prices, was developed. Some of them we apply to the assets that we investigate. The question of the presence of jumps is essential for the purposes of analysis of stock dynamics as they can significantly distort the results.

We build a model of the stock price on a fixed probability space (Ω, F_t, P), where filtration $\{F_t, t \in [0,T]\}$ –corresponds to the information that is available to market participants. If there are no jumps in the dynamic of stock price $S(t)$, we suggest it can be described by the standard lognormal model:

$$d\ln(S(t)) = \mu(t)dt + \sigma(t)dW(t), \tag{1}$$

where $W(t)$ is F_t-adapted standard Brownian motion, while processes $\mu(t)$ and volatility $\sigma(t)$ are F_t-adapted processes, such that $S(t)$ is Ito's process with continuous trajectories.

If there are jumps in the stock price dynamic, then

$$d\ln(S(t)) = \mu(t)dt + \sigma(t)dW(t) + Y(t)dJ(t), \tag{2}$$

where $J(t)$ is a counting process independent of $W(t)$, which is responsible for the appearance of jumps. $Y(t)$ determines the size of jump at time t and does not depend on the previous jumps or $W(t)$.

Observations of $S(t)$ are available at discrete time points $0 = t_0 < t_1 < \ldots < t_n = T$, furthermore $\Delta t = t_k - t_{k-1}$ is constant for all k.

We consider local dynamics of the process within the "window" of K consecutive observations that are used to determine the statistics defined below

Definition 1. Statistics $L(k)$, which allows us to check if there was a jump during the time interval $(t_{k-1}, t_k]$, is defined as follows

$$L(k)=\frac{\ln\left(\frac{S(t_k)}{S(t_{k-1})}\right)-\hat{m}_k}{\hat{\sigma}_k},$$

where $\hat{m}_k=\frac{1}{K-1}\sum_{j=i-K+1}^{i-1}\ln\left(\frac{S(t_j)}{S(t_{j-1})}\right)$ and $\hat{\sigma}_k^2=\frac{1}{K-2}\sum_{j=i-K+2}^{i-1}\left|\ln\left(\frac{S(t_j)}{S(t_{j-1})}\right)\ln\left(\frac{S(t_{j-1})}{S(t_{j-2})}\right)\right|$.

[1] provides proof of the following theorem, which allows us to build a statistical test for identification of jumps in the stock price dynamics by discrete observations.

Theorem 1. Let $L(i)$ meet Definition 1, with the size of the "window" $K=O_p\left(\Delta t^{\alpha}\right)$, where $-1<\alpha<-0,5$. Assume that the process $S(t)$ is described by equation (1) or (2). Let A_n' be a set of $k\in\{1,\dots,n\}$ such that for time interval $\left(t_{k-1},t_k\right]$ there are no jumps. If $\Delta t\to 0$, then

$$\frac{\max_{i\in A_n'}|L(i)|-C_n}{S_n}\to\xi,$$

where ξ has the distribution function $F(x)=\exp\left(-e^{-x}\right)$

$$C_n=\frac{(2\ln(n))^{\frac{1}{2}}}{c}-\frac{\ln(\pi)+\ln(\ln(n))}{2c(2\ln(n))^{\frac{1}{2}}},\ c=\frac{\sqrt{2}}{\sqrt{\pi}}\approx 0{,}7979,\ S_n=\frac{1}{c(2\ln(n))^{\frac{1}{2}}}$$

and n is the number of observations.

Based on this theorem we can construct a statistical test to check the hypothesis

H_0*: during time interval* $\left(t_{k-1},t_k\right]$ *there were no jumps in the stock price dynamic* against the alternative

H_1*: during time interval* $\left(t_{k-1},t_k\right]$ *there was at least one jump in the stock price dynamic and this dynamic is described by model* (2).

Let α be the significance level. If $\frac{|L(i)|-C_n}{S_n}\le-\ln\left(-\ln(\alpha)\right)=\beta$, then we accept the null hypothesis, otherwise we accept the alternative hypothesis. The parameters used for the purposes of our analysis of jumps in the stock dynamics are given in Table 2.

Table 2. Parameters for analysis of jumps in the stock dynamics

UX-1d
$K=255$
$\alpha=0,95$
$\beta=-\ln\left(-\ln(0,95)\right)=2,9702$
$n=8000$
$C_n=3,8442$
$S_n=0,2956$

After all the necessary calculations, according to the test, we can see that there are jumps in dynamic of the Ukrainian Stock Exchange Index. More precisely, 49 jumps took place during the period analysed.

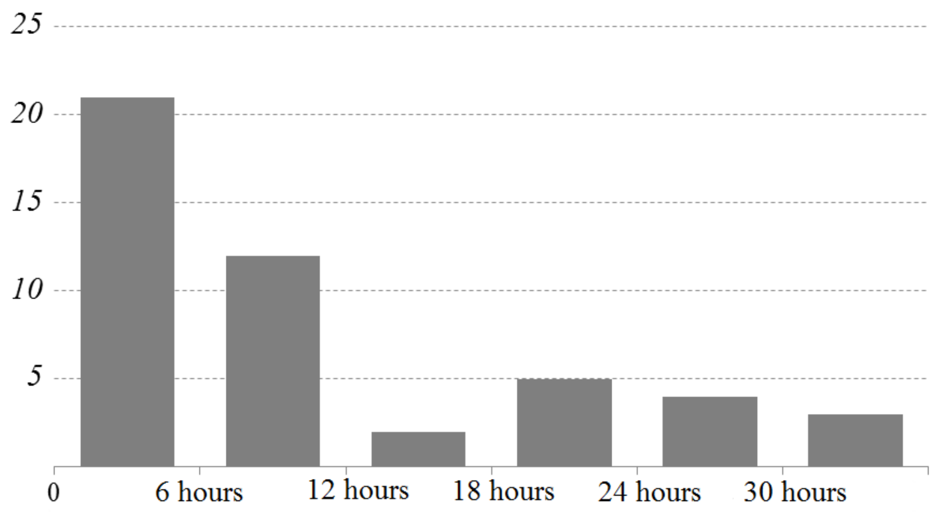

Figure 2. The histogram of time gaps between two consecutive jumps in the dynamics of the Ukrainian Stock Exchange

As mentioned earlier, jumps can have a significant impact on the results of the analysis of the dynamics of the Ukrainian Stock Exchange Index. Thus, for the purposes of further analysis, corresponding values were excluded from the stock dynamics. The following figure shows the dynamics of the Ukrainian Stock Exchange Index during the analysed period after the exclusion of the jumps.

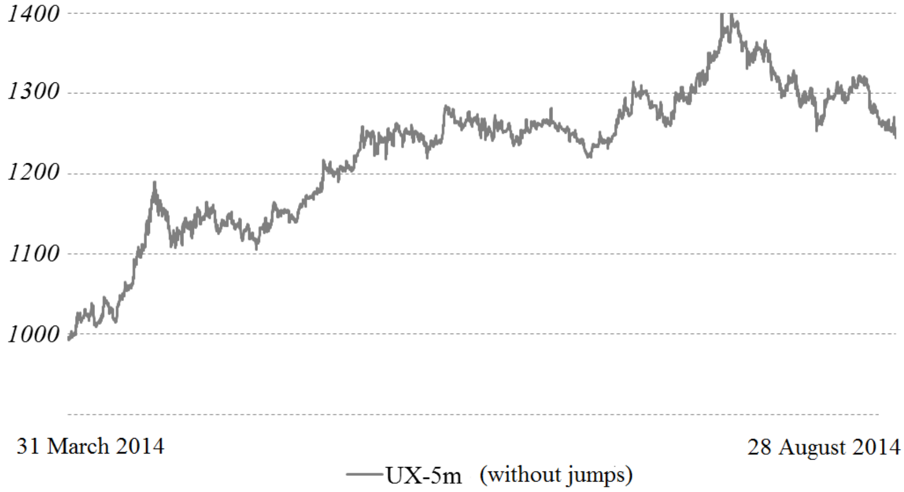

Figure 3. The dynamics of Ukrainian Stock Exchange index during the period analysed, with jumps removed.

3. Fractional Brownian motion and the analysed model

As part of this paper we consider a stochastic model of the Ukrainian Stock Exchange Index based on fractional Brownian motion, which is described in detail below.

Fractional Brownian motion is a generalization of the Wiener process. However, unlike the case of the Wiener process, the increments of Fractional Brownian motion are correlated (and therefore dependent). Processes of this type were firstly considered in the work of Mandelbrot and Van Ness in 1968.

Definition 2. A random process $B_H(t)$ defined on the time interval $[0,T]$ is called a standard fractional Brownian motion with a Hurst coefficient $H \in [0,1]$, if:

(1) $B_H(t)$ is a Gaussian process;

(2) $B_H(0)=0$;

(3) $EB_H(t)=0$;

(4) $EB_H(s)B_H(t)=\frac{1}{2}\left(t^{2H}+s^{2H}-|t-s|^{2H}\right)$, for any $t,s\in[0,T]$.

The value of the Hurst coefficient H determines the type of process $B_H(t)$:

- If $H=\frac{1}{2}$, then the process $B_H(t)$ is a Wiener process;
- If $H>\frac{1}{2}$, then increments of $B_H(t)$ are positively correlated;
- If $H<\frac{1}{2}$, then increments of $B_H(t)$ are negatively correlated.

Note, that the process of increments of the fractional Brownian motion $X_H(t)=B_H(t+1)-B_H(t)$ is called fractional Gaussian noise and

$$EX_H^2(t)=E(B_H(t+1)-B_H(t))^2=1.$$

Fractional Brownian motion has a number of important properties, most of which are listed in the following proposition.

Proposition 1. Fractional Brownian motion $B_H(t)$ has the following properties:

(1) Process $B_H(t)$ is self-similar, namely, in terms of distributions $B_H(at)\sim a^H B_H(t)$. Among all Gaussian processes only fractional Brownian motion has the property of self-similarity.

(2) $B_H(t)$ is a process with stationary increments, i.e.:

$$B_H(t)-B_H(s)\sim B_H(t-s)$$

(3) Trajectories of fractional Brownian motion $B_H(t)$ are almost everywhere non-differentiable. However, almost all trajectories $B_H(t)$ have a Holder exponent strictly less than H, i.e., for each such path there is a constant C, such that for $0<\varepsilon<H$:

$$|B_H(t)-B_H(s)|\le C|t-s|^{H-\varepsilon}$$

(4) For fractional Brownian motion it is possible to define stochastic integrals, also known as fractional stochastic integrals.

We consider the following lognormal model of the Ukrainian Stock Exchange Index:

$$Y(t)=Y_0\exp(\mu t+\sigma B_H(t)),$$ where

$Y(t)$ is the value of Ukrainian Stock Exchange Index at moment t;

$B_H(t)$ is fractional Brownian motion with a Hurst coefficient H;

μ and σ are coefficients of drift and volatility respectively.

First, we consider the problem of estimation of the Hurst coefficient H by discrete observations of process $Y(t)$ at time points:

$$t_i=\frac{iT}{N}, i=1,\ldots,N.$$

4. Estimation of the Hurst coefficient.

4.1. Definition of the Hurst coefficient and its evaluation using scalable rank

The Hurst coefficient is a measure of long-term memory, which is used in the analysis of time series. Below we provide a precise definition.

Definition 3. The Hurst coefficient H is defined in terms of the asymptotic behavior of the rescaled range as a function of the time span of a time series as follows:

$$E\left[\frac{R(n)}{S(n)}\right]\sim n^H, n\to\infty$$

where $R(n)$ is the range and $S(n)$ is the variance of the first n observations.

This definition allows us to build a simple estimation procedure for the Hurst coefficient based on the observed values of the time series $\{X_k, k=1,\ldots,N=2^m\}$ using the following algorithm [2].

For any $n=2^k, k=1,\ldots,m$ it is necessary to calculate rescaled range of the time series $\{X_k, k=1,\ldots,n\}$. Thus, consider the auxiliary time series $\left\{Z_k=\sum_{i=1}^{k}(X_i-\mu), k=1,\ldots,n\right\}$, where $\mu=\frac{1}{n}\sum_{k=1}^{n}X_k$ is corresponding average. Then the estimator of the rescaled range can be found as

$$\frac{R(n)}{S(n)}=\frac{\max(Z_1,\ldots,Z_n)-\min(Z_1,\ldots,Z_n)}{\sqrt{\frac{1}{n}\sum_{k=1}^{n}(X_k-\mu)^2}}.$$

Now an estimator of the Hurst coefficient can be found by applying a regression analysis to the following linear model

$$\ln\left(\frac{R(n)}{S(n)}\right)=C+H*\ln(n)$$

where C is some constant.

We apply this method to actual data. Let $\{Y_k, k=1,\ldots,n\}$ be a time series which represents stock price at moments t_k. Then the input data for the purposes of analysis is an auxiliary time series $\left\{X_k=\ln\left(\frac{Y_{k+1}}{Y_k}\right), k=1,\ldots,n-1\right\}$, which reflects the dynamics of changes in profitability in the lognormal model. Using the R software environment we made the necessary calculations and the conducted regression analysis. A scatter plot of corresponding regression model is presented below.

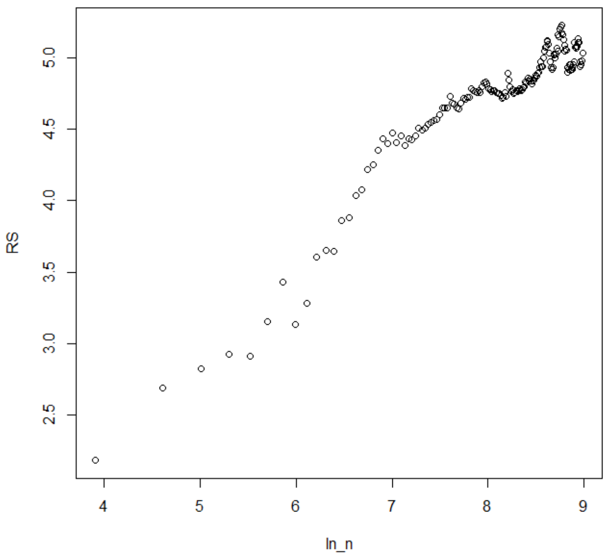

Figure 4. Scatter Plot

From the results of the analysis we found the estimator of Hurst coefficient to be

$$\hat{H}_1=0.5252907.$$

We also calculated upper and lower bounds of 95-percent confidence interval for H

$$\hat{H}_1^{Up}=0.588715,$$
$$\hat{H}_1^{Down}=0.461867.$$

Thus we see that the Hurst coefficient for the observed values of the Ukrainian Stock Exchange Index is greater than 1/2, indicating a positive long-term correlation between increments of Ukrainian Stock Exchange Index. Thus, after high values Ukrainian Stock Exchange Index increases are more likely, and vice versa.

This can be partially explained by influence on the stock market of various macroeconomic factors and the overall economic situation (we observe a serious long-term decline since 2008, resulting from the long-term global economic crisis).

However, given the fact that we consider a Ukrainian Stock Exchange Index with a high frequency of observations (in our case - every 5 minutes), we can see that the above factors have much less impact, and short-term random factors have much more significant effect.

4.2. Estimation of the Hurst coefficient in the analysed model using quadratic variations

For the analysed model the Hurst coefficient can be estimated using the property of self-similarity of fractional Brownian motion. For this purpose we use the following theorem, which is a consequence of results of Dozzi, Mishura and Shevchenko [5].

Theorem 2. Let $B_H(t)$ be a fractional Brownian motion with Hurst coefficient H that is observed at moments $t_j = \frac{jT}{N}, j = 0, \dots, N$. Then

$$\sum_{j=l}^{N} \left(B_H(t_j) - B_H(t_{j-l})\right)^2 \sim T^{2H} l^{2H} N^{1-2H}, N \to \infty.$$

Based on this theorem we can prove the following proposition.

Proposition 2. Let the process $Y(t)$ satisfy the following model

$$Y(t) = \exp(\mu t + \sigma B_H(t))$$

where $B_H(t)$ is a fractional Brownian motion with Hurst coefficient H, μ and σ are constant, and this process is observed at moments $t_j = \frac{jT}{N}, j = 0, \dots, N$. Then

$$\sum_{j=l}^{N} \left(\ln \left(\frac{Y(t_j)}{Y(t_{j-l})} \right) \right)^2 \sim \sigma^2 T^{2H} l^{2H} N^{1-2H}, N \to \infty.$$

Proof. By the definition of process $Y(t)$, we obtain

$$\sum_{j=l}^{N} \left(\ln \left(\frac{Y(t_{j+1})}{Y(t_j)} \right) \right)^2 = \sum_{j=1}^{N} \left(\mu \frac{t}{N} + \sigma \left(B_H(t_j) - B_H(t_{j-1})\right) \right)^2 =$$

$$\sum_{j=1}^{N} \left(\mu \frac{T}{N} \right)^2 + \sum_{j=1}^{N} \mu\sigma \frac{T}{N} \left|B_H(t_j) - B_H(t_{j-1})\right| + \sigma^2 \sum_{j=1}^{N} \left(B_H(t_j) - B_H(t_{j-1})\right)^2 \leq$$

$$\leq \sum_{j=1}^{N} \left(\mu \frac{T}{N} \right)^2 + \sum_{j=1}^{N} C\mu \left(\frac{T}{N} \right)^{1+H-\varepsilon} + \sigma^2 \sum_{j=1}^{N} \left(B_H(t_j) - B_H(t_{j-1})\right)^2$$

Where in the last inequality we used property (4) of fractional Brownian motion from Proposition 1. It follows from Theorem 2 that the last sum is

$$\sigma^2 \sum_{j=l}^{N} \left(B_H(t_j) - B_H(t_{j-l})\right)^2 \sim \sigma^2 T^{2H} l^{2H} N^{1-2H}.$$

The first two sums go to zero faster than N^{1-2H} for $\varepsilon < 1 - H$. Thus, we have

$$\sum_{j=l}^{N}\left(\ln\left(\frac{Y(t_{j+l})}{Y(t_j)}\right)\right)^2 \sim \sigma^2 T^{2H} l^{2H} N^{1-2H}.$$

Proposition 2. allows us to build a simple estimator for the Hurst coefficient H, which is derived from observations of process $Y(t)$ at points $t_j = \frac{jT}{N}, j = 0,\ldots,N$ and based on the ratio of quadratic variations

$$\hat{H}_2 = \frac{1}{2\ln(2)}\ln\left(\frac{\sum_{j=2}^{N}\left(\ln\left(\frac{Y(t_j)}{Y(t_{j-2})}\right)\right)^2}{\sum_{j=1}^{N}\left(\ln\left(\frac{Y(t_j)}{Y(t_{j-1})}\right)\right)^2}\right).$$

Obviously the consistency of this estimator is a direct consequence of Proposition 1.

We also applied this estimation procedure to the actual values of the Ukrainian Stock Exchange Index for the period analysed, and determined that the estimate of the Hurst parameter is

$$\hat{H}_2 = 0.5236212.$$

The two different estimation methods that we considered give almost identical results. For further analysis we use estimator $\hat{H}_2$. Now we can consider the problem of testing the hypothesis of whether the observed dynamics of the Ukrainian Stock Exchange Index correspond to the analysed model.

5. Test for the verification of whether the observed data can be described by the analysed model

5.1. Presentation of the Wiener process as a fractional stochastic integral

Fractional Brownian motion can be presented in the form of a stochastic integral from a Wiener process and vice versa. In particular, in [4] Ilkka Norros, Esko Valkeila and Jorma Virtamo considered kernel, fractional stochastic integrals which make it possible to convert fractional Brownian motion into a Wiener process. This transformation is described in detail below.

Consider the following function

$$w(t,s) = \begin{cases} c_1 s^{\frac{1}{2}-H}(t-s)^{\frac{1}{2}-H}, & s \in (0,t) \\ 0, & s \notin (0,t) \end{cases}$$

where $c_1 = \left(2HB\left(\frac{3}{2}-H, \frac{1}{2}+H\right)\right)^{-1}$.

The following theorems are proved in [4].

Theorem 3. Let $B_H(t)$ be a fractional Brownian motion with Hurst coefficient H. Then the centered Gaussian process

$$M_t = \int_0^t w(t,s)\,dB_H(s)$$

has independent increments and

$$EM_t^2 = \frac{c_H^2}{4H^2(2-2H)} t^{2-2H}$$

where $c_H = \left(\frac{2H\Gamma\left(\frac{3}{2} - H\right)}{\Gamma\left(\frac{1}{2} + H\right)\Gamma(2-2H)} \right)^{\frac{1}{2}}$. In particular M_t is a martingale.

Theorem 4. Let M_t be a stochastic integral which was defined in Theorem 3 and define

$$W_t = \frac{2H}{c_H} \int_0^t s^{H-\frac{1}{2}} dM_s.$$

Then W_t is a Wiener process.

Thus, the fractional Brownian motion can be transformed into Wiener process through simple integral transformation. Note that this property is characteristic for fractional Brownian motion. Thus, if the process W_t, constructed as above, is a Wiener process, then the process $B_H(t)$ is fractional Brownian motion. This feature allows to build a test that verifies whether a time series can be described by the analysed lognormal model.

5.2. Statistical test of correspondence to the analysed model

Our task is to build a test based on the observations of time series $Y(t)$ at points $t_k = \frac{kT}{N}, k = 0, \ldots, N = N_r^3$ which allows us to check the following hypothesis:

$$H_0: \ Y(t) \textit{ is described by model } Y(t) = Y_0 \exp\left(\mu t + \sigma B_H(t)\right),$$

where $B_H(t)$ is a fractional Brownian motion with Hurst coefficient H, and μ and σ are the coefficients of drift and volatility, respectively.

In our case time series $Y(t)$ represents the Ukrainian Stock Exchange Index.

Consider the following linear transformation of the time series $Y(t)$:

$$M_i = \sum_{k=0}^{iN_r - 1} w\left(\frac{iN_r T}{N}, \frac{kT}{N}\right)\left(\ln\left(Y\left(\frac{(k+1)T}{N}\right)\right) - \ln\left(Y\left(\frac{kT}{N}\right)\right)\right), i = 1, \ldots, N_r^2$$

$$m_i = \sum_{k=0}^{iN_r} w\left(\frac{iN_r T}{N}, \frac{kT}{N}\right)\frac{T}{N}, i = 1, \ldots, N_r^2.$$

and further the transformation of the obtained time series:

$$L_i = \sum_{k=0}^{iN_r - 1} \left(\frac{kN_r T}{N}\right)^{H-\frac{1}{2}} \left(M_{k+1} - M_k\right), i = 1, \ldots, N_r.$$

$$l_i = \sum_{k=0}^{iN_r - 1} \left(\frac{kN_r T}{N}\right)^{H-\frac{1}{2}} \left(m_{k+1} - m_k\right), i = 1, \ldots, N_r.$$

By Theorem 4, for sufficiently large N, the corresponding sums converge to integral transformations and

$$L_i = \mu l_i + W_{\frac{iT}{N_r}}, i = 1, \ldots, N_r$$

where W is a Wiener process. Thus, when the null hypothesis holds, the following regression model is a standard Gaussian regression:

$$L_{i+1} - L_i = \mu\left(l_{i+1} - l_i\right) + \varepsilon_i, i = 0, \ldots, N_r - 1$$

where ε_i is a sequence of independent normal variables.

Thus in order to confirm the null hypothesis, it is sufficient and necessary to confirm the hypothesis of normality of residuals in the above-mentioned regression model. For these purposes, we will use the Jarque–Bera test (see [6]-[7]). Note that as it was shown in the previous section, if the Jarque–Bera test shows that the residuals are normally distributed, then $Y(t)$ satisfies the analysed model. Thus, for a large number of observations the probabilities of Type I and Type II errors for our test coincide with the corresponding probabilities of the Jarque–Bera test for residuals. After conducting all necessary transformations, we had 20 observations of L_i and l_i and built the corresponding regression model. In order to apply the Jarque–Bera test, we calculated the value of the Jarque–Bera statistic for the number of observations $N_1 = 20$ and the number of regressors $k = 1$:

$$JB = \frac{19}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right)$$

where S and K are the coefficients of skewness and kurtosis of of the residuals of the regression model.

A Histogram of the residuals of the regression model is presented in the following figure.

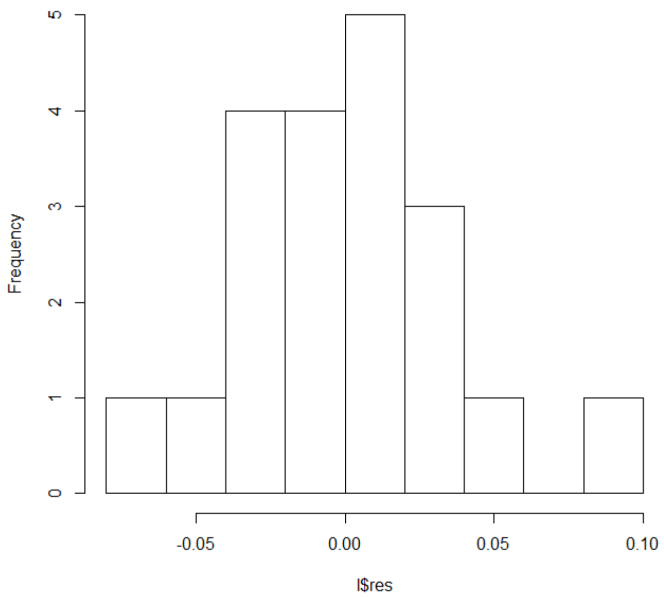

Figure 5. Histogram of the residuals in the analysed regression model

The calculated value of Jarque–Bera statistic is

$$JB = 1.434582.$$

Thus, for significance level $\alpha = 0.05$ the threshold value for the test is $Q^{\chi^2(2)}(1-0.05) = 5.991465$. The actual value of the statistic is less than the threshold value, and we accept the null hypothesis. Thus we can conclude that the proposed model can be used to describe the dynamics of assets on the Ukrainian Stock Exchange, at least in the short term perspective.

6. Conclusions

The problem of researching the Ukrainian stock market has recently become more important and urgent. Compared to the stock markets of the economically developed countries of Europe and America, where the corresponding mathematical apparatus has been developing since the middle of the twentieth century, the Ukrainian market is comparatively young. The purpose of this paper is to make the first steps towards conducting the necessary analysis.

In this paper we analysed the presence of jumps in the dynamic of assets on the stock market and statistically confirmed their existence. The results are important and can be used for a wide range of purposes. In particular, it is necessary to consider the effect of jumps during the calculation of the fair price of derivatives on the assets of the Ukrainian Stock Exchange. For the purposes of further analysis, jumps were excluded from consideration.

Also in this study we evaluated the Hurst coefficient for the logarithm of the price of the analysed assets by two different methods. In particular, we showed that the assets on the market might have long-term memory, but given a large number of random factors affecting the value of an asset at every moment, the estimated value of the Hurst coefficient appears to be close enough to $\frac{1}{2}$.

The problem considered in this paper is extremely relevant to modern Ukrainian realities, because a developed and open stock market is one of the necessary factors for the economic development of a country. The results of this work form a basis for further analysis.

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BIRŽOS KAINŲ GAUSO MODELIŲ PARAMETRŲ VERTINIMAS IR STATISTINIŲ HIPOTEZIŲ TIKRINIMAS

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Santrauka. Sudaromi Ukrainos vertybinių popierių rinkos aktyvų kainų modeliai ir, tikrinant atitinkamas statistines hipotezes realiems duomenims, tiriamas jų tinkamumas. Įvairių vertybinių popierių kainų dinamikoje tiriamas šuolių buvimas ir vertinamas vertybinių popierių kainos logaritmo Hursto indeksas dviem skirtingais metodais.

Reikšminiai žodžiai: Ukrainos vertybinių popierių rinka, Ukrainos vertybinių popierių birža, trupmeninis Brauno judesys, Hursto indekso vertinimas.