HARALD CRAMÉR – A GREAT STATISTICIAN

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Abstract. Harald Cramér is one of the great persons in the history of mathematical statistics and insurance mathematics in the twentieth century. He is perhaps the most well known Swedish statistician in modern time. The editor of "Lithuanian Journal of Statistics" has kindly asked me to write some notes on Harald Cramér, may be because my supervisor was one of Cramér's pupils. I am not the best person to do so, but I will do my best. At the end I will also give some references for those who want to read more about Cramér.

Keywords: pedagogic performance, insurance mathematics, mathematical methods of statistics, administrative capacities and social skills.

1. My first recollection

The first time I saw Harald Cramér, I was a young undergraduate. He was going to give a seminar at our department on level-crossings and extreme values of stationary processes. I was curious about the famous person. I had read some of his books and he had been supervisor of our professor Gunnar Blom, but otherwise I had no preconceptions. He turned out to be a well-groomed and well-dressed person who spoke very clearly and distinctly. At a quarter past one he started to write in the upper left hand corner of the blackboard. He kept the time schedule exactly. At three o´clock sharp when he said his concluding remarks he had filled the blackboard completely and just written the last formula in the lower right hand corner without ever having to rub something out or to add any balloons with extra text. All this had been done without any manuscript. It had been an interesting lecture, everything very clear and to the point, and I felt that there had been no unnecessary words. It was a really good and pedagogic performance. I have heard him many times since then, and my impression has always been the same.

2. Early days

Cramér was born September 25, 1893 and died October 5, 1985. When he first went to Stockholm University College in 1912, he did not intend to become a statistician or a probabilist at all. His main interests were chemistry and mathematics. For a short period he was a research assistant in biochemistry and wrote several scientific papers in that area (some of them with Hans von Euler, who later became a Nobel laureate). After that turned his interest to mathematics. His PhD thesis 1917 [5] dealt with Dirichlet series but he was also interested in other parts of mathematics. For instance he wrote several papers on the distribution prime numbers.

3. Institute of insurance mathematics

In 1919 Cramér got interested in insurance mathematics. During the late 19th century many insurance companies went bankrupt since they had too small reserves for the bad years having paid large dividends during the good ones. Laws to prevent this had been passed but there was no good theoretic basis. Cramér realised not only that insurance needed a firm basis of probability but also that probability needed a firm basis of mathematics. In the twenties he and the insurance companies made the government finance an institute on that topic in Stockholm together with a chair in insurance mathematics to which he was later appointed. Later he also made the insurance companies finance a building for its activities. The institute soon received a good reputation and Cramér could recruit many good students from all over the country. As an example, I may mention that an uncle of mine, Ronald Thorburn, who had first studied

mathematics in Lund, was tempted by the institute and he moved there to complete his studies. My uncle later became the responsible actuary at one the Swedish insurance companies. As I said earlier, Cramér was a very good speaker. His lectures at the institute were not only visited by his group of students but also by many outsiders from other subjects, the insurance companies and elsewhere. Many people came just to enjoy his talks. Cramér was also a sociable person. He invited many great persons to visit the institute. After the seminars the discussions often continued in a more relaxed and pleasant atmosphere at his home.

Cramér had many famous people and pupils at the institute; Herman Wold, Ulf Grenander, Bertil Matérn and Gunnar Blom graduated there, just to mention four of his ten PhDs. But many persons from all over the world want to claim that they are his inheritors ([11]). During that time the influence of the supervisor was not what it is today. His role was to create a good atmosphere, invite nice guests and perhaps to approve the topic. Gunnar Blom told me once that Cramér looked at his thesis only once, when it was almost finished, and said that he was allowed to put it forward. If the student wanted to discuss his thesis during the work, he had to talk to his fellow students or other persons.

In the thirties a brown cloud spread over Germany and later Europe. William Feller came as a refugee from Germany to Cramér as a guest of the institute and department. During the six years he was in Stockholm he contributed to the free and vivid academic discussions. In August 1939 he preferred to move to a safer country farther away from Germany. He took one of the last boats to the USA. Feller and his German university traditions may have contributed to Cramér's strong feeling for academic freedom.

4. Cramér's work in mathematical statistics

Even though Cramér made many important original works of his own both in insurance mathematics and mathematical statistics, I think that his largest contribution was his capacity to highlight other people's important results and to popularise and present them in an elegant and digestible, but still rigorous, way. One example of this is an understandable and comprehensible description of Filip Lundberg's work on collective risk theory [6]. Lundberg´s original work was very difficult to penetrate. This classical model is nowadays referred to as the Cramér-Lundberg model (or the Compound Poisson Risk Model). They assumed that the premiums arrive at a constant rate but the claims are of random sizes according to a Poisson process. They computed the probability that the company eventually will be ruined, i.e. the reserves become negative at some future time. Collective risk theory became one of the cornerstones of the research at the institute together with many recent ideas. One might say that Cramér was one of the very first persons to develop methods in stochastic finance. He also was quite early to appreciate and discover new results like Kolmogorov's work on probability and Khinchin's work on random processes. He made his students read their papers and use their results in their own work.

In 1936 Cramér and his student Herman Wold published a paper ([10]) containing the device, which bears their names. The Cramér-Wold device says that a sequence of multivariate random variables converges in distribution if and only if all one-dimensional linear combinations converge in distribution. It was a powerful tool and they used it when proving the continuity theorem and the Central Limit Theorem.

Sweden had the good fortune to be able to stay out of World War II. Cramér could work on his own and he could complete his book "Mathematical methods of Statistics" during that time, while most other statisticians were inhibited by the war. The book did not only contain some interesting new results but he made a rigorous and coherent theory out of all the scattered statistical results that were around at that time. The book was published in 1945 by a Swedish book company [7]. One year afterwards Princeton took over the responsibility for the book. It was systematic, well written and practically free from errors and even typos. Within eight years it had been translated into both Russian and Spanish and later to even more languages. When I was a PhD student in the seventies it was a standard reference book for us and it is still a very readable book. We referred to it as "Stora (great) Cramér". There was also an elementary introductory book in Swedish on probability theory which we called "Lilla (little) Cramér" ([8]), which we had used as undergraduates. One may say that this book was a predecessor to all modern elementary text books on mathematical probability.

Mathematical methods of Statistics had a profound influence on the future of statistics. It showed in an elegant way that probability theory is needed to describe statistics theoretically and Cramér also showed that this should be done in rigid mathematical terms. I think that today he would have been amazed by the possibilities to use computers in statistics and to handle large data sets, But he would also have been sorry to see the mass of articles who, instead of

treating statistical results and procedures mathematically, motivate the suggested methods with only simulations and other ad hoc arguments.

In the thirties the statistical society had started to realise the difference between parameters, random variables and observations and also of the importance of studying the distribution of the estimators as random variables conditional on the parameters. Neyman´s work is an example. Cramér also made many contributions to this. One of his best known results is the Information inequality, which is usually referred to as the Cramér-Rao inequality. It says that the variance of an unbiased estimator can never be less than the inverse of the Fisher information (the mean of the second derivative of the density evaluated at the true parameter value). He was not the only one who proved this result; apart from Rao, also Frechét and Darmois and others have been credited with it, but Cramér´s version was the one that first received international attention.

5. Administrative duties and later work

Cramér is best known as a theoretical statistician and not as someone working with real applications. This does not mean that he was not interested in applications. Much of his early work on insurance mathematics had its origin in real life. He encouraged many of his students to work with applications e.g. Herman Wold in econometrics, Ove Lundberg in insurance and Bertil Matern in forestry. He also had plans to add a section on applied statistics to the institute. But he got other duties and the idea was later carried out by a group of his students. When I read some of his work on insurance mathematics and other topics, I have the feeling that he is thinking on the reality that he is modeling. This is reality transformed into to a model that is easy to grasp and possible to handle mathematically. He did not end his treatments with the mathematical model. There should always be an inference part, i.e. how to draw conclusions from data. In this sense even the book Mathematical methods deals with applied statistics. He was also very early in talking about inference for stochastic processes.

Cramér was also respected for his administrative capacities and for his social skills. As I mentioned above he persuaded the insurance companies to support an institute in insurance mathematics and finance a building for it. In 1950 he was made "Rector Magnificus" (President) of Stockholm University. Even though this at that time was a more honorary position than today, it meant a lot of administrative and representative work. At that time Stockholm was a municipally-owned university college and Cramér made efforts to transform it into a full state-owned university. In 1958 his period ended and about one year later his efforts bore fruit and Stockholm finally became a full university. The expanding college was spread all over the city and a continuing problem was to find more buildings in the neighbourhood. After eight years as rector he became the chancellor of all Swedish Universities. During that period he fought for academic freedom and for independence of science against the Government. After three years he handed in his resignation to the government as a protest against their policy in this respect.

When he had resigned he was 68 years old. He then went to Research Triangle Institute at the invitation of Gertrude Cox and started a new scientific carrier. He came in contact with a young research assistant, Ross Leadbetter. Together they explored the properties of stationary processes, like differentiability, level crossings and extreme values. At the age of 72 he and Leadbetter ([9]) collected their results in a book. As graduate students in Lund we read and enjoyed the book. Two of my fellow students, Georg Lindgren and Holger Rootzén, became so interested that they have continued Cramér´s research in this area (see e.g. [14]).

6. Final words

Cramér was a prominent member of the Swedish statistical society. When I became its chairman in 1981, we were worried that a number of members had not paid their fees even after several reminders. We decided to expel all members who had not paid their fees for the last two years. Among them were Cramér. But in his case we decided to make an exception. We made him a permanent honorary member instead. The moral is that if you do not pay your fees you may become an honorary member.

During his old age Cramér was almost deaf. The last talks that I heard from him, were historical expositions. During those he was always accompanied by his son, Thomas Cramér. After the talks the audience were allowed one or two questions, which his son tried to communicate to him. Once Cramér had understood the questions his answers were

even then very clear and well organised. But sometimes he misunderstood the question and gave a very clear, lucid and informative answer to something else.

There exist many good biographies of Cramér. Those interested to learn more may read the excellent article by Cramér´s student Gunnar Blom ([2]) or volume 1994:1 of the Scandinavian Actuarial Journal, which was dedicated to the Harald Cramér Symposium. The volume contains several articles about Cramer and his work e.g. by Grenander ([12]) and Leadbetter ([13]). Bohman ([4]) and Lundberg ([15]) are obituaries concentrating on his work in actuarial sciences. A complete bibliography of Cramér´s works may be found in Blom and Matern ([3]) containing 114 international works and 38 works in Swedish.

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HARALDAS KRAMERIS – DIDIS STATISTIKAS

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Santrauka. Haraldas Krameris yra vienas žymiausių asmenų dvidešimtojo amžiaus matematinės statistikos ir draudimo matematikos istorijoje. Jis yra tikriausiai žinomiausias naujųjų laikų Švedijos statistikas. „Lietuvos statistikos darbų" redaktorius manęs maloniai paprašė parašyti keletą minčių apie Haraldą Kramerį, galbūt todėl, kad mano vadovas buvo vienas iš Kramerio mokinių. Nors ir nesu tinkamiausias asmuo šiam darbui, aš darau jį kaip galiu geriau. Tiems, kurie norėtų paskaityti apie Kramerį plačiau, pabaigoje pateikiu keletą literatūros šaltinių nuorodų.

Reikšminiai žodžiai: pedagogo vaidmuo, draudimo matematika, matematiniai statistikos metodai, administraciniai gebėjimai ir socialiniai įgūdžiai.